



Mathematical model for the Berth Allocation Problem in ports with cargo operation limitations along the pier

Modelo matemático para o Problema de Alocação de Berços em portos com limitações de operação de carga ao longo do cais

Ivan Bridi Gimenes Rodrigues¹
Rodrigo de Alvarenga Rosa¹
Thiara Cezana Gomes¹
Glaydston Matos Ribeiro²

Abstract: Oil exploration in Brazil is performed by platforms at sea that require several loads carried by vessels. Thus, it is necessary to have ports to handle these vessels' operation and, therefore, the ports assign specific segments along the pier to operate certain type of cargo, increasing the complexity of vessels berthing. Thus, this paper proposes a mathematical model for the Continuous Berth Allocation Problem that differs from the others because it considers that some cargos can be operated only in specific segments of the pier. We used real data from the Companhia Portuária de Vila Velha (CPVV) to evaluate the model. CPLEX 12.6 was used to run the model and instances up to 147 vessels with 440 meters of pier were optimally solved. The results are presented and compared to those achieved by the CPVV's manual method showing that it is possible to obtain significant gains.

Keywords: Berth Allocation Problem; Port operation; Oil and gas logistics.

Resumo: A exploração de petróleo no Brasil é realizada por plataformas em alto-mar que demandam diversas cargas levadas por navios. Os portos para atender esses navios têm de manusear vários tipos de carga e, por conta dessa variedade, trechos para movimentar cada tipo de carga são determinados ao longo do cais, aumentando a complexidade do planejamento da atracação dos navios. Visando aumentar a eficiência na operação desses portos, este artigo propõe um modelo matemático para o problema de alocação de berços contínuos que difere das demais por apresentar restrições nas operações de cargas ao longo do cais. Utilizaram-se dados reais da Companhia Portuária de Vila Velha (CPVV) para avaliar o modelo. Utilizou-se o CPLEX 12.6 para executar o modelo e instâncias de até 147 navios com 440 metros de cais foram resolvidas de forma ótima. Os resultados são apresentados e comparados com os alcançados pelo método manual atual, evidenciando ganhos importantes.

Palavras-chave: Problema de Alocação de Berços; Operação portuária; Logística do petróleo e gás.

1 Introduction

Oil exploration in Brazil is based mainly on exploration in deep waters. For this type of exploration, offshore platforms are positioned at the sea far from the coast, demanding resources equal to the demand of a medium-sized city. These platforms are supplied mainly by vessels that travel between the port and the platforms. Thus, it is essential to reduce the operation time at the port to avoid any shortage on any platform and consequently the interruption of oil exploration caused by the lack of supplies.

Thus, specialized ports to berth these vessels that supply the platforms become important for the oil exploration process. An important feature of these ports is that they handle bulk materials (cement, diesel oil and water) and general cargo (pipes, boxes, and other equipment). Thus, because of this variety of cargos, these ports assign segments of the pier to handle only certain types of cargo. So, planning the berthing of vessels in the port must take into account the cargo that the vessel will operate (load / unload) at the time of assigning a berth position to berth the

¹ Postgraduate Civil Engineering Program, Universidade Federal do Espírito Santo – UFES, Avenida Fernando Ferrari, 514, Goiabeiras, CEP 29075-910, Vitória, ES, Brazil, e-mail: ivanbridi@gmail.com; rodrigo.a.rosa@ufes.br; thiaracezana@hotmail.com

² Postgraduate Transportation Engineering Program, Centro de Tecnologia, Universidade Federal do Rio de Janeiro – UFRJ, Bloco H, Sala 106, Cidade Universitária, CEP 21949-900, Rio de Janeiro, RJ, Brazil, e-mail: glaydston@pet.coppe.ufrj.br

vessel. This situation makes the planning process a complex task.

To optimize the sequence of service of vessels by the port, it was proposed in the literature the Berth Allocation Problem (BAP) that deals with the problem of planning the sequence of service of a set of vessels, within a time horizon, in a pier with a specific layout, respecting the operational and commercial constraints of the port. The most common objective is to minimize the total time spent at the port by all vessels. At the ports where the vessels can berth at any position of the pier, the BAP is called Continuous Berth Allocation Problem (CBAP) (Bierwirth & Meisel, 2010). Some authors have developed extensive literature review about the BAP, such as: Meersmans & Dekker (2001), Vis & Koster (2003), Steenken et al. (2004), Vacca et al. (2007), and Stahlbock & Voß (2008), Bierwirth & Meisel (2010) and Rashidi & Tsang (2013).

In order to satisfy an operational characteristic of the specialized ports to supply the oil platforms, but also common to many other ports in the world, where certain cargos can only be handled in certain berthing positions along the pier, this paper presents a mathematical model for the CBAP with cargo operating limitations along the pier. The proposed model was applied to the real problem of the port Companhia Portuária de Vila Velha (CPVV) to assess the quality of the solution reached. CPVV

is the largest port specialized to logistics support of offshore platforms of the state of Espírito Santo, Brazil, operating per month, an average of 25 tons of general cargo, 20,000 cubic meters of clean water, 6,000 cubic meters of diesel and 4,000 tons of cement (CPVV, 2015).

Figure 1 shows an aerial view of CPVV that has a mooring pier with 320.0 m. CPVV has a feature common to many other ports where some cargos can only be handled in specific segment along the pier. Figure 1 presents these segments of the pier where each cargo can be operated. Cement vessels may berth between 0.0 m and 80.0 m due to the limitation of the tubes that transfer the cement of ground tanks to the vessel. General cargo vessels can berth between 0.0m and 245.0 m position. Anchorage vessels, known as Anchor Handling Tug (AHT), carrying flexible tubes on reels, that berth by stern, must be berthed between 246.0 m and 265.0 m. Finally, diesel vessels can berth between 266.0 m and 320.0 m (CPVV, 2015).

The mathematical model was solved by the solver CPLEX 12.6 and the solutions were compared with CPVV results. After a literature review, no articles were found proposing a model that treats the CBAP with operating limitations for some cargos along the pier extension.

The paper is organized as follows. Section 2 details the BAP and presents a literature review. Section 3 presents the proposed mathematical model. In Section

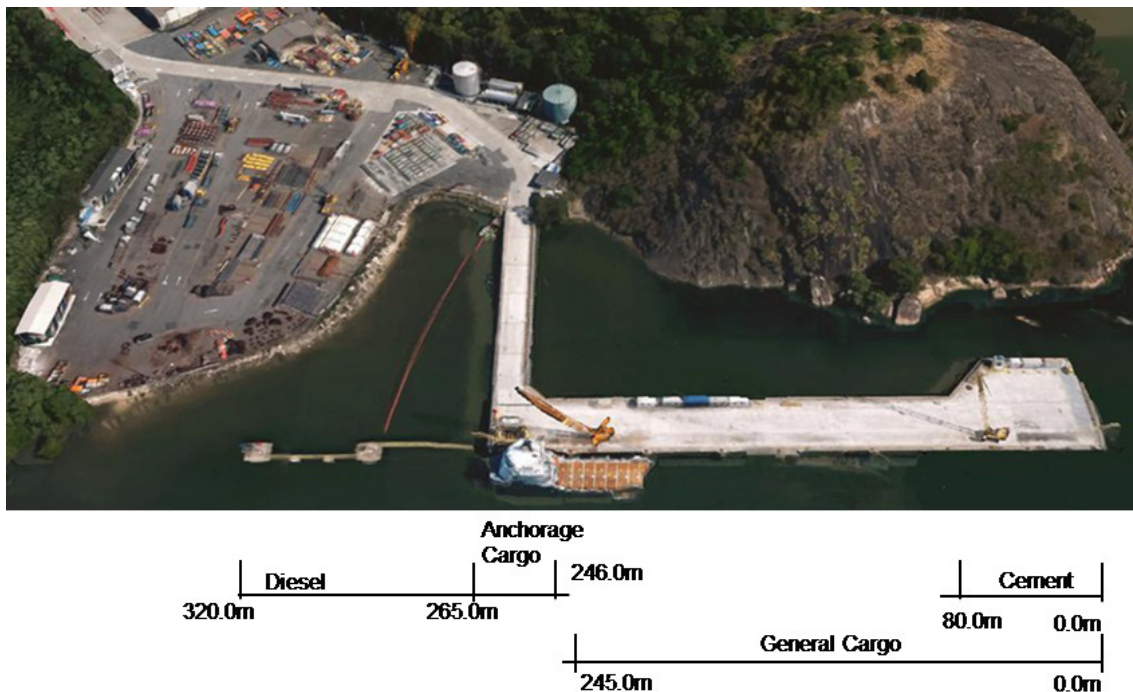


Figure 1. Aerial view of CPVV.

4, the computational experiments and analysis of the results are presented. In the last section, the conclusions are presented.

2 Berth Allocation Problem (BAP)

The Berth Allocation Problem (BAP) refers to the problem of assigning spaces to berth vessels in the berthing areas of the port over a planning horizon. Thus, in the BAP, the major decisions involve the location and the time where the vessels should berth (Cordeau et al., 2005).

In general, this type of problem deals with two types of restrictions: 1) spatial restrictions and 2) temporal restrictions. The spatial restrictions are related to the dimensions of the port, the size of the vessels, the water depth and the division of the pier. The temporal restrictions are related mainly to the arrival date of the vessel and the berthing date (Stahlbock & Voß, 2008; Bierwirth & Meisel, 2010; Cordeau et al., 2005).

Based on the spatial restrictions, the main types of BAP are: 1) BAP with discrete layout (DBAP), 2) BAP with hybrid layout (HBAP) and 3) BAP with continuous layout (CBAP). In DBAP the berthing area is divided into sections called berths, where only one vessel can be berthed in a certain period of time, respecting the restriction that the vessel's length cannot be greater than the berth length. In HBAP there is also a partitioned area in the berths, however, there is the possibility that a larger vessel occupies more than one berth, and also a berth can be occupied by more than one vessel. Finally, CBAP where the pier is not divided, i.e., vessels can berth at any position along the pier (Bierwirth & Meisel, 2010; Imai et al., 2005). Figure 2 shows a graphical representation of the berthing area for different layouts.

Based on the time constraints, the BAP is classified into: 1) Static Arrival and 2) Dynamic Arrival. In the statistic arrival, it is considered that all vessels are in the anchorage area ready to berth and therefore the date of arrival of all the vessels are equal and

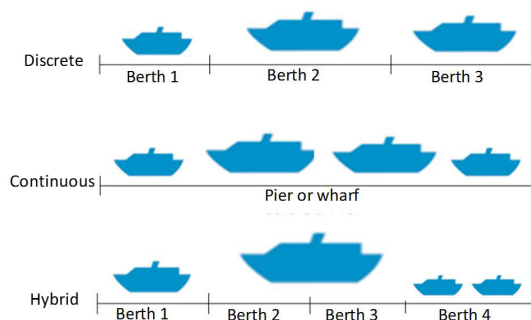


Figure 2. DBAP, CBAP and HBAP representation.

can be disregarded. In contrast, in the dynamic arrival, the vessels have different arrival dates over a planning horizon and therefore cannot berth before your arrival date at the port. The BAP with dynamic arrival represents more accurately the needs of the port administration (Bierwirth & Meisel, 2010; Imai et al., 2001).

Based on both spatial and time restrictions, it is expected as a BAP result a graph representing the sequence service of the vessels like the ones in Figure 3, respectively, a solution for the DBAPD, CBAP and HBAP. It is possible to observe that the space-time graph provides the planning of berthing where the horizontal axis represents the time horizon and the vertical axis represents the pier length. Furthermore, each rectangle represents a berthed vessel.

Thus, the width of this rectangle, parallel to Y axis, represents the total length of the vessel and the length of the rectangle, parallel to X axis, represents the total operation time of the vessel. It should be noted that if occur overlapping rectangles (vessels), then the solution is considered infeasible. Moreover, such vessels must meet the space limitations and specified arrival time (Umang et al., 2013; Bierwirth & Meisel, 2010; Stahlbock & Voß, 2008; Imai et al., 2005).

Different studies have highlighted different characteristics and specificities related to the BAP. They emphasize new mathematical formulations that consider both the discrete and continuous cases, in addition, they applied their models to real cases. The most cited papers about the BAP are shown next.

Guan et al. (2002), Park & Kim (2003), Guan & Cheung (2004) proposed solutions for BAP with the objective to minimize the vessels total time spent in the port. Lim (1998; 1999), Tong et al. (1999) and Goh & Lim (2000) proposed as an objective function the reduction of the total occupied space of the pier. Elwany et al. (2013) proposed a solution based on Simulated Annealing for CBAP with dynamic arrival and draft restrictions. Tang et al. (2009) also dealt with CBAP with dynamic arrival and they proposed two mathematical models and a Lagrangian relaxation heuristic.

Imai et al. (2005) and Chang et al. (2008) studied the CBAP considering that the loading time is dependent on the berthing position of the vessel in the continuous berth. Park & Kim (2002) and Kim & Moon (2003) solved the CBAP by Lagrangian relaxation. Moon (2000), Kim & Park (2004), Kim & Moon (2003) and Briano et al. (2005) proposed an objective function that minimizes the vessels' delays. Brown et al. (1994), Brown et al. (1997) and Lee & Chen (2008) proposed the possibility of moving the vessels from one berth to other berth during operation. Cordeau et al. (2005) studied the DBAP and the CBAP, proposing a Tabu Search metaheuristic to

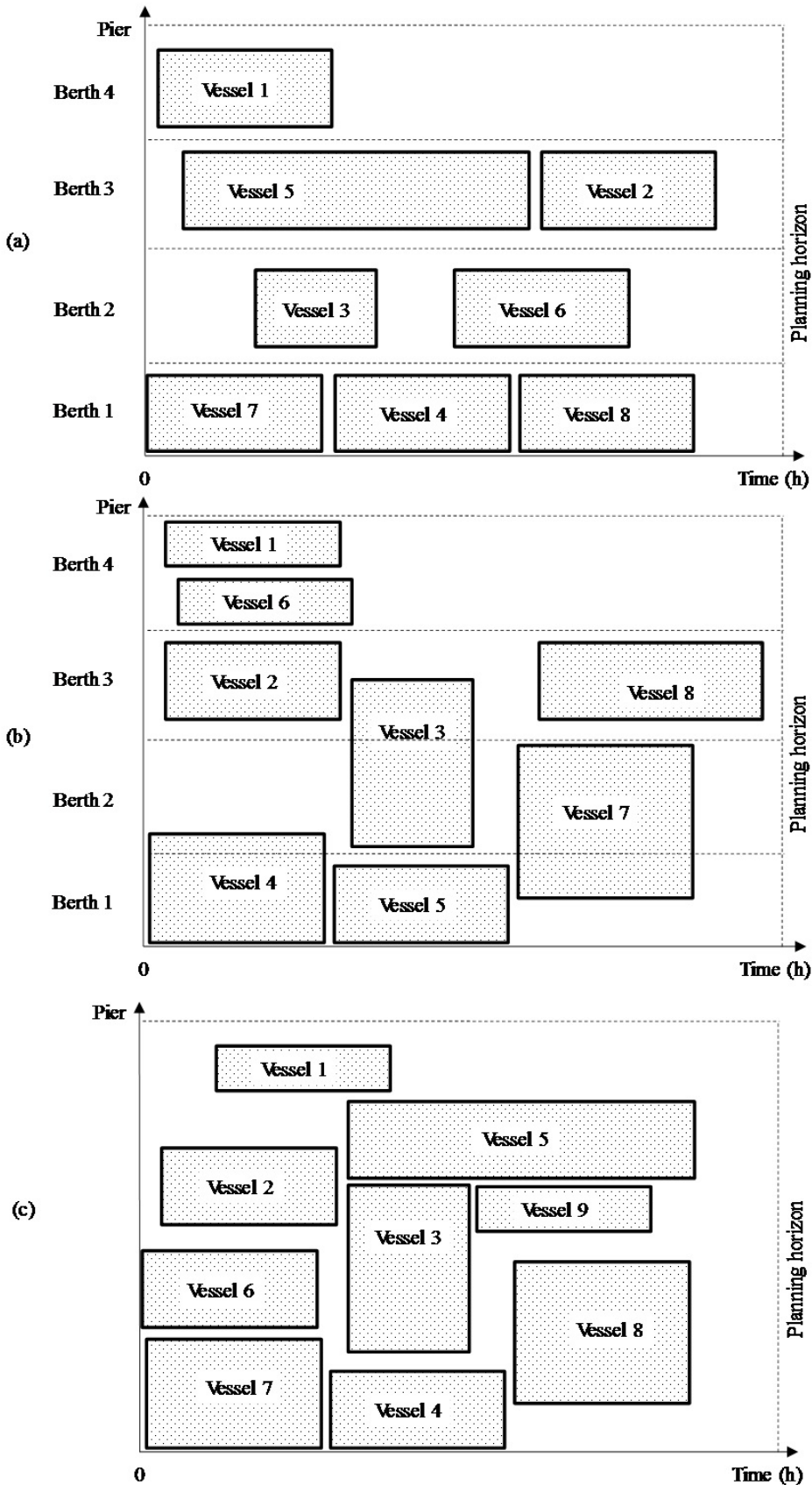


Figure 3. Service sequence of the vessels: (a) DBAP; (b) HBAP e (c) CBAP.

solve the two types of the BAP. Sabar et al. (2015) proposed an algorithm called Differential Evolution (DE) that is inspired by nature to solve the CBAP. Babazadeh et al. (2015) used the Particle Swarm Optimization (PSO) in their research, however, they applied to the case of CBAP and reached computation times smaller than a Genetic Algorithm (GA).

Lopes et al. (2011) implemented a Greedy Randomized Adaptive Search Procedure (GRASP) with Path-Relinking for the DBAP. Buhrkal et al. (2011) proposed an exact method for the DBAP applied to container terminals with better solutions than the ones found by Lopes et al. (2011), Imai et al. (2007), Imai et al. (2001) and Theofanis et al. (2007) analyzed the DBAP with static arrival and the objective function was the minimization of the total time that the vessel stayed at the port and the deviations between the sequence of arrival and berthing of vessels sequence. Hansen & Oguz (2003) have proposed a more compact mathematical model for the same problem. Imai et al. (2001), Monaco & Sammarra (2007) and Imai et al. (2003) studied the DBAP with dynamic arrival.

Zhou & Kang (2008) and Han et al. (2010) worked with DBAP with dynamic arrival and stochastic date of arrival and stochastic operation time. Mauri et al. (2010) proposed for the same problem an approach based on Column Generation. Mauri et al. (2008) proposed a solution using Population Training Algorithm / Linear Programming (ATP/PL). Imai et al. (2008) proposed a solution through Genetic Algorithm to minimize the number of vessels rejected for not being berthed before the maximum deadline. In the mathematical model proposed by Golias et al. (2007) the arrival date and the vessels operation time are considered stochastic variables. Hansen & Oguz (2003) and Hansen et al. (2008) proposed a Variable Neighborhood Search (VNS) metaheuristic which showed better results than those reported by Nishimura et al. (2001). Zhou et al. (2006) and Han et al. (2006) considered stochastic the arrival date and considered also a waiting time restriction for berthing. They used Genetic Algorithm to solve the problem.

Hu (2015) proposed a multi objective Genetic Algorithm for the DBAP. The two objectives considered were the preference for daytime operations and the other was the minimization of the workloads. Ting et al. (2014) proposed a PSO metaheuristic to the dynamic DBAP. They obtained small computational times when compared to other metaheuristics. Lin et al. (2014) proposed an Iterated Greedy (IG) metaheuristic to solve the dynamic DBAP minimizing the total service time. Zhen (2015) proposed a stochastic programming formulation for the DBAP that can handle any probability distributions that represent the vessels' operational times. Banos et al. (2016)

proposed a mathematical model and a Simulated Annealing metaheuristic to the DBAP that consider the operation time dependent on the carried cargo and also the berthing position. Vervloet & Rosa (2016) proposed a mathematical model for the DBAP considering similar commercial conditions as the ones established by a Charter Party.

Chen & Hsieh (1999) proposed a Mixed Integer Linear Programming (MILP) formulation for the HBAP considering dynamic arrivals. Moorthy & Teo (2006) also studied the same problem considering that the arrival date is stochastic. Nishimura et al. (2001) and Cheong et al. (2010) added to their models the draft restrictions and the dynamic arrival. Imai et al. (2007) considered that the operation time has dependence on the berthing position of the vessel. Dai et al. (2008) proposed a Simulated Annealing metaheuristic to solve the HBAP. Hoffarth & Voß (1994), Nishimura et al. (2001), Cordeau et al. (2005), Imai et al. (2007) and Cheong et al. (2007) studied the HBAP with different objective function and different restrictions. Umang et al. (2013) presented the HBAP with dynamic arrival applied to bulk ports in order to minimize the total service time of the vessels. They used exact and heuristic methods. Yan et al. (2015) proposed a space-time matrix approach to HBAP called Dynamic Vessel Arrivals and Flexible berth space BAP (DFBAP). Rodrigues et al. (2015) proposed a mathematical model applied to a CBAP with discontinuity of the pier seeking to minimize the total stay time of the vessels at the port.

Other approaches integrating the CBAP with cranes allocation were proposed by Bierwirth & Meisel (2009) who analyzed the integrated CBAP with the crane allocation problem for container terminals. Kim & Park (2004) used the GRASP metaheuristic to solve the CBAP considering the cranes which serve the vessels. Wang & Lim (2007) sought to minimize the penalty costs for rejected vessels and showed a heuristic that can solve instances with high amount of vessels. Ganji et al. (2010) proposed a model where vessels have a desired position for berthing and penalties are applied if this position is not respected.

After the literature review, as far as we know, it was not found any paper that dealt with the CBAP with the berth position of the vessels limited by the cargo that the vessel carries to specific segment of the pier. It was also not found any paper that dealt with the CBAP applied to ports specialized in the offshore logistics for the oil exploration logistics.

3 Proposed mathematical model

The proposed mathematical model for the CBAP with the positioning of the vessels limited by the cargos to specific segments of the pier formulated as

a Mixed Integer Linear Programming (MILP) model is based on the two-dimensional packaging problem. This model takes into account the one proposed by Guan & Cheung (2004) which considers the total extension of the pier without any operational restrictions, however, the proposed model in this paper considers several operational restrictions and seeks to minimize the vessels' total stay time at the port, calculated as the difference between the time of unberthing and the time of arrival of the vessel at the port. The vessels' arrival dates are known a priori.

Figure 4 shows a schematic drawing presenting the main parameters and decision variables of the model. In the space-time diagram, a vessel i is symbolized by a rectangle where one side is the length l_i and the other one is the operation time t_i . The vessel arrives at the port at the time e_i and berths at time b_i . So, it waits to berth $b_i - e_i$ time units and unberth at time

$u_i = b_i + t_i$. Its berthing position is p_i and occupies the pier from p_i to $p_i + l_i$.

The problem addressed in this paper assumes that the pier has specific segments for the operation of certain cargos. Thus, it is defined the concept of cargo operating zone that can be seen in Figure 1. To exemplify the cargo operating zone, Figure 5 shows two specific zones to operate certain types of cargos. Zone 1 starts at the beginning of the pier, α_1 , and finishes at β_1 . Zone 2 starts at α_2 and finishes at β_2 . It is important to highlight that there is an overlap of the Zone 1 with the Zone 2, i.e., the Zone 2 starts before the end of the Zone 1. This happens because in this overlapping section both types of cargo can be operated.

Considering the previous explanations, the mathematical model is presented in five parts: sets, parameters, decision variables, objective function and constraints.

Sets

N : Set of vessels;

C : Set of cargos operated at the port; and

F : Set of draft zones of the pier.

Parameters

A : Length of the pier;

T : Planning time horizon;

α_c : Initial position of the cargo operating zone that can handle cargo $c \in C$;

β_c : Final position of the cargo operating zone that can handle cargo $c \in C$;

γ_f : Initial position of the draft zone $f \in F$;

δ_f : Final position of the draft zone $f \in F$;

t_i : Operation time of vessel $i \in N$ expressed in time unit (hour);

l_i : Length of the vessel $i \in N$ expressed in metric unit (meter) including the clearance between vessels;

e_i : Arrival time of vessel $i \in N$;

d_i : Draft zone of vessel $i \in N$; and

c_i : Type of the cargo $c \in N$ that each vessel $i \in N$ operates.

Decision variables

b_i : Berthing time of vessel $i \in N$;

p_i : Berthing position of vessel $i \in N$;

u_i : Unberthing time of vessel $i \in N$;

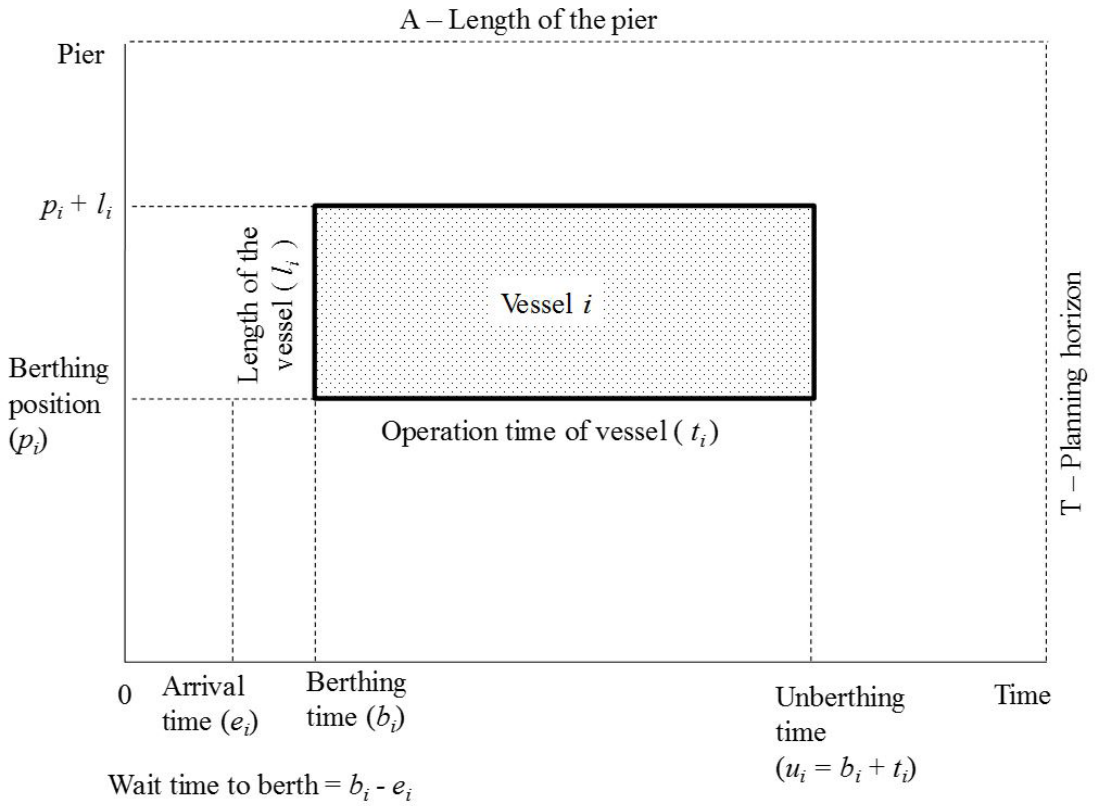


Figure 4. Schematic drawing showing the main parameters and decision variables.

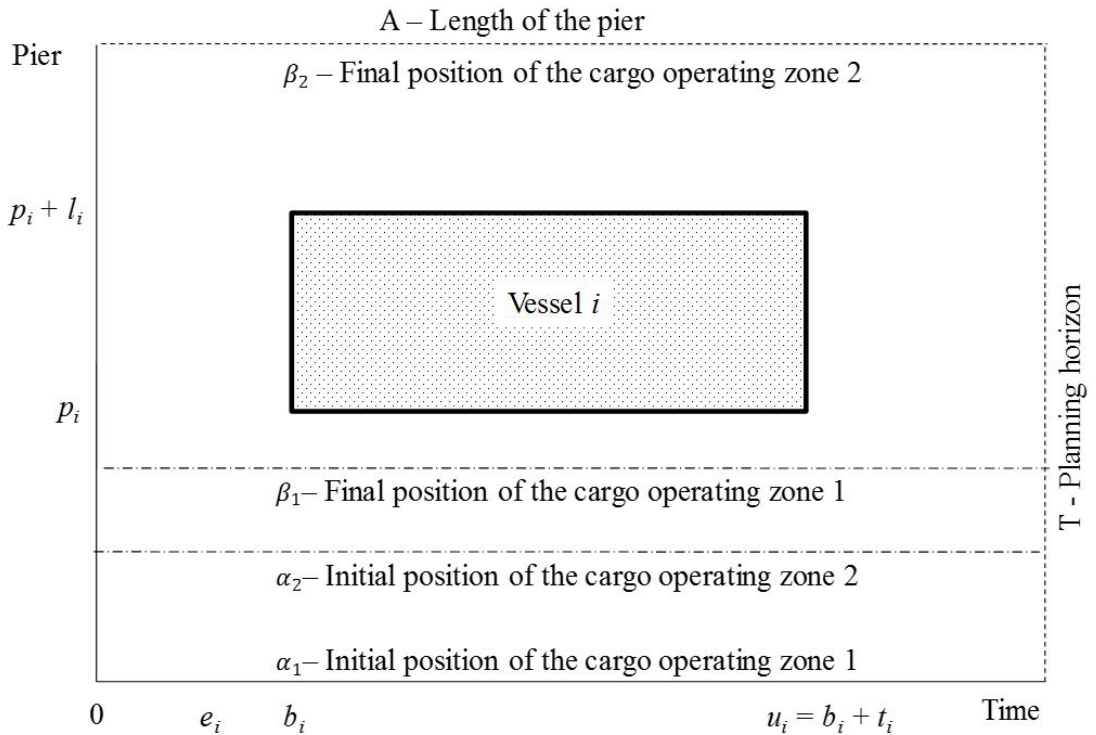


Figure 5. Schematic view of the cargo operating zone.

σ_{ij} : binary variable that assumes value equal to 1 if the vessel $j \in N$ berths totally to the right of the time occupied by vessel $i \in N$ in the space-time diagram, such that the two vessels do not overlap. Otherwise, it assumes value equal to 0; and

θ_{ij} : binary variable that assumes value equal to 1 if the vessel $j \in N$ berths totally above of the space occupied by vessel $i \in N$ in the space-time diagram, such that the two vessels do not overlap. Otherwise, it assumes value equal to 0.

After the presentation and explanation of the main sets, parameters and decision variables, the objective function and the constraints of the mathematical model are presented next.

Objective function

Minimize

$$\sum_{i \in N} (u_i - e_i) \quad (1)$$

Constraints

$$b_j - b_i - t_i - (\theta_{ij} - 1)T \geq 0 \quad \forall i, j \in N, i \neq j \quad (2)$$

$$p_j - p_i - l_i - (\sigma_{ij} - 1)A \geq 0 \quad \forall i, j \in N, i \neq j \quad (3)$$

$$\sigma_{ij} + \sigma_{ji} + \theta_{ij} + \theta_{ji} \geq 1 \quad \forall i, j \in N, i \neq j \quad (4)$$

$$b_i + t_i = u_i \quad \forall i \in N \quad (5)$$

$$e_i \leq b_i \leq (T - t_i) \quad \forall i \in N \quad (6)$$

$$\gamma_{d_i} \leq p_i \leq (\delta_{d_i} - l_i) \quad \forall i \in N \quad (7)$$

$$\alpha_{c_i} \leq p_i \leq (\beta_{c_i} - l_i) \quad \forall i \in N \quad (8)$$

$$\sigma_{ij} \in \{0, 1\} \quad \forall i, j \in N \quad (9)$$

$$\theta_{ij} \in \{0, 1\} \quad \forall i, j \in N \quad (10)$$

$$u_i, v_i, c_i \in \mathfrak{R}^+ \quad \forall i \in N \quad (11)$$

The objective function, Equation 1, represents the total time that the vessels stayed in the port which must be minimized. This time is calculated by subtracting the unberthing time by the arrival time at the port.

Constraints 2 ensure that there is no overlap of vessels in the time dimension in the space-time diagram. Constraints 3 ensure that there is no overlap of vessels in the length of the pier in the same diagram. Constraints 4 require that at least one of these variables is equal to 1, thereby ensuring the berthing of the vessel and that there is no overlap of vessels in both dimensions: time and length, thus, the rectangles representing the vessels do not overlap each other.

To illustrate Constraints (4), Figure 6a shows an overlap of vessel i and j . In this case, because i is not

fully to the left of j then, $\sigma_{ij} = 0$ and j is not fully to the left of i , then, $\sigma_{ji} = 0$. Still in the same situation, as i is not fully below of j then $\theta_{ij} = 0$ and also j is not fully below of i , then $\theta_{ji} = 0$. Thus, $\sigma_{ij} + \sigma_{ji} + \theta_{ij} + \theta_{ji} = 0$ and therefore this solution is not feasible. However, for the situation shown in Figure 6b, vessel i is fully to the left of j then $\sigma_{ij} = 1$, and j is not fully to the left of i , then $\sigma_{ji} = 0$. Still in the same situation as i is fully above of j then $\theta_{ij} = 1$, and j is not totally above of i , then $\theta_{ji} = 0$. Thus, $\sigma_{ij} + \sigma_{ji} + \theta_{ij} + \theta_{ji} = 2 \geq 1$ and therefore the solution 6 (b) is feasible.

Constraints (5) ensure the vessel's unberthing time calculation that is the sum of the vessel the berthing time and the vessel operation time. Constraints 6 ensure that the vessel is berthed within the planning

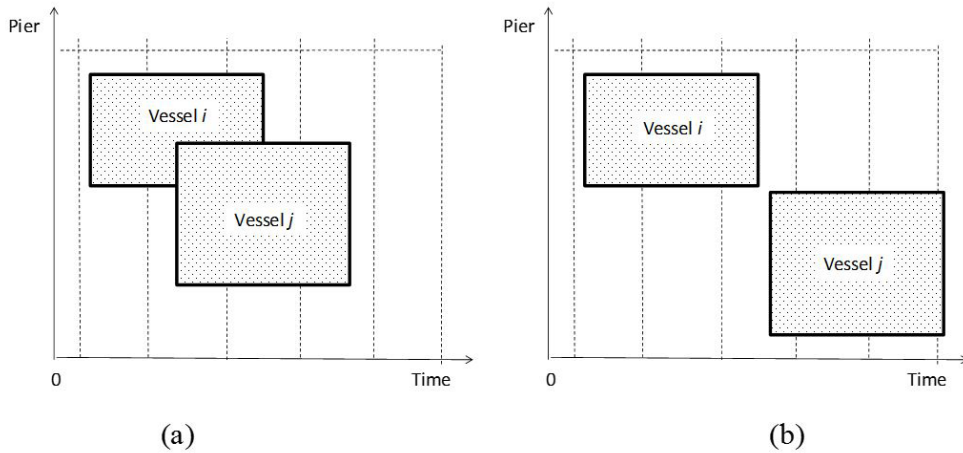


Figure 6. Constraints 4 (a) overlapping vessels; (b) no overlapping vessels.

horizon, i.e., after the vessel’s arrival time and before the end of the planning horizon. Constraints 7 require that the vessel $i \in N$ berths in a position between the initial position and the final position of a draft zone that is compatible with the draft zone of the vessel $i \in N$. Constraints 8 ensure that the vessel is berthed at a position that is compatible with its cargo, i.e., their berthing position is a position between the initial position and the final position of a cargo operating zone that operates the type of cargo $c \in C$ that the vessel is carrying. Each vessel $i \in N$ carries one type of cargo $c_i \in C$. Constraints 9-11 define the scope of the decision variables.

4 Results and analysis

The CPVV’s planning team gave real data from the port operations to evaluate the proposed mathematical model. The data refer to the first six months of 2013 and besides the port planning team plans for each week, it was decided to test the model in a bigger planning horizon, ranging from 14 to 21 days, aiming to show the model’s capability to solve bigger instances and to provide the planning team a tool that facilitates the port planning for a longer planning horizon.

Thus, eight instances were developed to test the model, they can be seen in Table 1. For the real instances from 1 to 6, it was raised the vessels’ estimated date of arrival, operating times and pier characteristics. For those instances, the pier has 320.0m (length currently available). It was always chosen the two busiest consecutive weeks in each month of the reporting period. The last two instances, 7 and 8, were created to analyze a possible growth of the total extension of the pier, from 320.0m to 440.0m, and also a possible growth of the total number of served vessels. For all eight instances shown in Table 1, the

Table 1. Test instances for the propose model.

Instance	Length of the Pier (m)	Planning Horizon (h)	Number of vessels
1	320	360	83
2	320	408	97
3	320	456	102
4	320	480	113
5	320	528	123
6	320	456	101
7	440	528	123
8	440	576	147

cargo operating limitations were considered along the pier as shown in Figure 1.

The Instance 1 represents the first two weeks of January when 83 vessels were berthed and so it was decided to establish a 15-day planning horizon (360 hours). The Instance 2 represents the second and third weeks of February, when 97 vessels were berthed. Since the number of vessels in Instance 2 is higher than in Instance 1, it was established a planning horizon a larger than the one adopted in Instance 1 in order to allow vessels with a late arrival time to be berthed and to be operated within the planning horizon. Thus, the Instance 2 had its planning horizon defined as 17 days (408 hours).

Instance 3 represents the last two weeks of March, when 102 vessels were berthed. It was established a period of 19 days (456 hours) for its planning horizon. Instance 4 represents the first two weeks of April, when 123 vessels were berthed. It was established a period of 20 days (480 hours) for its planning horizon. Instance 5 represents the second and third weeks of May, when 113 vessels were berthed. It was

established a period of 22 days (528 hours) for its planning horizon. Instance 6 represents the last two weeks of June when 101 vessels were berthed. It was established a period of 19 days (456 hours) for its planning horizon.

Instance 7 was created to analyze a project carried by CPVV to expand its pier in 120.0 m to operate general cargo. Instance 5 was created evaluate a larger number of vessels to be berthed changing only the length of the pier. Instance 8 was based on Instance 7 considering an increasing of 20% in the number of vessels to be berthed due to the new platforms that will work at the pre-salt exploration in the Espirito Santo basin. Thus, Instance 8 considered 147 vessels to be berthed and a 24-day planning horizon (576 hours).

The tests were performed on an Intel *i7* computer with 16GB of RAM memory running the solver CPLEX 12.6 and no runtime limit was established for each instance. The results were compared with the CPVV's planning process, that is done manually. The CPVV's planning process is named in this paper as Real Case. Table 2 shows the results found with CPLEX and with CPVV team. It also presents the reduction in the waiting time to berth obtained by CPLEX when compared to the Real Case.

In Table 2, the Instance column shows the name of the instance, the GAP column shows the gap after running the CPLEX and the Runtime column shows the CPLEX runtime in seconds. CPLEX - Wait time to berth column shows the waiting time to berth, in hours, calculated as the difference between the berthing time and the time of arrival of the vessel. Column Real Case - CPVV - Wait time to berth shows the wait time to berth registered by the CPVV's planning team. The last two columns, Reduction - Wait time to berth (hours) and Reduction - Wait time to berth (percentage), show the difference between the waiting time to berth in hours and in percentage found by CPLEX when compared to the Real Case. For Instances 7 and 8, the last three columns are not

fulfilled because these instances refer to test instances. Thus, there is no way to compare the results obtained by CPLEX with the Real Case.

CPLEX solved all instances optimally with GAP equal to zero, in a reasonable runtime, given the size of the instances. For Instance 1, with 14 days and 80 vessels, CPLEX reached the optimal solution in a runtime of 26.41 seconds. Instance 8, the biggest one, with 14 days of arrival of vessels and 147 vessels had the biggest runtime, 7439.56 seconds (2 hours and 4 minutes). Still, it can be considered that 2 hours and 4 minutes to carry out the optimal planning to berth vessels along 24 days is an acceptable time in practical terms.

Continuing the analysis of Table 2, it can be noted that even with an increase of vessels in the different instances, the wait time to berth remains very close when one instance is compared to another. This situation happens because there is a single port operator in the port and this operator is also the owner of the offshore platforms. Therefore, the logistics team of this operator already defines the voyages of the vessels in regular periods. Another fact that should be emphasized was the larger difference between the values found by CPLEX and those of the Real Case. This difference can be seen in the graph of Figure 7. Because of this situation, it was decided to return to the port to present the results and to better analyze the CPVV's results previously provided for the study.

Thus, after a meeting with the port planning team, it was detected that many vessels were waiting in the anchorage area and did not berth due to the lack of cargo at the port to be loaded. Based on this information, for each vessel in this situation, it was determined the total amount of hours they were waiting in the anchorage area waiting to berth because of the lack of cargo. Thus, the values of the Real Case were calculated removing these times and they can be seen in Table 3, which has exactly the same structure of Table 2.

Table 2. CPLEX's Results and Real Case's Results.

Instance	CPLEX (Mathematical Model)			Real Case - CPVV	Reduction CPLEX x Real Case	
	GAP (%)	Runtime (s)	Wait time to berth (h)	Wait time to berth (h)	Wait time to berth (h)	Wait time to berth (%)
1	0.00	26.41	5.89	7.53	-1.64	22%
2	0.00	30.03	12.32	21.32	-9.00	42%
3	0.00	997.14	14.23	19.17	-4.94	26%
4	0.00	2,718.92	15.49	27.46	-11.97	44%
5	0.00	4,345.17	16.08	45.13	-29.05	64%
6	0.00	993.45	13.56	17.32	-3.76	22%
7	0.00	6,345.78	10.34	-	-	-
8	0.00	7,439.56	15.45	-	-	-

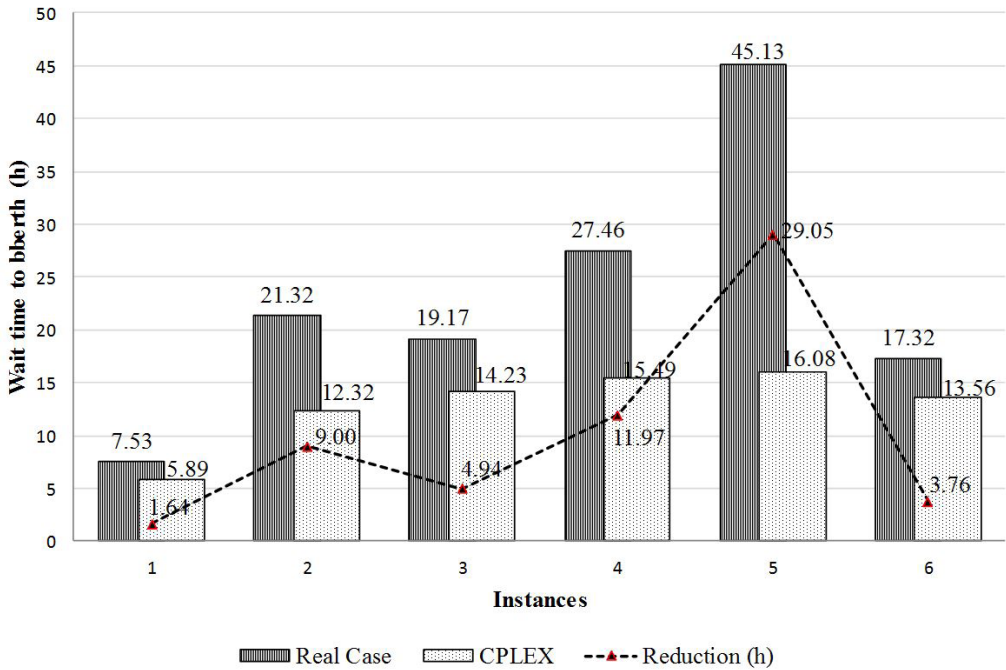


Figure 7. Comparison of the wait time to berth (CPLEX × Real Case).

Table 3. Comparison between CPLEX’s results and CPVV’s results without the wait time to berth due to the lack of cargo.

Instance	CPLEX (Mathematical Model)			Real Case - CPVV	Reduction CPLEX × Real Case	
	GAP (%)	Runtime (s)	Wait time to berth (h)	Wait time to berth (h)	Wait time to berth (h)	Wait time to berth (%)
1	0.00	26.41	5.89	6.43	-0.54	8.4%
2	0.00	30.03	12.32	13.35	-1.03	7.7%
3	0.00	997.14	14.23	14.54	-0.31	2.1%
4	0.00	2,718.92	15.49	16.10	-0.61	3.8%
5	0.00	4,345.17	16.08	18.12	-2.04	11.3%
6	0.00	993.45	13.56	13.67	-0.11	0.8%
7	0.00	6,345.78	10.34	-	-	-
8	0.00	7,439.56	15.45	-	-	-

Table 3 shows the reductions achieved by CPLEX after removing the wait time to berth at the anchorage area due to the lack of cargo. It can be noted that there was a decrease in the difference of the values found by CPLEX and the ones found by CPVV, Real Case. However, CPLEX still produces effective reductions in the wait time to berth, reaching a minimum reduction of 0.8% (0.11 hours) in Instance 6, and a maximum reduction of 11.3% (2.04 hours) in Instance 5. This can be seen analyzing the graph of Figure 8 that shows that CPLEX reached better results for the wait time to berth in all instances.

CPLEX has gained 4.64 hours in the wait time to berth in the 12 weeks analyzed when compared

with the actual manual plan of CPVV. It can be said that the port would reduce 9.28 hours to berth more vessels in the 24 weeks of one semester. It can also be made a projection knowing that the year has 48 weeks that the port would have gain 18.56 hours per year to berth more vessels. The average price of the vessel that berth at CPVV is about US\$ 50,000.00 per day which gives a cost of US\$ 2,085.00 per hour. Then, the owner of CPVV, that is the same owner of the offshore platforms, would have saved approximately US\$ 38,697.60 per year if the planning team had used the proposed mathematical model in this paper. This gain in time would also reflect in the number of trips done by the vessels, because since they stay

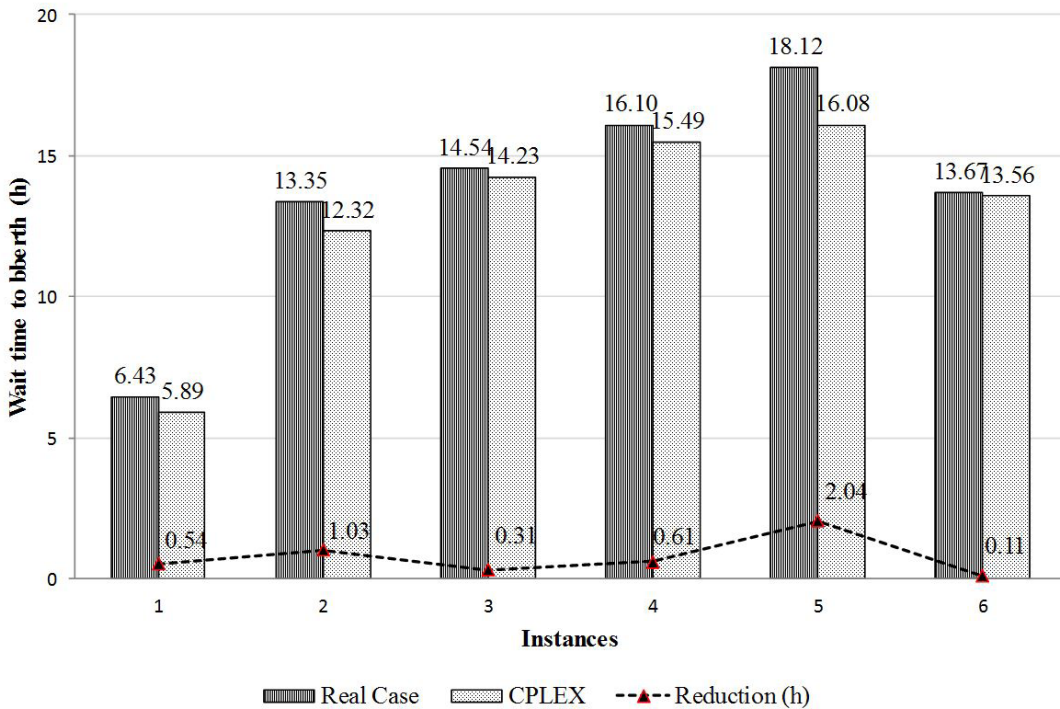


Figure 8. Comparison of the Wait time to berth (CPLEX × Real Case) without the wait time to berth due to the lack of cargo.

less time at the port, then they can make more trips and so the owner of the offshore platforms can rent less vessels, making the offshore logistics cheaper.

Analyzing Instance 7, Table 3, it can be noted that increasing the pier length from 320.0m to 440.0m led to a higher CPLEX runtime, 6.345,78s versus 4,345.17s, i.e., an increase of 46.0%. But CPLEX runtime is still an acceptable value considering that it is a 22-day port operation plan. CPLEX reached an optimal solution and the result of the waiting time to berth was reduced from 16.08 hours in Instance 5 to 10.34 hours in Instance 7. This was expected, since Instance 7 has a greater availability of pier to berth. It is important to emphasize that Instance 7 cannot be compared with the real case (it is a test instance).

CPLEX reached the optimal solution for Instance 8 although it took a long runtime due to the growth in the number of vessels to be berthed, the increase of the planning horizon and the expansion of the pier’s length. In terms of the mathematical model, there is an increase in the area where the rectangles, that represent the vessels, can be placed leading to an increase in the complexity of Instance 8. Thus, despite the increase in the runtime, CPLEX was able to solve the Instance 8 in 2 hours and 4 minutes, which is still an acceptable time to make a 24-days port operation plan.

Aiming to contribute to a more effective planning of the port, an application was developed in Matlab language to read the results from CPLEX and present these results in a graphical form. A partial view of the output of this application can be seen in Figure 9.

It is known through interviews with the port planners that the oil and gas sector has a very dynamic operation where several unplanned vessels require berthing at the port in order to take or bring a not scheduled cargo by the oil platforms. With the developed application, that shows the result graphically, Figure 9, the port planner can verify what are the periods that the pier has no scheduled vessel, i.e., when it is empty and so can receive new vessels. Thus, this graphical view of the scheduled vessels, may help the port planner to adjust the berthing of the unplanned vessel without changing the optimal planning calculated by CPLEX or, at least, reducing as little as possible the impact in the CPLEX’s optimal plan.

Finally, it can be seen by the results achieved by CPLEX running the proposed model that the results are better than the ones achieved by the actual manual process to plan the port operation. Thus, it can be said that the mathematical model is a reliable tool that can be used for planning the berthing of vessels for all the ports which are characterized by the condition that certain cargos can only be operated in certain

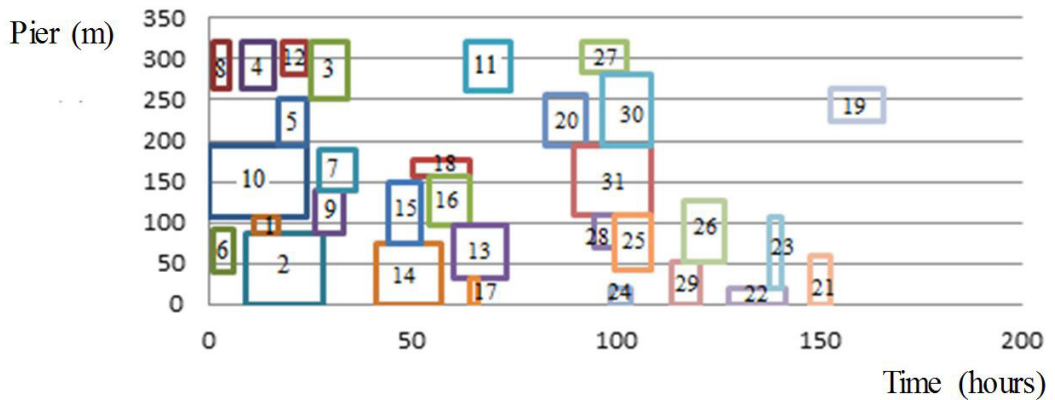


Figure 9. Matlab application with a partial graphical view of CPLEX results.

segments of the pier and that it may eventually bring possible operational and financial gains to the ports.

5 Conclusions

This paper proposed a mathematical model for the CBAP with cargo operating limitations along the pier to meet the operational characteristic of specialized ports to support the offshore oil platforms where certain cargos can only be operated in certain berthing positions along the pier. It is important to mention that the proposed model in this paper can be applied to the planning of the vessels' berthing sequence in any port that has operating restrictions of certain cargos in specific segments of the pier.

CPLEX was used to solve the mathematical model and real data from CPVV were obtained to test the model. The solutions were compared with the actual data from CPVV. Analyzing the results achieved by CPLEX running the proposed mathematical model, it can be noted that CPLEX found good solutions when compared to the actual results achieved by CPVV planning team. These gains were observed in all the real instances tested. Summing all these reductions achieved by CPLEX, the vessels stayed less time at the port, reducing 4.64 hours in the 12 weeks analyzed. Thus, it would be possible to the vessels to perform more trips to the platforms with a possible annual gain for the CPVV of US\$ 38,697.60.

A test instance was created to evaluate the model considering a future expansion of the pier. Another test instance was created considering a forecast of growth of vessels' berthing considering the new demands from the pre-salt exploration. CPLEX found good solutions for both of them proving to be also a good solution tool for the new expansion scenarios of the port.

Thus, CPLEX solving the mathematical model achieves good solutions for planning the vessels' berthing, including a 24-day planning horizon, bigger than the usual, which is 7 days. The mathematical model is also useful as an operational procedure for the port as it respects operating constraints like the limitations to operate some cargos in some segments of the pier and also the vessel's draft.

Finally, it is suggested as a future work, the development of a metaheuristic for the proposed mathematical model to solve larger instances. Also, it is suggested to consider uncertainty in the mathematical model, treating the operating times and arrival times at the port as stochastic.

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References

- Babazadeh, A., Shahbandi, M. G., Ganji, S. R. S., & Joharianzadeh, M. (2015). A PSO algorithm for continuous berth allocation problem. *International Journal of Shipping and Transport Logistics*, 7(4), 479-493. <http://dx.doi.org/10.1504/IJSTL.2015.069687>.
- Banos, R. S., Rosa, R. A., Mauri, G. R., & Ribeiro, G. M. (2016). Modelo matemático e meta-heurística Simulated Annealing para o problema de alocação de berços com múltiplas cargas. *Transportes*, 24(1), 51-62. <http://doi.org/10.14295/transportes.v24i1.980>
- Bierwirth, C., & Meisel, F. (2010). A survey of berth allocation and quay crane scheduling problems in container terminals. *European Journal of Operational*

- Research, 202(3), 615-627. <http://dx.doi.org/10.1016/j.ejor.2009.05.031>.
- Bierwirth, C., & Meisel, F. (2009). A fast heuristic for quay crane scheduling with interference constraints. *Journal of Scheduling*, 12(4), 345-360. <http://dx.doi.org/10.1007/s10951-009-0105-0>.
- Briano, C., Briano, E., & Bruzzone, A. G. (2005). Models for support maritime logistics: a case study for improving terminal planning. In Y. Merkuryev, R. Zobel & E. Kerckhoffs (Eds.), *Proceedings of the 19th European Conference on Modeling and Simulation – ECMS* (pp. 199-203). Riga: European Council for Modelling and Simulation.
- Brown, G. G., Lawphongpanich, S., & Thurman, K. P. (1994). Optimizing ship berthing. *Naval Research Logistics*, 41(1), 1-15. [http://dx.doi.org/10.1002/1520-6750\(199402\)41:1<1::AID-NAV3220410102>3.0.CO;2-L](http://dx.doi.org/10.1002/1520-6750(199402)41:1<1::AID-NAV3220410102>3.0.CO;2-L).
- Brown, G. G., Cormican, K. J., Lawphongpanich, S., & Widdis, D. (1997). Optimizing submarine berthing with a persistence incentive. *Naval Research Logistics*, 44(4), 301-318. [http://dx.doi.org/10.1002/\(SICI\)1520-6750\(199706\)44:4<301::AID-NAV2>3.0.CO;2-A](http://dx.doi.org/10.1002/(SICI)1520-6750(199706)44:4<301::AID-NAV2>3.0.CO;2-A).
- Buhrkal, K., Zuglian, S., Ropke, S., Larsen, J., & Lusby, R. (2011). Models for the discrete berth allocation problem: a computational comparison. *Transportation Research Part E, Logistics and Transportation Review*, 47(4), 461-473. <http://dx.doi.org/10.1016/j.tre.2010.11.016>.
- Chang, D., Yan, W., Chen, C.-H., & Jiang, Z. (2008). A berth allocation strategy using heuristics algorithm and simulation optimization. *International Journal of Computer Applications in Technology*, 32(4), 272-281. <http://dx.doi.org/10.1504/IJCAT.2008.021382>.
- Chen, C.-Y., & Hsieh, T.-W. (1999). A time-space network model for the berth allocation problem. In *19th IFIP TC7 Conference on System Modeling and Optimization*. Cambridge: Kluwer Academic Publishers.
- Cheong, C., Tan, K., Liu, D., & Lin, C. (2010). Multi-objective and prioritized berth allocation in container ports. *Annals of Operations Research*, 180(1), 63-103. <http://dx.doi.org/10.1007/s10479-008-0493-0>.
- Cheong, C. Y., Lin, C. J., Tan, K. C., & Liu, D. K. (2007). A multi-objective evolutionary algorithm for berth allocation in a container port. In *Congress on Evolutionary Computation 2007 (CEC 2007)* (pp. 927-934). Singapore: IEEE. <http://dx.doi.org/10.1109/CEC.2007.4424569>.
- Cordeau, J.-F., Laporte, G., Legato, P., & Moccia, L. (2005). Models and tabu search heuristics for the berth-allocation problem. *Transportation Science*, 39(4), 526-538. <http://dx.doi.org/10.1287/trsc.1050.0120>.
- Companhia Portuária de Vila Velha – CPVV. (2015). Vila Velha: CPVV. Recuperado em 17 de março de 2015, de <http://www.cpvv.com.br/cpvv/>.
- Dai, J., Lin, W., Moorthy, R., & Teo, C.-P. (2008). Berth allocation planning optimization in container terminals. In C. S. Tang, C. P. Teo & K. K. Wei (Eds.), *Supply chain analysis: a handbook on the interaction of information, system and optimization* (pp. 69-105). New York: Springer.
- Elwany, M. H., Ali, I., & Abouelseoud, Y. (2013). A heuristics-based solution to the continuous berth allocation and crane assignment problem. *Alexandria Engineering Journal*, 52(4), 671-677. <http://dx.doi.org/10.1016/j.aej.2013.09.001>.
- Ganji, S. R. S., Babazadeh, A., & Arabshahi, N. (2010). Analysis of the continuous berth allocation problem in container ports using a genetic algorithm. *Journal of Marine Science and Technology*, 15(4), 408-416. <http://dx.doi.org/10.1007/s00773-010-0095-9>.
- Goh, K. S., & Lim, A. (2000). Combining various algorithms to solve the ship berthing problem. In *Proceedings of the 12th IEEE International Conference on Tools with Artificial Intelligence – ICTAI'00* (pp. 370-373). Los Alamitos: IEEE Computer Society.
- Golias, M., Boile, M., & Theofanis, S. (2007). The stochastic berth allocation problem. In *Proceedings of the International Conference on Transport Science and Technology – TRANSTEC 2007* (pp. 52-66). Prague: Czech Technical University.
- Guan, Y., & Cheung, R. K. (2004). The berth allocation problem: models and solution methods. *OR-Spektrum*, 26(1), 75-92. <http://dx.doi.org/10.1007/s00291-003-0140-8>.
- Guan, Y., Xiao, W.-Q., Cheung, R. K., & Li, C.-L. (2002). A multiprocessor task scheduling model for berth allocation: heuristic and worst-case analysis. *Operations Research Letters*, 30(5), 343-350. [http://dx.doi.org/10.1016/S0167-6377\(02\)00147-5](http://dx.doi.org/10.1016/S0167-6377(02)00147-5).
- Han, M., Li, P., & Sun, J. (2006). The algorithm for berth scheduling problem by the hybrid optimization strategy GASA. In *Proceedings of the 9th International Conference on Control, Automation, Robotics and Vision – ICARCV'06* (pp. 1-4). Washington: IEEE Computer Society.
- Han, X., Lu, Z., & Xi, L. (2010). A proactive approach for simultaneous berth and quay crane scheduling problem with stochastic arrival and handling time. *European Journal of Operational Research*, 207(3), 1327-1340. <http://dx.doi.org/10.1016/j.ejor.2010.07.018>.
- Hansen, P., & Oguz, C. (2003). A note on formulations of static and dynamic berth allocation problems. *Les Cahiers du GERAD*, 30, 1-17.
- Hansen, P., Oguz, C., & Mladenovic, N. (2008). Variable neighborhood search for minimum cost berth allocation. *European Journal of Operational Research*, 191(3), 636-649. <http://dx.doi.org/10.1016/j.ejor.2006.12.057>.
- Hoffarth, L., & Voß, S. (1994). Berth allocation in a container terminal: development of a decision support system (In German). In H. Dyckhoff, U. Derigs, M. Salomon & H. C. Tijms (Eds.), *Operations research proceedings 1993* (pp. 89-95). Berlin: Springer.

- Hu, Z. H. (2015). Multi-objective genetic algorithm for berth allocation problem considering daytime preference. *Computers & Industrial Engineering*, 89, 2-14. <http://dx.doi.org/10.1016/j.cie.2015.04.035>.
- Imai, A., Nishimura, E., & Papadimitriou, S. (2001). The dynamic berth allocation problem for a container port. *Transportation Research Part B: Methodological*, 35(4), 401-417. [http://dx.doi.org/10.1016/S0191-2615\(99\)00057-0](http://dx.doi.org/10.1016/S0191-2615(99)00057-0).
- Imai, A., Nishimura, E., & Papadimitriou, S. (2003). Berth allocation with service priority. *Transportation Research Part B: Methodological*, 37(5), 437-457. [http://dx.doi.org/10.1016/S0191-2615\(02\)00023-1](http://dx.doi.org/10.1016/S0191-2615(02)00023-1).
- Imai, A., Sun, X., Nishimura, E., & Papadimitriou, S. (2005). Berth allocation in a container port: using a continuous location space approach. *Transportation Research Part B: Methodological*, 39(3), 199-221. <http://dx.doi.org/10.1016/j.trb.2004.04.004>.
- Imai, A., Nishimura, E., Hattori, M., & Papadimitriou, S. (2007). Berth allocation at indented berths for mega-containerships. *European Journal of Operational Research*, 179(2), 579-593. <http://dx.doi.org/10.1016/j.ejor.2006.03.034>.
- Imai, A., Nishimura, E., & Papadimitriou, S. (2008). Berthing ships at a multi-user container terminal with a limited quay capacity. *Transportation Research Part E, Logistics and Transportation Review*, 44(1), 136-151. <http://dx.doi.org/10.1016/j.tre.2006.05.002>.
- Kim, K. H., & Park, Y. M. (2004). A crane scheduling method for port container terminals. *European Journal of Operational Research*, 156(3), 752-768. [http://dx.doi.org/10.1016/S0377-2217\(03\)00133-4](http://dx.doi.org/10.1016/S0377-2217(03)00133-4).
- Kim, K. H., & Moon, K. C. (2003). Berth scheduling by simulated annealing. *Transportation Research Part B: Methodological*, 37(6), 541-560. [http://dx.doi.org/10.1016/S0191-2615\(02\)00027-9](http://dx.doi.org/10.1016/S0191-2615(02)00027-9).
- Lee, Y., & Chen, C.-Y. (2008). An optimization heuristic for the berth scheduling problem. *European Journal of Operational Research*, 196(2), 500-508. <http://dx.doi.org/10.1016/j.ejor.2008.03.021>.
- Lim, A. (1998). The berth planning problem. *Operations Research Letters*, 22(2), 105-110. [http://dx.doi.org/10.1016/S0167-6377\(98\)00010-8](http://dx.doi.org/10.1016/S0167-6377(98)00010-8).
- Lim, A. (1999). An effective ship berthing algorithm. In D. Thomas (Ed.), *Proceedings of the 16th International Joint Conference on Artificial Intelligence – IJCAI-99-vol-1* (pp. 594-599). San Francisco: Morgan Kaufmann Publishers.
- Lin, S. W., Ying, K. C., & Wan, S. Y. (2014). Minimizing the total service time of discrete dynamic berth allocation problem by an iterated greedy heuristic. *The Scientific World Journal*, 2014, 218925. PMID:25295295.
- Lopes, A. T., Schulz, V. M. L. & Mauri, G. R. (2011). GRASP com *path relinking* para o problema de alocação de berços. *PODes - Pesquisa Operacional para o Desenvolvimento*, 3(3), 218-229.
- Mauri, G. R., Oliveira, A. C. M., & Lorena, L. A. N. (2008). A hybrid column generation approach for the berth allocation problem. In J. van Hemert & C. Cotta (Eds.), *8th European Conference on Evolutionary Computation in Combinatorial Optimisation – EvoCOP 2008* (Lecture Notes in Computer Science, 4972, pp. 110-122). Berlin: Springer.
- Mauri, G. R., Oliveira, A. C. M., & Lorena, L. A. N. (2010). Resolução do Problema de alocação de berços através de uma técnica de geração de colunas. *Pesquisa Operacional*, 30(3), 547-562. <http://dx.doi.org/10.1590/S0101-74382010000300003>.
- Meersmans, P. J. M., & Dekker, R. (2001). *Operations research supports container handling* (Econometric Institute Report 234). Rotterdam: Erasmus University Rotterdam.
- Monaco, M. F. & Sammarra, M. (2007). The berth allocation problem: a strong formulation solved by a Lagrangean approach. *Transportation Science*, 41(2), 265-280.
- Moon, K. (2000). *A mathematical model and a heuristic algorithm for berth planning* (Doctoral thesis). Pusan: Pusan National University.
- Moorthy, R., & Teo, C.-P. (2006). Berth management in container terminal: the template design problem. *OR-Spektrum*, 28(4), 495-518. <http://dx.doi.org/10.1007/s00291-006-0036-5>.
- Nishimura, E., Imai, A., & Papadimitriou, S. (2001). Berth allocation planning in the public berth system by genetic algorithms. *European Journal of Operational Research*, 131(2), 282-292. [http://dx.doi.org/10.1016/S0377-2217\(00\)00128-4](http://dx.doi.org/10.1016/S0377-2217(00)00128-4).
- Park, K. T., & Kim, K. H. (2002). Berth scheduling for container terminals by using a sub-gradient optimization technique. *The Journal of the Operational Research Society*, 53(9), 1054-1062. <http://dx.doi.org/10.1057/palgrave.jors.2601412>.
- Park, Y. M., & Kim, K. H. (2003). A scheduling method for berth and quay cranes. *OR-Spektrum*, 25(1), 1-23. <http://dx.doi.org/10.1007/s00291-002-0109-z>.
- Rashidi, H., & Tsang, E. (2013). Novel constraints satisfaction models for optimization problems in container terminals. *Applied Mathematical Modelling*, 37(6), 3601-3634. <http://dx.doi.org/10.1016/j.apm.2012.07.042>.
- Rodrigues, P. H., Rosa, R. A., Arpini, B. P., Bissoli, A. L., & Ribeiro, G. M. (2015). Planejamento de atracação de navios por meio de um modelo matemático para o problema de alocação de berços contínuos com descontinuidade do cais. *Transportes*, 23(1), 37-46. <http://doi.org/10.4237/transportes.v23i1.861>. TRANSPORTES.
- Sabar, N. R., Chong, S. Y., & Kendall, G. (2015). A hybrid differential evolution algorithm: game theory for the berth allocation problem. In *Proceedings of the 18th*

- Asia Pacific Symposium on Intelligent and Evolutionary Systems* (vol. 2, pp. 77-87). Berlin: Springer.
- Stahlbock, R., & Voß, S. (2008). Operations research at container terminals: a literature update. *OR-Spektrum*, 30(1), 1-52. <http://dx.doi.org/10.1007/s00291-007-0100-9>.
- Steenken, D., Voß, S., & Stahlbock, R. (2004). Container terminal operation and operations research: a classification and literature review. *OR-Spektrum*, 26(1), 3-49. <http://dx.doi.org/10.1007/s00291-003-0157-z>.
- Tang, L., Li, S., & Liub, J. (2009). Dynamically scheduling ships to multiple continuous berth spaces in an iron and steel complex. *International Transactions in Operational Research*, 16(1), 87-107. <http://dx.doi.org/10.1111/j.1475-3995.2009.00662.x>.
- Theofanis, S., Boile, M., & Golias, M. (2007). An optimization based genetic algorithm heuristic for the berth allocation problem. In *IEEE Congress on Evolutionary Computation 2007 – CEC 2007* (4439-4445). Washington: IEEE Computer Society.
- Ting, C., Wu, K., & Chou, H. (2014). Particle swarm optimization algorithm for the berth allocation problem. *Expert Systems with Applications*, 41(4), 1543-1550. <http://dx.doi.org/10.1016/j.eswa.2013.08.051>.
- Tong, C. J., Lau, H. C., & Lim, A. (1999). Ant colony optimization for the ship berthing problems. In P. S. Thiagarajan & R. Yap (Eds.), *5th Asian Computing Science Conference – ASIAN'99* (Lecture Notes in Computer Science, 1742, pp. 359-370). Berlin: Springer.
- Umang, N., Bierlaire, M., & Vacca, I. (2013). Exact and heuristic methods to solve the berth allocation problem in bulk ports. *Transportation Research Part E, Logistics and Transportation Review*, 54, 14-31. <http://dx.doi.org/10.1016/j.tre.2013.03.003>.
- Vacca, I., Bierlaire, M., & Salani, M. (2007). Optimization at container terminals: status, trends and perspectives. In *Proceedings of the Swiss Transport Research Conference – STRC* (pp. 1-21). Ascona: Monte Verità.
- Vervloet, H. L. L., & Rosa, R. A. (2016). Modelo matemático para planejamento da atracação de navios sob o contrato de charter party. *Transportes*, 24(1), 84-94. <http://doi.org/10.14295/transportes.v24i1.976>.
- Vis, I. F. A., & Koster, R. (2003). Transshipment of containers at a container terminal: an overview. *European Journal of Operational Research*, 147(1), 1-16. [http://dx.doi.org/10.1016/S0377-2217\(02\)00293-X](http://dx.doi.org/10.1016/S0377-2217(02)00293-X).
- Wang, F., & Lim, A. (2007). A stochastic beam search for the berth allocation problem. *Decision Support Systems*, 42(4), 2186-2196. <http://dx.doi.org/10.1016/j.dss.2006.06.008>.
- Yan, S., Lu, C.-C., Hsieh, J.-H., & Lin, H.-C. (2015). A network flow model for the dynamic and flexible berth allocation problem. *Computers & Industrial Engineering*, 81, 65-77. <http://dx.doi.org/10.1016/j.cie.2014.12.028>.
- Zhen, L. (2015). Tactical berth allocation under uncertainty. *European Journal of Operational Research*, 247(3), 928-944. <http://dx.doi.org/10.1016/j.ejor.2015.05.079>.
- Zhou, P., Kang, H., & Lin, L. (2006). A dynamic berth allocation model based on stochastic consideration. In *Proceedings of the 6th World Congress on Intelligent Control and Automation – WCICA 2006* (vol. 2, pp. 7297-7301). Washington: IEEE Computer Society.
- Zhou, P., & Kang, H. (2008). Study on berth and quay-crane allocation under stochastic environments in container terminal. *Systems Engineering - Theory & Practice*, 28(1), 161-169.