

Fuzzy Goal Programming applied to the process of capital budget in an economic environment under uncertainty

Programação por Metas Fuzzy aplicada ao processo de orçamento de capital em um ambiente econômico sob incerteza

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Abstract: The Goal Programming (GP) is a multi-criteria approach of Operational Research that has been used for solving complex decision problems. This paper proposes a new Fuzzy Goal Programming (FGP) model to handle the process of capital budget of companies in an economic environment under uncertainty. For performance comparison purposes, the FGP and another recently published model developed for the same purposes were applied to data from a company that was the object of the study. The modeling and optimization were made with the GAMS software - 23.6.5 and using the CPLEX solver. The results obtained from the FGP model provided higher improvements than those obtained with the alternative model, as for example: increased profitability index, reduced payback and better application of the capital available in the budget. Furthermore, the FGP model has flexibility features that allow the manager to simulate, quickly and easily obtaining results about scenarios under uncertainty.

Keywords: Fuzzy Goal Programming; Capital budget; Economic environment under uncertainty.

Resumo: A Programação por Metas (Goal Programming - GP) é uma abordagem multicritério da Pesquisa Operacional que vem sendo empregada na solução de complexos problemas de decisão. Este trabalho propõe um novo modelo de Programação por Metas Fuzzy (Fuzzy Goal Programming - FGP) para tratar o processo de orçamento de capital de empresas em um ambiente econômico sob incerteza. Para fins de comparação de desempenho, o modelo FGP e outro modelo desenvolvido com os mesmos fins, recentemente publicado, foram aplicados considerando-se os dados de uma empresa que foi o objeto do estudo. A modelagem e otimização foram feitas com o software GAMS - 23.6.5 e utilizando-se o solver CPLEX. Os resultados do modelo FGP proporcionaram melhorias com relação aos obtidos com o modelo alternativo citado, por exemplo: aumento do índice de lucratividade, redução do payback e melhor aplicação do capital disponível no orçamento. Além disso, o modelo FGP tem características de flexibilidade que permitem ao gestor simular, obter resultados rapidamente e com facilidade, acerca de cenários sob incerteza de seu interesse.

Palavras-chave: Programação por Metas Fuzzy; Orçamento de capital; Ambiente econômico sob incerteza.

1 Introduction

In complex industrial problems, the decision taking involves inaccurate or incomplete information, multiple decision agents and, in general, multiple objectives that can be conflicting with each other (Deb, 2001). This is the case of investment project

selection problems, or capital budget problems, (Santos & Barros, 2011), according to Brigham et al. (2001), there are many ways to select the best set of investment projects, and the traditional investment assessment methods are (Abensur, 2012):

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- Net Present Value Method (NPV) – it allows comparing initial investments with future returns, in which, when the NPV is positive, it means that the capital invested will be recovered;
- Internal Rate of Return Method (IRR) – it considers the discount rate that equates the inflows and outflows of an investment, in which, for the project to be viable, the value of the IRR must be greater than or equal to the Minimum Attractive Rate of Return (MARR) adopted by the company;
- Payback Method – it analyzes the recovery period of the invested capital, that is, the period necessary to a certain investment can be paid;
- Profitability Index Method (PI) – it considers the ratio between the NPV of net cash inflows of the project and the initial investment value.
- Bakirli et al. (2013) combined the AHP method (Saaty, 1977) and a Fuzzy Goal Programming (FGP) model with the QFD (Quality Function Deployment) matrix (Lam & Lai, 2015), to select projects that offer the maximum benefits when performed within various budgets;
- Abensur (2013) evaluated the capital budget in a context in which the managers can establish the completion sequence of projects previously selected, which minimizes the need for investment;
- Khalili-Damghani et al. (2013) applied the FGP model combined with the TOPSIS - Technique for Order Preference by Similarity to Ideal Solution multicriteria method (Joshi & Kumar, 2016) to select multi-period projects in a context under uncertainty;
- Li & Wan (2014) developed a Fuzzy Linear Programming model for the selection of supply chain projects.

In this type of problem, the budget constraint plays important role with respect to limit the decisions that can be taken in the selection of the investment projects, and it is recommended the use of optimization techniques aiming at the optimal allocation of available financial resources (Abensur, 2012).

In this sense, an alternative is to apply to this type of complex and critical situation for companies, the models and techniques of the Goal Programming (GP). In fact, the GP is a method of Operational Research (OR), which allows the modelling and solution of multiobjective problems, including under the occurrence of uncertainties, as is typical in economic scenarios associated with decisions in investment portfolios (Silva & Marins, 2014). Some researchers developed interesting related works:

- Bradi et al. (2000) applied a binary GP model to the problem of project selection, without considering the occurrence of uncertainty;
- Lee & Kim (2000) combined the ANP (Analytic Network Process) (Saaty, 2006) with the GP to select information system projects;
- Santos et al. (2012) investigated the performance of business managers based on the knowledge about cost management and budget participation;
- Ghahtarani & Najafi (2013) combined the robust stochastic optimization (Mulvey et al., 1995) with the GP, developing the Robust Goal Programming (RGP) model, which was applied in selecting investment portfolios in the stock market;

Although there are many studies linked to the selection of investment projects, the use of Fuzzy Goal Programming (FGP) models in this type of problem is recent, corroborating therefore the importance of this research. So, this study aimed to develop and apply a FGP model to solve the capital budget problem. In addition, it was performed a comparison of its results with those obtained by Abensur (2012), who proposed, recently, an alternative model to the same class of investment problems.

Considering the classification proposed by Bertrand & Fransoo (2002), this research can be classified as an applied research, because it provides contributions for the current literature, possessing normative empirical objective, since the model aims to understand policies and strategies that allow actions to better understand a current situation. Besides, with respect to the form of approach of the problem, the research is quantitative type and the research method adopted is the modeling.

This article is organized in 5 sections. Section 2 briefly describes the theoretical foundations of the GP and, in particular, addresses the FGP model. Section 3 addresses the problem description; Section 4 presents the problem modeling and the results obtained; finally, Section 5 brings the conclusion and the recommendations for future researches, followed by the bibliography.

2 Goal programming

According to Charnes & Cooper (1961), Romero (2001, 2004), Silva & Marins (2014, 2015), Chang (2007, 2008), Jamalnia & Soukhakian (2009) and Yaghoobi & Tamiz (2007), the three most used types of deterministic models of the GP, are:

- Weighted Goal Programming (WGP) – weights are attributed to the (positive and negative) deviation variables with regard to the goals chosen for the objectives. This was the first model of GP developed and, due to this; it will be briefly described below;
- Lexicographic Goal Programming (LGP) or Preemptive Goal Programming – objective functions are ordered according to their importance established from a prioritization made by the managers;
- Minmax Goal Programming (Minmax GP) – objective functions, which consider the sum of the deviation variables, are included in the model.

2.1 Weighted Goal Programming

In the WGP models, the deviation variables present equivalent hierarchies, being the weighting what will distinguish the most important objectives. The decision-makers need to estimate the weights in such a way that a large part of the objectives is satisfied, which generates problems with the subjectivism in the estimation of these weights. To try minimizing the subjectivism, a method that has been very adopted in the prioritization of objectives is the Analytic Hierarchy Process – AHP (Saaty, 1977).

According to Martel & Aouni (1998), the original WGP model can be expressed by:

Equation 1 β e α are weights attributed, respectively, to d_i^+ and d_i^-

$$\text{Min } \sum_{i=1}^n (\beta_i d_i^+ + \alpha_i d_i^-) \tag{1}$$

Subject to:

Constraint 2 where x is the vector of the decision variables of the model x_i ; $f_i(x)$ is an objective function I , g_i is a goal value chosen to f_i , d_i^+ and d_i^- are, respectively, the auxiliary (positive and negative) deviation variables related to the goal g_i

$$f_i(x) - d_i^+ + d_i^- = g_i, \quad i = 1, 2, \dots, n. \tag{2}$$

Constraint 3 A and c are, respectively, a matrix of the LHS (Left Hand-Side) coefficients and a RHS (Right Hand-Side) vector for hard restrictions in the original multi-criteria model, and F is Feasible set.

$$Ax \leq c \tag{3}$$

Constraint 4 It is observed that, by only one of the deviation variable associated with each goal can have value different from zero.

$$x_i, d_i^+, d_i^- \geq 0, d_i^+ d_i^- = 0, i = 1, 2, \dots, n. x \in F \tag{4}$$

There are other deterministic GP models; however, they are not presented in this work, for more details it is recommended to consult a GP review by Silva & Marins (2015).

2.2 Fuzzy Goal Programming (FGP) Model

It should be noted that management decision problems have as natural characteristics the occurrence of uncertainty in their parameters, such as, for example, problems related to aggregate production planning and budgetary problems (Wang & Liang, 2004). The main GP models under uncertainty are (Silva & Marins, 2015):

- Multi-Choice Goal Programming (Chang, 2007);
- Revised Multi-Choice Goal Programming (Chang, 2008);
- Integer Multi-Choice Goal Programming (Silva et al., 2013a);
- Fuzzy Multi-Choice Goal Programming (Bankian-Tabrizi et al., 2012);
- Multi-Segment Goal Programming (Liao, 2009);
- Revised Multi-Segment Goal Programming - RMSGP (Chang et al., 2012a);
- Multi-Coefficients Goal Programming - MCGP (Chang et al., 2012b);
- Revised multi-choice Goal Programming - LHS (Silva et al., 2015);
- Fuzzy Goal Programming - FGP (Silva & Marins, 2014; Jamalnia & Soukhakian, 2009; Yaghoobi & Tamiz, 2007; Liang & Wang, 1993; Zimmermann, 1978);
- Robust stochastic optimization (Mulvey et al., 1995).

In this research, it was applied the FGP model, because it is the oldest GP model under uncertainty, having a wide range of real applications (Silva & Marins, 2014, 2015). Concepts and results relating to the inclusion of uncertainty occurrence in the problem data, the contribution of the GP, and the basic theory of Fuzzy sets to handle these situations are presented (Chang, 2007).

Zimmermann (1978) used the triangular fuzzy number to characterize linguistic values and Liang & Wang (1993) justified the use of triangular fuzzy functions, because they properly characterize the human trials, and also allow simulating uncertainty occurrence in the data and parameters involved.

This was the option adopted in this work that is going to be detailed subsequently.

In this line, according to Jamalnia & Soukhakian (2009), and Yaghoobi & Tamiz (2007), there are three most common types of pertinence functions, denoted by μ_{z_k} for a given objective function G_k , when we are working with triangular fuzzy numbers, as show the expressions (5)-(7) and illustrate the Figures 1-3. It was adopted the notation $[\sim]$ to represent a fuzzy goal value (desired and imprecise) g_k chosen to a fuzzy objective function G_k :

$$G_k(x) \lesssim g_k, \quad k = 1, \dots, m, \tag{5}$$

Equation 6 there is a fuzzy objective function of the type “the higher its value in relation to g_k , the better it is”

$$G_k(x) \gtrsim g_k, \quad k = m + 1, \dots, n, \tag{6}$$

Equation 7 there is a objective function that is wanted to be achieved exactly at the value g_k chosen.

$$G_k(x) \equiv g_k, \quad k = n + 1, \dots, l. \tag{7}$$

The fuzzy goals can be identified as fuzzy sets defined on the feasible set of solutions associated with a pertinence function. The linear pertinence functions are the functions most adopted both in the theoretical and practical works (Jamalnia & Soukhakian, 2009). For the fuzzy constraints (5) and (6), by adopting the triangular fuzzy numbers, in which L_k and U_k , respectively, are the minimum and maximum values chosen by the decision-maker to be attributed to the fuzzy goal g_k , the linear pertinence functions can be expressed by (8)-(10):

$$\mu_{z_k}(x) = \begin{cases} 1 & \text{se } G_k(x) \leq g_k \\ \frac{U_k - G_k(x)}{U_k - g_k} & \text{se } g_k \leq G_k(x) \leq U_k \\ 0 & \text{se } G_k(x) \geq U_k \end{cases} \quad k = 1, \dots, m \tag{8}$$

$$\mu_{z_k}(x) = \begin{cases} 1 & \text{se } G_k(x) \geq g_k \\ \frac{G_k(x) - L_k}{g_k - L_k} & \text{se } L_k \leq G_k(x) \leq g_k \\ 0 & \text{se } G_k(x) \leq L_k \end{cases} \quad k = m + 1, \dots, n \tag{9}$$

$$\mu_{z_k}(x) = \begin{cases} 0 & \text{se } L_k \leq G_k(x) \leq g_k \\ \frac{U_k - G_k(x)}{U_k - g_k} & \text{se } g_k \leq G_k(x) \leq U_k \\ 0 & \text{se } G_k(x) \geq U_k \end{cases} \quad k = n + 1, \dots, l \tag{10}$$

Other form to visualize this pertinence functions is in Figures 1-3:

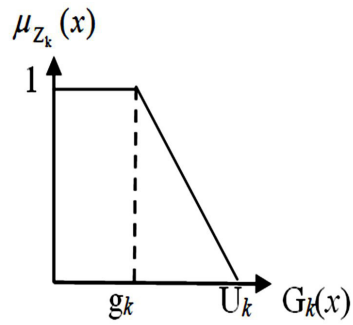


Figure 1. $G_k(x) \lesssim g_k$.

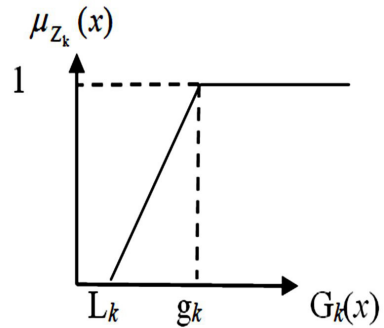


Figure 2. $G_k(x) \gtrsim g_k$.

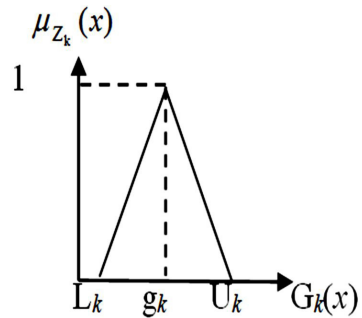


Figure 3. $G_k(x) \equiv g_k$.

It is observed that, usually, the limits L_k and U_k are subjectively chosen by the decision-makers, or are associated with the tolerances existing in a technical process. The choice of these tolerance limits is very important, once they directly influence the model optimization (Silva & Marins, 2014).

The model of the Fuzzy Goal Programming (FGP), proposed by Yaghoobi & Tamiz (2007), using the triangular pertinence functions, can be expressed by:

Equation 11 objective function aims to maximize the total degree of fuzzy achievement and, when $\lambda = 1$, it means that all fuzzy goals were fully satisfied or met.

$$\text{Max } \lambda \tag{11}$$

Subject to:

Constraint 12 represents a situation for which it is desired to penalize the positive deviation.

$$G_i(X) - d_i^+ \leq g_i, \quad i = 1, 2, \dots, i_0 \quad (12)$$

Constraint 13 represents a situation for which it is desired to penalize the negative deviation.

$$G_i(X) + d_i^- \geq g_i, \quad i = i_0 + 1, 2, \dots, j_0 \quad (13)$$

Constraint 14 represents a situation for which it is desired to penalize both positive and negative deviations.

$$G_i(X) + d_i^- - d_i^+ = g_i, \quad i = j_0 + 1, 2, \dots, K \quad (14)$$

Constraints 15 to 17 are related to the limitations imposed to the degree of achievement of the scenarios associated to constraints (5)-(7).

$$\lambda + \frac{1}{\Delta_{iU}} d_i^+ \leq 1, \quad i = 1, 2, \dots, i_0 \quad (15)$$

$$\lambda + \frac{1}{\Delta_{iL}} d_i^- \leq 1, \quad i = i_0 + 1, 2, \dots, j_0 \quad (16)$$

$$\lambda + \frac{1}{\Delta_{iL}} d_i^- + \frac{1}{\Delta_{iU}} d_i^+ \leq 1, \quad i = j_0 + 1, 2, \dots, K \quad (17)$$

Constraints 18 to 19 indicate the domains of variables.

$$d_i^+, d_i^- = 0, \quad i = 1, 2, \dots, K \quad (18)$$

$$\lambda, d_i^+, d_i^- \geq 0, \quad X \in F \text{ (F is a feasible set)} \quad (19)$$

where $\lambda = \sum_i \lambda_i$ is the degree of achievement of the fuzzy goals, g_i is the level of aspiration (or desired value) for the objective function G_i , Δ_{iL} and Δ_{iU} are, respectively, the difference between the minimum value (L) and the maximum value (U) with respect to the goal g_i .

It can be observed that $\lambda_i \in [0,1]$ and, when the value of $\lambda_i = 1$, it means that the fuzzy, goal g_i was fully achieved. In this

In the sequence, it is presented the capital budget problem and the steps followed for its resolution.

3 Problem description and research steps

In this article, the FGP model was applied to the capital budget problem, considering the economic environment in which there is occurrence of uncertainties in the data and parameters involved in the decision taking. The research steps were:

- **Problem identification** – It was chosen the capital budget problem proposed and studied by Abensur (2012), in which it is desired to select, among a set of 45 investment projects, which projects should be executed;
- **Data collection** – The data used were those available in Abensur (2012) and they are in Table 1;
- **Modeling, model solution and comparison of results** – The FGP model was developed and implemented using the GAMS software - version 23.6.5, and was solved using the CPLEX Solver, as described in Section 4 (GAMS, 2014).

For the comparison between the results of the FGP model and the model of Abensur (2012), three objective functions were considered, related to:

- The Profitability Index (PI) – given by the ratio between the NPV and the initial payout value;
- The Total Discounted Payback (TDP) – associated with the time taken to recover the capital invested;
- The Degree of Total Financial Leverage (DTFL) – is a risk measure to evaluate investment projects.

The constraints considered in the model by Abensur (2012) are relating to:

- Mutually exclusive relationships and relationships of dependence of projects;
- Investment limits foreseen for the projects;
- Additional relationships that ensure that the projects of the optimal solution have Modified Internal Rate of Return (MIRR) above the Minimum Acceptable Rate of Return (MARR), PI above 1 and TDP lower or equal to the useful life (economic life or their duration) of projects.

The model by Abensur (2012) also has as premises that:

- All projects begin their activities on the same initial date;
- The project groups are independent of each other;
- There are groups with mutually exclusive projects;
- There are independent projects;
- There are projects with relationship of dependence;
- The capital budget constraint occurs only once on the initial date of analysis of the projects;

Table 1. Projects analyzed and their respective indicators of profitability, risk and return.

Group	Project	DI [Brazilian Reais - R\$]	TMA [% per year]	N [year]	VPL [Brazilian Reais - R\$]	IL [%]	MTIR [%]	PBD [year]	GAFT [%]
A	1	1,000	10	4	39	3.91	11.06	2.90	15.57
	2	1,000	10	4	53	5.35	11.44	4.70	19.32
B	3	1,000	12	4	58	5.80	13.59	4.60	20.06
	4	1,000	12	4	39	3.99	13.10	3.70	15.88
C	5	22,000	12	6	3,860	17.55	15.06	5.30	33.58
	6	17,500	12	6	3,057	17.47	15.05	5.30	33.49
D	7	10,000	12	5	814	8.14	13.77	5.10	22.41
	8	25,000	12	5	1,675	6.70	13.46	5.10	20.77
E	9	300,000	9	5	-43,883	-14.43	5.66	20.00	8.29
	10	120,000	9	5	253,406	211.17	36.78	2.40	250.13
F	11	68,000	10	10	84,385	124.10	19.24	4.20	156.74
	12	28,000	10	5	44,783	159.94	33.16	2.70	193.09
G	13	5,000	8	4	701	14.03	11.60	4.60	28.13
	14	10,000	8	3	970	9.70	11.39	3.80	22.59
	15	10,000	8	4	3,248	32.49	15.87	3.80	48.29
	16	12,000	8	3	885	7.38	10.59	3.70	19.80
	17	8,000	8	2	2,699	33.74	24.90	2.50	48.71
	18	5,000	8	2	-216	-4.32	5.64	10.00	6.64
	19	6,000	8	4	2,153	35.89	16.60	3.90	52.31
H	20	100	10	2	38	38.84	29.61	2.40	54.75
	21	80	10	2	32	41.01	30.62	2.40	57.17
I	22	100	10	2	17	17.36	19.16	2.80	31.22
	23	100	10	2	16	16.74	18.85	2.40	29.81
J	24	480	9	7	170	35.46	13.83	5.90	53.90
	25	620	9	7	92	14.97	11.19	6.80	30.59
	26	750	9	7	192	25.60	12.61	6.30	42.67
K	27	10	10	2	21	214.05	94.94	0.60	248.97
	28	5	10	2	16	321.49	125.83	2.00	371.05
	29	5	10	2	11	238.84	102.48	2.00	278.52
L	30	5,000	10	5	1,338	26.76	15.34	4.50	42.93
	31	8,000	10	10	1,794	22.43	12.35	6.90	40.26
M	32	1,500	10	5	-610	-40.68	2.16	10.00	35.37
	33	1,500	10	5	766	51.07	19.46	5.20	72.63
	34	1,500	10	5	796	53.09	19.78	5.20	74.33
	35	1,500	10	5	779	51.95	19.60	4.40	72.06
N	36	85,000	20	4	18,549	21.82	26.07	3.90	38.92
	37	150,000	20	4	51,921	34.61	29.26	3.60	53.51
	38	250,000	20	4	87,577	35.03	29.36	4.50	58.03
	39	378,000	20	4	19,337	5.12	21.51	4.10	18.52
O	40	100,000	10	5	84,337	84.34	16.94	5.20	111.19
	41	70,000	10	5	52,891	75.56	16.37	5.50	101.13
P	42	80,000	10	5	-9,339	-11.67	7.30	10.00	5.41
	43	20,000	15	7	2,399	12.00	16.88	6.50	29.07
	44	500	20	10	128	25.70	22.78	7.00	47.23
	45	200	20	10	219	109.62	29.22	3.82	145.39
Total	45	1,805,450			672,213	2.271			3.160
Min		5	8	2	-43,883	-40.68	2.16	0.60	5.41
Max		378,000	20	10	253,406	321.49	125.83	20.00	371.05

Source: Abensur (2012).

- It is attributed, in the second stage of the model, the same weight to all components of the objective function.

Besides, to consider the situation of economic environment under uncertainty, the values of the upper (U) and lower (L) limits, desired for the fuzzy goals to PI, TDP and DTFL (see Table 2), were chosen by the researchers involved with this work, seeking to provide a better use of the budget, that is, seeking the reduction of the gap that occurred in the solution proposed by Abensur (2012).

- **Analysis of Results and Conclusions** – These issues are developed in Sections 4 and 5.

In the next section, the FGP model is customized for solving the investment problem studied by Abensur (2012).

4 Problem modeling

Initially, the indices, sets, parameters and variables considered in the FGP model proposed are presented: Indices and Sets:

j Projects $j \in J, J = \{1, 2, \dots, 45\}$.

i Objective functions, $i \in I, I = \{1, 2, 3\}$.

Parameters:

ε_j : Initial payout of the project *j*;

ρ_j : Minimum Acceptable Rate of Return of the project *j*;

γ_j : Profitability Index of the project *j*;

η_j : Modified Internal Rate of Return of the project *j*;

θ_j : Discounted Payback of the project *j*;

ρ_j : Degree of Leverage of the project *j*;

τ_j : Useful life of the project *j*.

Decision variables:

x: Binary variable associated with the selection of the project *j*.

Auxiliary variables:

d_i^+ : Positive deviation variable for the *i*th goal

d_i^- : Negative deviation variable for the *i*th goal

λ_i : Degree achievement fuzzy function with $0 \leq \lambda_i \leq 1$.

Fuzzy goal objective functions

The expressions (20)-(22) are related to the maximization of the liquidity index (function Z_1) under uncertainty, being adopted the values of the upper and lower limits given in Table 2:

$$\max Z_1 = \sum_{j \in J} x_j \gamma_j \tag{20}$$

$$\sum_{j \in J} x_j \gamma_j \gtrsim 1,700 \tag{21}$$

$$\mu_{Z_1} = \begin{cases} 1 & \text{se } Z_1 \geq 1,700 \\ \left[\frac{G_1 - 1,700}{2,000 - 1,700} \right] & \text{se } 1,700 \leq Z_1 \leq 2,000 \\ 0 & \text{se } Z_1 \leq 1,700 \end{cases} \tag{22}$$

Figure 4 illustrates the behavior of the fuzzy goal 1 (liquidity index accumulated), being considered a linear function of triangular pertinence:

The expressions (23)-(25) are bound to the minimization of the payback (function Z_2) under the situation with uncertainty, being adopted the values of the upper and lower limits given in Table 2:

$$\min Z_2 = \sum_{j \in J} x_j \theta_j - \sum_{j \in J} x_j \tau_j \tag{23}$$

$$\sum_{j \in J} x_j \theta_j - \sum_{j \in J} x_j \tau_j \lesssim 4 \tag{24}$$

$$\mu_{Z_2} = \begin{cases} 1 & \text{se } Z_2 \leq 4 \\ \left[\frac{4 - G_2}{4 - 0} \right] & \text{se } 0 \leq Z_2 \leq 4 \\ 0 & \text{se } Z_2 \geq 4 \end{cases} \tag{25}$$

Figure 5 describes the behavior of the fuzzy goal 2 (total payback), as a linear triangular pertinence function:

The expressions (26)-(28) are related to the maximization of the DTFL (function Z_3) under uncertainty, being adopted the values of the upper and lower limits given in Table 2:

$$\max Z_3 = \sum_{j \in J} x_j \rho_j \tag{26}$$

$$\sum_{j \in J} x_j \rho_j \gtrsim 2,200 \tag{27}$$

$$\mu_{Z_3} = \begin{cases} 1 & \text{if } Z_3 \geq 2,200 \\ \left[\frac{G_3 - 2,200}{2,500 - 2,200} \right] & \text{if } 2,500 \leq Z_3 \leq 2,200 \\ 0 & \text{if } Z_3 \leq 2,200 \end{cases} \tag{28}$$

Table 2. Goals and Lower (L) and Upper (U) Bounds chosen for the objective functions.

Goals	L	U
PI	1,700	1,800
TDP	1	4
DTFL	2,200	2,500

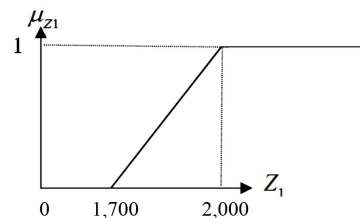


Figure 4. Function of triangular pertinence for the fuzzy goal associated to the accumulated liquidity index.

Figure 6 shows the behavior of the fuzzy goal 3 (DTFL), as a linear triangular pertinence function:

Finally, considering the data in Table 1, the FGP model can be formulated for the situation proposed by Abensur (2012):

FGP Model

Equation 29 is the objective function, which aims the maximization of the degree of achievement of the fuzzy goals associated to each objective.

$$Max \sum_{i \in I} \lambda_i \tag{29}$$

Subject to:

Hard constraints (Abensur, 2012)

Constraint 30 is the constraint that concerned to the budget use.

$$\sum_{j \in J} x_j \varepsilon_j \leq 452,000 \tag{30}$$

Constraints 31 to 32 are constraints to the mutually exclusive projects.

$$-x_{32} - x_{34} + x_{42} \leq 1 \tag{31}$$

$$\sum_{j=13}^{19} x_j + \sum_{j=32}^{35} x_j \leq 1 \tag{32}$$

Constraint 33 is the constraint that establishes that the total modified internal rate of return must be higher or equal to the total minimum acceptable rate of return.

$$\sum_{j \in J} x_j \eta_j \geq \sum_{j \in J} x_j \varphi_j \tag{33}$$

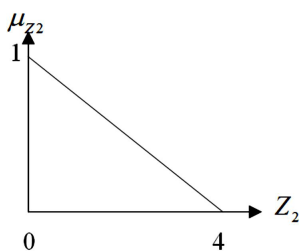


Figure 5. Triangular Pertinence Function for the fuzzy goal associated to the minimization of the total payback.

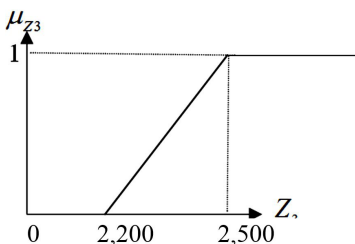


Figure 6. Triangular Pertinence Function for the fuzzy goal associated to the maximization of DTFL.

Fuzzy constraints (new)

Constraints 34 to 35 are fuzzy constraints that associate the maximization of liquidity to the goal value R\$1,700.00.

$$\sum_{j \in J} x_j \gamma_j + d_1^- \geq 1,700 \tag{34}$$

$$\frac{1}{300} d_1^- + \lambda_1 \leq 1 \tag{35}$$

Constraints 36 to 37 are fuzzy constraints that associate the minimization of the total discounted payback to the goal value established in 4 years.

$$\sum_{j \in J} x_j \theta_j - \sum_{j \in J} x_j \tau_j - d_2^+ \leq 4 \tag{36}$$

$$\frac{1}{3} d_2^+ + \lambda_2 \leq 1 \tag{37}$$

Constraints 38 to 39 are fuzzy constraints that associate the maximization of the degree of total financial leverage to the goal value R\$2,200.00.

$$\sum_{j \in J} x_j \rho_j + d_3^- \geq 2,200 \tag{38}$$

$$\frac{1}{200} d_3^- + \lambda_3 \leq 1 \tag{39}$$

Constraints 40 is the constraint associated to the domain of the variables.

$$x_j \in \{0, 1\}, \quad \forall j \in J, \lambda_\phi \geq 0, d_i^+ \geq 0, d_i^- \geq 0, d_i^+ d_i^- = 0, \quad \forall i \in I. \tag{40}$$

Tables 3 and 4 summarize the results obtained with both models (FGP Model and Abensur Model), being highlighted that the Abensur model does not contemplate the occurrence of uncertainty and that the resolution times of the FGP model, for any scenarios, were on average 4 seconds that facilitate the generation of alternative scenarios of interest for the problem manager.

It can be observed in Tables 3 and 4 that:

- There was an increase of 380% in the number of projects selected by FGP Model (24 projects) in comparison to Abensur Model (only 05 projects). This represents that FGP Model offers a higher flexibility to the manager;
- It was generated a higher value of PI (= R\$1,660.50) by FGP Model than by Abensur Model (PI = R\$854.17), corresponding to an increase of 94.4%. This means that the FGP model provided a better remuneration of invested capital;

- It was generated a higher value of DTFL (=R\$2,181.38) by FGP Model than by Abensur Model (DTFL = R\$1,031.72), corresponding to an increase of 111.4%. This means that the FGP model allows a better financial turnover;
- Using FGP Model, the value of TDP (=0.02 year) was substantially lower (99.5%) than by Abensur Model (TDP = 4.4 years), despite the occurrence of uncertainty in the data. This means that FGP Model allows shorter time to investment recovery;
- There was a better use of the budget available (R\$452,000.00) by FGP Model (investing R\$446,450.00, almost 98.77% of the total resources available for investment) than by Abensur Model (investing R\$149,705.00, only 33,12% of the total resources available for investment);
- The degree of achievement λ_i for TDP was equal to 1.0. This means that, with the FGP model, it was obtained a value of TDP below the chosen fuzzy goal (4 years);
- The degree of achievement λ_i for PI was equal to 0.87. This means that, with the FGP Model, the value of PI was only 13% below the chosen fuzzy goal (R\$1,700.00), which is fully acceptable due to the uncertainties were incorporated to the data;

- The degree of achievement λ_i for DTFL was equal to 0.91. This means that, with the FGP Model, the value of DTFL was only 9% below the chosen fuzzy goal (R\$ 2,200.00), which is fully acceptable due to the uncertainties were incorporated to the data.

As an additional exercise to verify the sensitivity of the FGP model, two scenarios were generated with respect to the requirements for PI and DTFL values. In the first scenerly it was considered a decrease in those values – it means a less strict project selection, and in the second scenerly it was considered an increase in the values - it means a stricter project selection, as explained below, and the new solution results are in Tables 5 and 6:

- First scenerly – the fuzzy goal for PI was decreased from R\$1,700.00 to R\$1,600.00 and the fuzzy goal for DTFL was decreased from R\$2,200.00 to R\$2,100.00;
- Second scenerly – the fuzzy goal for PI was increased from R\$1,700.00 to R\$1,800.00 and the fuzzy goal for DTFL was increased from R\$2,200.00 para R\$2,300.00.

As can be observed in Tables 5, for both new scenarios with uncertainty, the number of projects selected was the same previously obtained by FGP Model, i.e. 24 projects, but note that projects 6, 11 and 43 were selected only for the first scenerly.

Table 3. Results of the Abensur Model and FGP model.

Item	Abensur Model	FGP Model	Difference [%]
Number of selected projects	5	24	380
Chosen Projects	10-12-28-35-45	1-2-3-4-5-7-10-12-18-20-21-22-23-24-25-26-27-28-29-30-40-41-42-45	
PI [Brazilian Reais - R\$]	854.17	1,660. 50	94.4
TDP [year]	4.4 years	0.02 years	-99.5
DTFL [Brazilian Reais -R\$]	1,031.72	2,181.38	111,4
Utilized Resources [Brazilian Reais -R\$]	149,705.00	446,450.00	198.2

Table 4. Analysis with respect to degree of achievement fuzzy functions and deviation variables values.

Function	Degree achievement fuzzy function λ_i	Positive deviation variables d_i^-	Negative deviation variables d_i^+
PI	0.87	39.5	0
TDP	1	0	0
DTFL	0.91	18.62	0

As justification for this different selection, it is verified that for the second scenery are selected projects with the highest goals values, which is not the case of these three projects mentioned.

In Table 5, it can be observed, also, that, although for the first scenery has presented lower values of PI and DTFL, it presents the higher value of payback and provides the best allocation of the available resources.

Observing Table 6, for the first scenery (with uncertainty), all fuzzy goals were fully achieved ($\lambda_i = 1$), and it presents the best allocation (almost 99% were invested) of the available resources, while the Abensur Model proposed to invest only 33% for a situation without uncertainty.

For the second scenery, the values of PI, TDP and DTFL were the same previously got by FGP Model, but the degree of achievement of the fuzzy goals associated to PI and DTFL objectives were substantially lower, respectively, 0.54 and 0.41, than those previously got by FGP Model, respectively, 0.87 and 0.91.

The sensitivity analysis shows the existence of trade-offs between objectives, enabling managers assess the impact on the solution caused by changing goals of an objective function. It also allows the decision maker identifies which are the most important objective functions for certain projects, for example,

for the shareholder would be desirable to maximize profitability, and, for other hand, the company would can be interested in to obtain the return on investment in the project in a shortest possible time.

5 Conclusions

The project selection of different nature is a very difficult task, because usually there are many criteria (objectives) to be optimized, the traditional investment analysis method are restricted to a mono-objective function, and they do not consider the occurrence of uncertainty. This was the main motivation to this work that used multi-objective GP models under uncertainty.

As conclusions of this paper, it can be affirmed that the application of the FGP model was viable and relevant to solve the capital budget problem, presenting advantages in relation to the adoption of classical optimization methods (deterministic). In the case described here, the improvements found were, mainly, with regard to the increase in the number of projects included in the selected portfolio, in PI and DTFL values, in the reduction of the payback and in a better resources allocation.

In fact, the FGP model had a better performance than Abensur Model, offering better PI, TDP and DTFL values in all optimizations carried out, allowing more

Table 5. Previous and new (Sensibility Analysis) results from FGP model.

	Previous results - FGP Model	First Scenery	Second Scenery
Number of projects	24	24	24
Projects	1-2-3-4-5-7-10-12-18-20-21-22-23-24-25-26-27-28-29-30-40-41-42-45	2-3-4-5-6-7-10-11-12-18-20-21-22-23-24-25-26-27-28-29-30-41-42-43	1-2-3-4-5-7-10-12-18-20-21-22-23-24-25-26-27-28-29-30-40-41-42-45
PI [Brazilian Reais - R\$]	1,660.50	1,616.2	1,660.50
TDP [year]	0.02	0.1	0.02
DTFL [Brazilian Reais -R\$]	2,181.38	2,128.53	2,181.50
Budget [Brazilian Reais -R\$]	446,450.00	450,750.00	446,450.00

Table 6. New degree of achievement fuzzy function and deviation variables values (Sensibility Analysis).

	Degree achievement fuzzy function λ_i	Positive deviation variables d_i^-	Negative deviation variables d_i^+
First Scenery			
PI	1	0	0
TDP	1	0	0
DTFL	1	0	0
Second Scenery			
PI	0.54	139.50	0
TDP	1	0	0
DTFL	0.41	118.62	0

flexibility to the managers with regard to the use of the resources, allowing testing variations in the goals.

Finally, it should also be highlighted that the FGP model allows aggregating the occurrence of uncertainty in the problem of capital budget, as it is much verified in the practice, not occasioning higher mathematical and computational complexity both in the modeling phase and model solution, presenting low solution times.

As proposals for future researches, it is suggested to apply to the capital budget problems:

- Data Envelopment Analysis - DEA Models (Silva et al., 2013b);
- The Revised Multi-Choice Goal Programming - RCMGP- LHS (Silva et al., 2015);
- Monte Carlo simulation combined with GP models under uncertainty (Silva et al., 2014);
- Models proposed by Ekel et al. (2008) and Pereira et al. (2015) combined with GP models under uncertainty.

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