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Lot sizing and scheduling in the molded pulp packaging industry

Planejamento da produção na indústria de embalagens de polpa moldada

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Abstract: This paper addresses the problem of production planning and scheduling in the pulp molded packaging industry, considering especially a Brazilian plant that produces molded packs for eggs and fruits. The production process involves utilizing some molding patterns through which the different products are produced. Thus, decisions related to the production planning and scheduling involve the choice of which molding pattern will be used, how long they will be used on each production line, and how they should be sequenced. For representing this problem, a mathematical model based on the General Lot Sizing and Scheduling Problem (GLSP) with sequence-dependent setup times and costs was proposed. Results show that the production plans obtained by the model is advantageous compared with the company plan, because it involves lower capacity consumption, lower total setup time, and better inventory control, besides reducing the total cost of the proposed plan in approximately 36%.

Keywords: Production planning and scheduling; Molding patterns programming; Lot sizing; Molded pulp packaging industry.

Resumo: *Este trabalho aborda o problema de planejamento e programação da produção na indústria de embalagens de polpa moldada, particularmente o sistema de produção de uma fábrica de embalagens para acondicionamento de ovos e frutas. O processo de produção envolve a utilização de padrões de moldagem, através dos quais são produzidos os diferentes produtos demandados. Desta forma, as decisões no planejamento da produção envolvem a escolha dos padrões de moldagem a serem utilizados, o tempo de produção de cada um deles em cada linha de produção, e a forma como devem ser sequenciados. Para representar o problema, foi proposto um modelo matemático baseado no Problema de Dimensionamento e Sequenciamento de Lotes Geral (GLSP), com tempos e custos de preparação dependentes da sequência. Os resultados do modelo sugerem planos de produção significativamente melhores que os planos de produção da fábrica em estudo, sendo que, em todos os experimentos realizados com dados reais, a demanda é atendida com menor consumo de capacidade, menor tempo total dedicado às operações de setup e melhor controle nos níveis de estoque, além de uma redução de aproximadamente 36% dos custos totais envolvidos.*

Palavras-chave: *Planejamento e programação da produção; Programação de padrões de moldagem; Dimensionamento de lotes; Indústria de embalagens de polpa moldada.*

1 Introduction

Packages solutions have an important role in the industrial sector, since they protect and preserve products and food over shipping and handling activities. In Brazil, the packaging sector has registered high growth in last years. According to the Brazilian Packaging Association (ABRE, 2014), the net incomes of this sector in 2013 achieved R$51.8 billion, exceeding the R$46.7 billion in 2012. For 2014, forecasts are positive since a volume of production 1.5% greater than the previous year is expected.

The production planning and scheduling in the molded pulp packaging industry involves important concerns related to the production systems where these products are manufactured, as well as economic and environmental aspects. Production environment concerns imply in dealing with a high and heterogeneous demand, high setup costs and

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times. Regarding the economic aspect, packaging production has to deal with low sale prices and high operational costs, which requires an effective high scale production system capable of keeping this activity economically feasible. This economic concern reinforces a need for planning the sources of the system efficiently. As to environmental issues, since packages are quickly discarded and may generate large quantities of waste, they must be designed by aiming at an easy disposal and an efficient use of natural sources (Pereira & Silva, 2010).

This study approaches a production environment that produces molded pulp packages for fruit and eggs. The decision making process is described based on a case study in a Brazilian molded pulp plant located in the São Paulo state. This problem comprises lot sizing and scheduling decisions in parallel machines, which can use different molding patterns that must be scheduled in order to meet the demand without backlogging. The objective is to find production schedules by minimizing total setup costs, which are sequence-dependent, total inventory holding costs, and penalties associated to deviations from minimum and maximum target inventory levels.

This paper is organized as follows: the next section presents a brief literature review about lot sizing and scheduling applications in different industrial settings. Section 3 describes the production process and planning decisions in molded pulp packaging industry, particularly the production process considered in the case study. Section 4 presents some assumptions of the modeling approach. Section 5 presents some computational experiments and results, as well as some comparisons with a real schedule. Finally, concluding remarks and future research directions are provided.

2 Literature review

The lot sizing and scheduling problem answers the questions about what, when, and how much to produce of each product, it defines the use of the production sources, and determines inventory levels by minimizing total costs (Karimi et al., 2003; Drexl & Kimms, 1997). Several classical formulations and different approximated solution methods have been used to represent and solve lot sizing and scheduling problems in industrial applications. Some of them can be found in the tobacco industry (Pattloch et al., 2001), textile (Silva & Magalhaes, 2006), yogurt (Marinelli et al., 2007), soft drinks (Toledo et al., 2007, 2009, 2011; Ferreira et al., 2008, 2009), electrofused-grains (Luche & Morabito, 2005; Luche et al., 2009), animal feed (Toso & Morabito, 2005; Toso et al., 2008, 2009; Clark et al., 2010; Augusto et al., 2014), foundry (Araujo et al., 2004, 2007; Luche & Morabito, 2005), glass industry (Almada-Lobo et al., 2008), among other industrial settings. These applications report some adaptations and extensions of the classical formulations in the literature, so that characteristics and decisions of real systems are included in their mathematical models. Results show that solution obtained by these approaches are advantageous when compared with real solutions.

Pattloch et al. (2001) researched the production planning in a tobacco company whose production environment comprises identical parallel machines and multiple products. The authors propose a mathematical formulation to minimize the total setup costs.

Silva & Magalhaes (2006) studied the lot sizing and scheduling in a company which produces acrylic fibers to the textile industry. Their paper considers a parallel machine production system and a specificity related to setup operations, since it is possible to incur in setup costs for changeovers between two lots of the same product. It must be taken into account because the need of replacing wear tools.

Marinelli et al. (2007) studied the lot sizing and scheduling problem in a yogurt company. They proposed mathematical models based on the Capacitated Lot sizing Problem (CLSP) and the Continuous Setup Lot Sizing problem (CSLP) to represent the storing and processing steps.

In the soft drinks industry, Ferreira et al. (2012) analyzed the lot sizing and scheduling problem for a two-stages production process. The first stage was related to the syrup production and the second stage was the bottling process. The authors proposed four single-stage formulations to approach this problem and the synchrony between the two stages, based on the classical GLSP and ATSP (Asymmetric Travelling Salesman Problem) formulations. Results showed than single-stage formulations have a better performance than the two-stages formulations proposed in Ferreira, Morabito and Rangel 2009. Heuristics and metaheuristics methods were also proposed to solve the lot sizing problem in this industry, such as Mixed Integer Programming (MIP) heuristics (Ferreira et al., 2008), multiple-populational genetic algorithms (Toledo et al., 2009) and a Tabu Search algorithm (Toledo et al., 2011).

In this context, this study presents an unexplored application of the lot sizing and scheduling problem which takes place in molded pulp packaging industry. In this production system, the production volumes of packages are determined by the use of molding patterns, so that planning decisions are related to the lot sizing and scheduling of these molding patterns. In the next section we describe the production process and the use of the molding patterns to produce different types of packages.

3 Characteristics of the molded pulp packaging production system

The whole production process of molded pulp packages can be divided into two different processes: the molding process and the printing or customizing process. The molding process comprises such steps as blending, molding, drying, and pressing. The customizing process includes printing, sorting, and packing. Figure 1 presents all the steps of the production process. Even though both the molding and customizing processes comprise several steps, each one can be considered as a single stage process, since there is a continuous flow without intermediate inventory between each step. This study focuses on the molding process because it is considered as the bottleneck of the production systems and its production planning decisions are more challenging.

The first step consists in making the pulp by blending the raw materials in a specific equipment named Hydrapulper. The raw materials include many types of post-consumed papers, which are blended in hot water and other chemicals until the desired humity and color conditions are achieved. Next, the pulp goes through a set of vibrating sieves that remove its impurities, such as plastic and metal dross. Finally, the pulp goes to a storage tank that supplies the molding step.

The molding step is considered the most important one in the process since it is where products are formed. In this step, the pulp is formed by a rotary machine that molds the pulp by a dinamic pressing and vacuum system. The molds used to form the products are attached to the molding machine and this combination of molds is called " molding pattern". A molding pattern may include several types of molds at the same time, so several products may be produced simultaneously. Some patterns may produce the same mix of products but at differrent production rates. As an example, consider two molding patterns which simultaneously produce packages for six (Product A) and twelve eggs (Product B). The first one produces 100,000 units of A and 200,000 units of B per hour, meanwhile the other one produces 150,000 of each product per hour. If we use each molding pattern for one hour, different volumes of products A and B will be obtained. Thus, although these patterns produce the same products (A and B), they are deemed different because the amount of products is also different after the same production time.

The drying step takes place after the molding. The material goes through an industrial oven, when it is heated by temperatures between 180 °C and

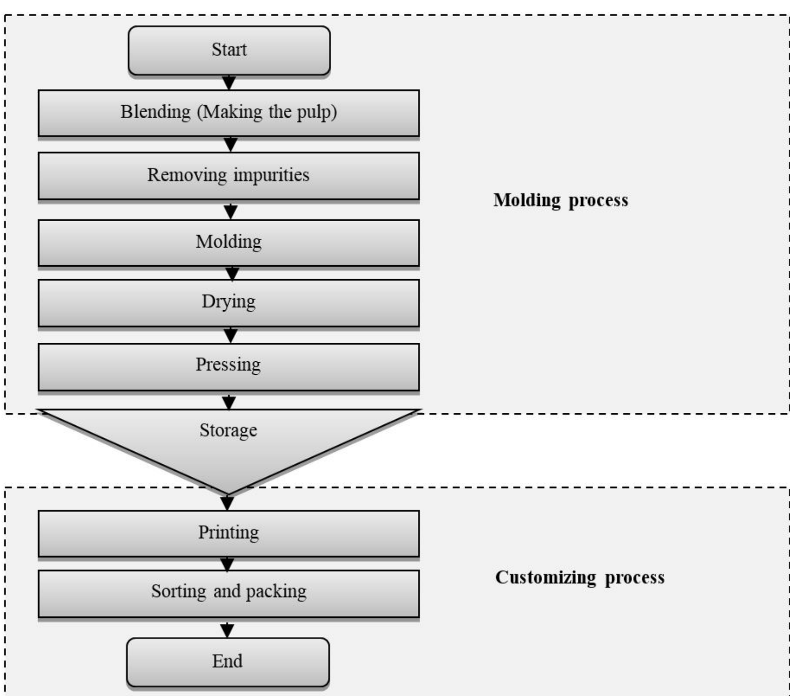

Figure 1. Flowchart of the production process in molded pulp packages industry.

240 °C for 10-15 minutes. After that, packages are pressed in order to provide better resistance and finishing. This is the last step of the molding process, after which products are stored and ready for the customizing process.

The customizing process starts with the printing step, where packages are printed according to specific layouts provided by each customer. Next, packages are sorted and arranged in pallets to be shipped to the final customers.

The production planning in both the molding and customizing processes is made separately. In the molding process it is made based on demand forescast provided by the comercial department for a specific planning horizon. Production planning in the customizing process, however, is made based on direct customers' orders, who also specify due dates.

As mentioned before, this study focuses on the planning decisions in the molding process. This production environment usually comprises several parallel machines, to which all the possible molding patterns can be attached. The main decisions include defining which molding pattern to attach to each molding machine, how long each molding pattern should be used and what sequence they should follow. Besides that, minimum and maximum inventory target levels must be considered, since there are penalties costs associated to the deviation from these levels.

As several products can be produced simultaneously by different production rates, the inventory levels must be carefully managed in order to avoid producing high levels of low demand products and backlogs of high demand products. To avoid solutions of this type, minimum and maximum inventory target levels are defined according to the demand of each product and penalties are defined for eventual deviations from these levels.

Changeovers between molding patterns imply sequence-dependent setup times and costs. In this industry, setup times are triangular and can take from 30 minutes up to 48 hours. Besides that, setup operations also require specialized labour which increase significantly the setup costs. In this way, all decisions about lot sizing and sequencing molding patterns must be made in order to minimize the total setup and inventory holding costs, as well as penalties associated to deviation from the inventory target levels.

4 Mathematical model

The mathematical approach proposed to represent the studied problem is based on the classical GLSP formulation proposed by Meyr (2002) and Ferreira et al. (2012). It considers a set of parallel machines which have the same technical characteristics, speed, and setup times. It is assumed that each machine can use any of the possible molding patterns at any time.

Each time period represents a week of the planning horizon so that each one is divided into several sub-periods of variable sizes. Only one molding pattern can be set up and used in each sub-period. Parameters such as total capacity and the production rate of each molding pattern are provided in hours. Setup times and costs are triangular and sequence-depedent. The setup state is preserved between time periods, so it is considered as setup carry-over.

Indices

- k Products
- i, j Molding patterns
- l Machines
- t Time periods
- s Sub-periods

Parameters

- N Number of all possible molding patterns
- K Number of products
- L Total machines
- T Total time periods over the planning horizon
- S Total sub-periods over the planning horizon
- S_t Total sub-periods in period t.
- I_{k0} Initial inventory level of product k
- $I_{k(\min)}$ Minimum inventory target level of product k
- $I_{k(\max)}$ Maximum inventory target level of product k
- h_k Unit inventory holding cost of product k
- α_k Unit penalty for the amount of inventory greater than the maximum target level for product k
- β_k Unit penalty for the amount of inventory levels less than the minimum target level for product k
- d_{kt} Demand of product k in period t
- Q_{lt} Capacity of machine l in period t (hours)
- p_{ki} Units of product k obtained by molding pattern i (units/hour).
- st_{ij} Setup time for changeovers from molding pattern i to molding pattern j

c_{ij} Setup costs for changeovers from molding pattern i to molding pattern j

M_{lit} Upper bound for variable x_{lis}.

Variables

x_{lis} Production time of molding pattern i in machine l, in sub-period s (hours)

y_{lis} 1, if machine l is set up to molding pattern i at the beginning of sub-period s; 0, otherwise

z_{lijs} 1, if there is a changeover from molding pattern i to j in machine l, in sub-period s; 0, otherwise

I_{kt} Inventory of product k at the end of period t

E^+_{kt} Amount of inventory of product k greater than the maximum inventory target level.

E^-_{kt} Amount of inventory of product k less than the minimum inventory target level

$$\text{Minimize} \sum_{t=1}^{T}\sum_{k=1}^{K} h_k I_{kt} + \sum_{l=1}^{L}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=1}^{S} c_{ij} z_{lijs} + \sum_{t=1}^{T}\sum_{k=1}^{K} (\alpha_k E^+_{kt} + \beta_k E^-_{kt}) \quad (1)$$

Subject to

$$I_k = I_{k(t-1)} + \sum_{l=1}^{L}\sum_{i=1}^{N}\sum_{s\in S_t} p_{ki} x_{lis} - d_{kt} \quad \forall\ t=1,...,T;\ k=1,...,K \quad (2)$$

$$\sum_{i=1}^{N}\sum_{s\in S_t} x_{lis} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s\in S_t} st_{ij} z_{lijs} \le Q_{lt} \quad \forall\ t=1,...,T;\ l=1,...,L \quad (3)$$

$$x_{lis} \le M_{lit} y_{lis} \quad \forall l=1,...,L;\ i=1,...,N;\ t=1,...,T; s\in S_t \quad (4)$$

$$\sum_{i=1}^{N} y_{lis} = 1 \quad \forall\ l=1,...,L;\ s=1,...,S \quad (5)$$

$$\sum_{i=1}^{N} z_{lijs} \le y_{ljs} \quad \forall l=1,...,L;\ j=1,...,N;\ s=1,...,S \quad (6)$$

$$y_{li(s-1)} = \sum_{j=1}^{N} z_{lijs} \quad \forall l=1,...,L;\ i=1,...,N;\ s=1,...,S \quad (7)$$

$$\sum_{j=1}^{N} z_{ljis} = y_{lis} \quad \forall l=1,...,L;\ i=1,...,N;\ s=1,...,S \quad (8)$$

$$I_{kt} + E^-_{kt} - E^+_{kt} \ge I_{k(\min)} \quad \forall k=1,...,K;\ t=1,...,T \quad (9)$$

$$I_{kt} + E^-_{kt} - E^+_{kt} \le I_{k(\max)} \quad \forall k=1,...,K;\ t=1,...,T \quad (10)$$

$$I_{kt}, x_{lis}, E^+_{kt}, E^-_{kt} \ge 0; y_{lis}, z_{lijs} \in \{0,1\} \quad \forall\ i,j=1,....,N;\ l=1,..,L;\ k=1,...,K;\ s=1,...,S;\ t=1,...,T \quad (11)$$

The objective function (1) minimizes the total costs involved, which comprise the inventory holding costs, setups costs, as well as penalties associated to deviations from the inventory target levels.

Constraints (2) are the demand balance constraints, which relate the quantities produced the inventory levels and the demand. Differently from the other classical formulations for lot sizing and scheduling problems, the lots of products are obtained by parts, each part by using a specific molding pattern. Thus, there is not just one lot of products in each period, but there is only one lot of process represented by the production time of each molding pattern.

Constraints (3) are about the capacity consumption in each time period. Note that capacity is consumed by production times of each molding pattern and setup times.

Constraints (4) guarantee that a molding pattern is used if, and only if, the machine is set up to this in sub-period s. As backlogs are not allowed, if molding pattern i is used, the maximum production time in each period is at most M_{lit}, which may be approximated to equation (12).

$$M_{lit} = \min\left\{ Q_{lt}, \max_{k:p_{ki}\neq 0} \frac{\sum_{h=t}^{T} d_{kh}}{p_{ki}} \right\} \quad \forall\ l=1,...,L;\ i=1,...,N;\ t=1,...,T \quad (12)$$

Equations (12) determine an upper bound for the production time of each molding pattern in each period. In this way, each molding pattern may use at most the total capacity of the period, or the time required to meet the remaining demand of the products that are produced by this pattern. As an example, it considers that the pattern i produces 20,000 units/hour of product A and 30,000 units/hour of product B, simultaneously. The capacity of machine l in period t is 168 hours and the remaining demand is 500,000 units of product A and 1,200,000 units of product B. Thus, the upper bound $M_{lit} = \min\left\{168,\ \max\left\{\frac{500,000}{20,000}; \frac{1,200,000}{30,000}\right\}\right\}$. It means that this molding pattern will be used at most for 40 hours in the production line l in period t.

Constraints (5) guarantee that each machine is set up to only one molding pattern in each sub-period. Constraints (6), (7) and (8) relate the setup states between two consecutives sub-periods, so that they determine changeovers and preserve the setup state.

Inequalities (9) and (10) determine how many units of each product are out of the inventory target levels, in each period. Note that the inventory quantities are defined by variable I_{kt}; however, variables E^{+}_{kt} indicate the amount of inventory greater than the maximum inventory target level and E^{-}_{kt} indicate the deviation related to the minimum inventory target level.

Finally, (11) defines the type of variable of the problem. Note that the production time of each molding pattern is a positive variable. Although this does not guarantee that the amount of products are integer quantities, this approximation is acceptable in this production environment, since production volumes are large and minimum inventory levels are defined.

5 Computational experiments

All computational tests to validate and analyze the model results were implemented in GAMS (*General Algebraic Modeling System*) version 22.6 and solved by CPLEX 11.0, on a computer Intel Core i7-26000, 3.40 GHz and 16 GB RAM memory. The experiments comprise several instances based on real data provided by a Brazilian plant. A single instance was used to compare production plans provided by solving the model and by the production planner of the company. Other computational tests were executed for 12 real instances and other random instances in order to analyze the performance of the model.

Some detailed information for one particular instance was collected to compare the model solutions and the planner's solution. Besides the parameters of the problem, for this instance the initial setup states of the machines and the maintenance activities scheduled over the planning horizon were considered. The number of sub-periods was defined based on the maximum number of changeovers defined by the company, ideally up to 4 per period. The limit elapsed time to solve the model is 3 hours.

The results provided by the GLSP model for this instance define a schedule plan which specifies the molding patterns to be used over the planning horizon and how long each one is used. Table 1 presents the total capacity consumption and the total setup times of the solution of GLSP model and the company's schedule. Table 2 presents some information about the amount of products produced over the planning horizon, the total inventory at the end of the planning horizon and the deviations from the inventory target levels.

Note that the main advantages of the schedule provided by solving the proposed model are related to the capacity consumption, reductions of the total setup times and control of the inventory levels. The GLSP model provided a solution that uses less capacity (approximately 8% less), at the same time reducing approximately 9 hours of the total setup times in all the three machines. Although this reduction is not much over the one-month planning horizon, it has a better effect on the total setup costs, since setup costs in this company are quite high.

Note that the model solution meets all the demand without backlogs. However, the company solution does not meet the demand of 811,058 units at the end of the planning horizon. This evidences how hard it is to choose and schedule molding patterns so that demand requirements are fully met. In general, the model solution produced a lower quantity of products and inventory levels at the end of the planning horizon. It suggests that the model solution manages inventory and production levels in a better way than the company schedule, so that in this solution the inventory levels are closer to the target levels. Note that in the company schedule 45.54% of the total inventory is out of the target levels at the end

Table 1. Capacity consumption and total setup times in the production plans provided by the company and the GLSP model.

Schedules	Capacity consumption (Total setup time)			
	Machine 1	Machine 2	Machine 3	Total in the production system
Company	100% (5.5h)	100% (20h)	100% (10.5h)	100% (36h)
GLSP model	97.34% (5.5h)	94.91% (2h)	85.88% (11.5h)	92.47% (27h)

Table 2. Comparisons between the company production plan and the model solution.

Elements of the schedule	Company	GLSP model
Total production volume (units)	16,157,858	14,952,093
Inventory at the end of the planning horizon (units)	4,579,267	2,562,202
Backlogs at the end of the planning horizon (units)	811,058	0
Units above the maximum inventory target level	1,521,469	575,611
Units below the minimum inventory target levels	472,375	75,850

of the planning horizon. Meanwhile in the schedule provided by solving the model it is only 25.42%.

Some of the advantages of the model solution related to the costs involved are illustrated in Figure 2, which presents the total costs for each schedule.

Note that the schedule provided by the model's solution incurred in approximately 35.96% less than the total costs of the company's schedule. The main costs in the production process are the setup and the inventory holding costs, since they represent more than 70% of the total in both the schedules. Note that the setup costs in the model's solution is approximately 27% less than the setup costs in the company's schedule. In the same way inventory holding costs are also reduced about 33.1%.

Setup and inventory holding costs are lower in the model's schedule, and penalties related to the inventory target levels were reduced as well. That was expected since the volumes of production and inventory levels are lower in the model's solution than in the company's schedule.

To analyze the performance of the GLSP model, a set of 12 instances was created based on real information provided by the company. These instances represent real data related to the number of products, possible molding patterns, production rates of each molding pattern, inventory target levels, setup times and costs, and other parameters, as presented in Table 3.

The 12 instances represent demand requirements for 12 different months of 4 weeks each. The production environment comprises 3 machines available 7 days per week, 24 hours per day. The number of sub-periods in each period was defined based on the maximum number of changeovers desired by the planners, so that each period has 4 sub-periods. Each instance comprises information about demand and initial inventory level for 14 products, which can be obtained by 19 different molding patterns, as Table 4 shows. More details about input data can be found in Martínez (2013).

Table 5 presents the results of the model after 3 hours. This table shows the incumbent solution after the limit time, the best upper bound, the gap provided by CPLEX, total elapsed time and the approximated time that the incumbent solution was found. As a general remark, the model was able to find feasible solutions for all the instances. However, its performance is not uniform for all of them, so that in some cases optimal solutions are found and in others the solutions have a high gap.

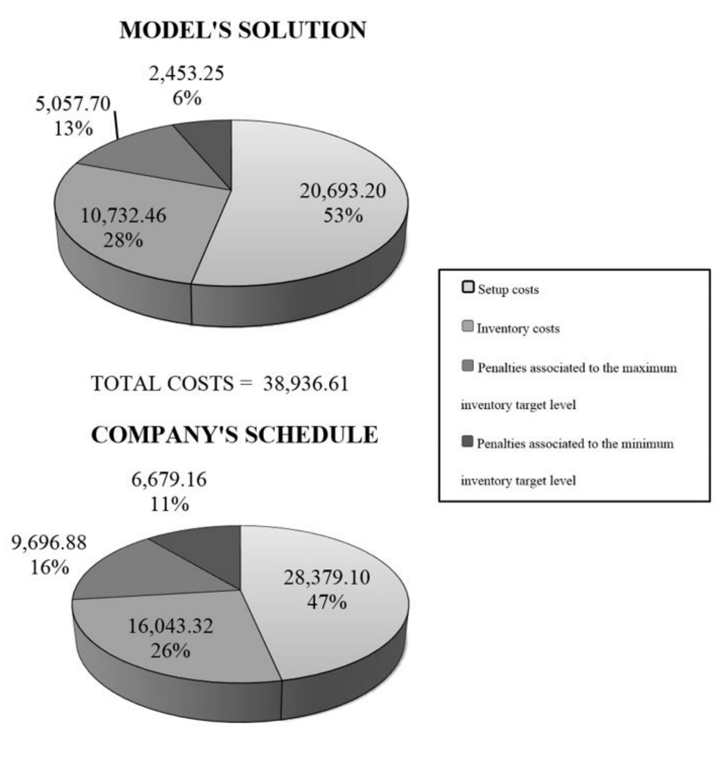

Figure 2. Total costs in the company's schedule and the model's solution.

Table 3. Production rates of the molding patterns (units per hour).

Products	Molding patterns																		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
1	8,791	0	0	0	0	0	0	0	0	0	7,326	5,862	0	0	7,326	5,862	0	0	4,395
2	0	8,791	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	8,791	0	0	0	0	0	4,395	0	0	0	0	0	0	0	0	0	0
4	0	0	0	8,791	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	8,791	0	0	0	0	7,326	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	8,791	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	8,791	4,395	0	0	0	0	7,326	5,862	0	0	7,326	5.862	0
8	0	0	0	0	0	0	0	3,295	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	3,295	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	1,465	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	1,465	2,929	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	1,465	2.929	0	0	0	0	0	0	1,465
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,465	2,929	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1,465	2,929	0	0	2,929

Note than 50% of the instances were solved optimally in elapsed times which vary from 1 minute to 1 and a half hours. Among the instances which were not solved optimally, some solutions have a gap less than 6.5%. However, instances 1 and 10 were not easy to solve, since the incumbent solutions have gaps of 64.45% and 24.2%, respectively. Note in the last column of Table 5 that, on average, the model gets the best solution in the first minutes for most of the instances. However, sometimes it spends a lot of time improving lower bounds.

To make a better analysis of the model performance, some random instances were also tested. These data was generated based on Haase (1996) and Fleischmann & Meyr (1997) and comprise 4 different size groups of 10 instances. The characteristics of these random instances are presented in Table 6 and the results in Table 7.

Note that these instances are smaller in size than the real instances provided by the studied company. Results show than the proposed model is able to provide feasible solutions for all the instances. However, optimal solutions only were found in the smaller size groups 1 and 2, which comprise only 2 machines, 6 molding patterns, 5 products, and 5 time periods. In the cases with more possible molding patterns, the performance of the model gets worse, since most of the instances have high gaps, which can exceed 50%.

It is worth mentioning that in the random instances the number of sub-periods for each period was determined as the number of possible molding patterns, i.e. $S = N*T$. Considering that in the real instances the number of sub-periods in each period was smaller, results suggest that the GLSP model has a better performance in those real instances, where the number of sub-periods are defined based on the expertise of the company's planners. Thus, it evidences that the model performance depends, among other parameters, on the number of sub-periods which is

Table 4. Initial inventory levels for the 12 instances.

Products	Inst. 1	Inst. 2	Inst. 3	Inst. 4	Inst. 5	Inst. 6	Inst. 7	Inst. 8	Inst. 9	Inst. 10	Inst.11	Inst. 12
1	288,543	530,620	527,890	487,323	500,242	346,987	456,231	345,280	657,987	768,945	921,865	698,627
2	144,010	1,498,168	1,143,836	1,484,517	1,605,017	1,629,929	747,670	345,238	105,768	42,862	183,053	59,064
3	126,782	1,310,897	1,000,847	1,944,050	759,292	1,426,188	29,701	302,083	350,587	295,544	211,700	631,584
4	36,052	374,542	285,956	186,815	216,941	38,855	353,584	252,976	82,521	66,794	339,794	447,509
5	360,569	213,356	1,151,031	159,621	460,872	523,153	458,681	1,236,914	269,614	112,348	350,025	717,061
6	54,655	561,813	428,935	833,164	325,411	334,753	806,847	655,935	123,781	100,191	31,243	488,508
7	198,627	117,346	633,067	87,792	1,267,205	287,734	56,000	484,028	357,503	271,007	54,495	309,662
8	36,766	23,560	12,489	68,978	53,567	39,878	23,458	89,712	23,678	18,542	177,850	38,380
9	20,577	15,679	15,679	296,878	29,678	296,878	12,879	12,879	10,076	10,076	10,173	10,173
10	45,349	40,567	35,321	56,789	49,987	42,234	78,654	67,543	47,851	37,823	31,005	163,724
11	0	0	0	0	0	0	0	0	0	0	0	0
12	97,856	196,754	249,445	316,246	250,787	319,978	352,397	263,872	325,925	314,602	179,125	48,305
13	0	0	0	0	0	0	0	0	0	0	0	0
14	188,724	327,924	415,742	527,077	417,978	533,296	587,329	439,787	543,209	524,337	491,431	88,137

Table 5. Performance of the GLSP model.

Instance	Incumbent/ optimal solution	Lower bound	*Gap*	Elapsed time (s)	Time to find the incumbent/ optimal solution (s)
1	71,568.38	26,872.47	62.45%	10,801.2	6,971.4
2	22,562.54	22,562.54	0%	118.7	90
3	19,325.65	19,325.66	0%	5,856.9	3,858
4	38,031.73	38,031.73	0%	433.5	301.2
5	23,882.86	23,882.86	0%	2,277.3	2,250
6	36,290.37	35,485.97	2.27%	10,801.5	4,980
7	35,718.47	35,445.11	0.77%	10,802.2	3,610.8
8	20,432.04	20,432.04	0%	1,591	390
9	31,554.20	29,783.99	5.61%	10,800.9	3,780
10	25,568.23	19,379.25	24.20%	10,801.4	6,024
11	20,476.83	20,476.83	0%	6,362	1,044
12	26,727.71	25,037.12	6.32%	10,801.1	1,022.4

Table 6. Characteristics of the random isntances.

Group	# of instances	# of molding patterns (*N*)	# of products (*K*)	# of time periods (*T*)	# of machines (*L*)	% expected capacity consumption (*U*)
1	10	6	4	5	2	80%
2	10	6	4	5	2	90%
3	10	10	6	5	2	80%
4	10	10	6	5	2	90%

Table 7. Results of the GLSP model for random instances.

Inst.	GROUP 1			GROUP 2			GROUP 3			GROUP 4		
	Incumbent solution	*Gap*	Time (s)	Incumbent solution	*Gap*	Time (s)	Incumbent solution	*Gap*	Time (s)	Incumbent solution	*Gap*	Time (s)
1	700.45	18%	10,800	2,104,85	0%	126.2	2,041.95	39.30%	10,800	2,515.33	0%	10,072.04
2	1,098.81	0%	1,588.6	911,15	18.73%	10,800	1,401.08	37.23%	10,800	1,334.64	0%	10,800
3	740.15	37.17%	10,800	747,71	0%	10,010.7	1,738.88	55.80%	10,800	1,317.4	48.13%	10,800
4	648.11	0%	8,266.8	866,91	0%	63.6	1,442.77	52.00%	10,800	3,175.61	8.39%	10,800
5	742.64	0%	2,568.4	547,43	0%	707.3	1,552.04	57.58%	10,800	3,735.91	7.79%	10,800
6	731.22	17.88%	10,800	814,62	0%	147.8	1,859.39	26.24%	10,800	1,881.17	34.36%	10,800
7	711.24	0%	455.7	532,90	0%	2,219.3	1,530.67	27.09%	10,800	1,425.6	49.22%	10,800
8	799.22	0%	10,800	759,16	12.11%	10,800	1,817.05	28.59%	10,800	1,456.23	48.69%	10,800
9	1,184.36	0%	532.9	877,97	0%	3,206.9	1,433.98	51.20%	10,800	1,465.82	28.97%	10,800
10	749.89	0%	794.7	899,28	0%	6,943.5	1,402.48	30.15%	10,800	1,878.77	29.20%	10,800

determined in advance. This parameter can be defined either based on the experience of the planner or by preliminary computational experiments.

6 Concluding remarks and future research

This paper studied the production planning and scheduling in molding packaging industry by particularly considering the production environment of a Brazilian company which produces packages for fruit and eggs. The planning decisions are related to the selection of molding patterns, how long they are used, and how they are sequenced. To represent those decisions we have proposed an optimization model based on the classical General Lot Sizing and Scheduling Problem (GLSP).

Results of the computational tests showed that it is possible to find feasible solutions for all the instances by solving the model by CPLEX. For the real set of instances, 50% of them were solved optimally and approximately 33% have a gap of less than 10%. However, the performance of the model is highly influenced by the number of sub-periods defined for each period, which was defined based on the planners' expertise for the real set of instances. That suggests exploring alternative formulations that do not depend on parameters defined in advance, which can influence the performance of the model.

Some comparisons with real schedules for a particular instance were also presented in order to analyze the advantages of the model's solutions. These results evidenced how difficult it is to meet the total demand without backlogs for the company's planner. However, the model's solution provided a schedule that met the full demand by using less capacity, lower setup time, setup costs, and inventory costs than the company's schedule. Moreover, the model's solution kept the inventory levels closer to the target levels. All these advantages are evidenced in a reduction of 27.08% of the setup costs, 33.10% of inventory costs, which consequently implied in a reduction of 36.96% in the total costs.

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