

LEWIS CARROLL, EDUCATION AND THE TEACHING OF GEOMETRY IN VICTORIAN ENGLAND

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Abstract

Research partly motivated by Lewis Carroll's *Euclid and his modern rivals* (1879) portuguese translation, this paper presents some hermeneutical remarks taken as necessary to understand the context in which such book was produced. The paper focuses particularly on education, in general, and on the teaching of mathematics and Geometry in victorian England.

Key-words: Lewis Carroll, Euclid and his modern rivals, history of education, mathematics education, geometry.

LEWIS CARROLL, A EDUCAÇÃO E O ENSINO DE GEOMETRIA NA INGLATERRA VITORIANA

Resumo

Parte da pesquisa motivada pela tradução para o português do livro *Euclides e seus rivais modernos*, publicado por Lewis Carroll em 1879, este artigo se inscreve numa série de estudos que visam a um exame hermenêutico dessa obra. São discutidos temas relacionados com a educação, a educação matemática e o ensino de Geometria na Inglaterra vitoriana.

Palavras-chave: Lewis Carroll, Euclides e seus rivais modernos, história da educação, educação matemática, geometria.

LEWIS CARROLL, LA EDUCACIÓN Y EL ENSINODE GEOMETRÍA EN LA INGLATERRA VICTORIANA

Resumen

Parte de la investigación motivada por la traducción al portugués del libro *Euclides y sus enemigos modernos*, publicado por Lewis Carroll en 1879, este artículo se inscribe en una serie de estudios que tienen por objetivo hacer un examen hermenéutico de la obra. Son aquí discutidos temas relacionados como la educación, la educación matemática y la enseñanza de Geometría en la Inglaterra victoriana.

Palabras-clave: Lewis Carroll, Euclides y sus enemigos modernos, historia de la educación, educación matemática, geometría.

LEWIS CARROLL, L'ÉDUCATION ET L'ENSEIGNEMENT DE GÉOMÉTRIE EN L'ANGLETERRE VICTORIENNE

Résumé

Faisant partie de la recherche motivée par la traduction en portugais du livre *Euclide et ses rivaux modernes*, publié par Lewis Carrol en 1879, cet article s'inscrit dans une série d'études dont le but est un examen herméneutique de cette oeuvre. On va tout particulièrement aborder des thèmes comme l'éducation, l'éducation mathématique et l'enseignement de la Géométrie en Angleterre victorienne.

Mots-clé: Lewis Carroll, Euclide et ses rivaux modernes, histoire de l'éducation, éducation mathématiques, géométrie.

This study comprises a series of essays sharing a common theme: general education, the teaching of mathematics and, more specifically, school geometry in Victorian England, and aims to develop a hermeneutic analysis of one of Lewis Carroll's books: *Euclid and his modern rivals*, written in 1879.

Lewis Carroll is best known for his writing of fiction and for two books to which his name is irrevocably linked: *Alice in wonderland* and *Through the looking glass, and what Alice found there*¹. It is known, however, that Carroll was a lover of mathematics and wrote many essays of didactic nature to his students, including a series of twelve texts on *Euclid's elements*, published between 1860 and 1888, thorough which he attempted to make the first two books of Euclid more accessible to learners (Cohen, 1998).

Carroll's *Euclid and his modern rivals*, whose preparation at times took between six and eight hours of daily work (Cohen, 1998), can be seen as the coming of age of short stories and plays that the author wrote to be performed by puppets and which entertained his younger siblings (Fisher, 2000). His writing style - Carroll writes as a sequence of acts and theatrical scenes - can be attributed to the author's acknowledgement of the importance of theatrical arts, which he admired (Gardner, 2002).

Euclid and his modern rivals is in short a play in four acts which introduces us to a teacher involved in the stressful job of correcting tests. Being exhausted, he falls asleep and in his dreams, he talks to the ghost of Euclid. From the encounter between Euclid and Minos², the exhausted teacher, a project of analyzing the geometry books of the time is developed for the purpose of an educational reform. These reform ideas popped out in increasingly significant numbers at the time, all with the intent of replacing *Euclid's elements*, which had been so far a par excellence text for the teaching of Geometry in English schools:

The *Elements*, consisting of 13 books, had ruled alone for twenty centuries, until the beginning of the Victorian era, when the market was flooded with books of anti-Euclidean orientation. The attempt to challenge the ideas of Euclid often resulted in misrepresentation, and the variety of ways to present the Euclidean geometry ended up with a great confusion in the study of this subject. Charles³ [...] felt challenged to stop this horde of Euclid corruptors, and did so by revitalizing the real euclidean geometry. (Cohen, 1998, p. 450)

In *Euclid and his modern rivals*, the latter, i.e., the new authors whose opinions countered some aspect of Euclid's systematization by offering different approaches to teaching in their manuals are gradually and thoroughly judged. The dialogues between characters make it clear that all rivals "show fallacies or inconsistencies in their work, and

¹ There have been different titles for the translation of the book. In this study we have opted for *Through the Looking Glass and What Alice Found There*, title of the annotated edition published by Zahar in 2002 consulted for this work.

² Rhadamanthus, Rhadamanthos or Rhadamanthys and Minos are characters who, along with Euclid's ghost, appear in the first two scenes of the first act of *Euclid and his modern rivals*. The former are two of the three infernal judges acting under the rule of Hades. In Greek mythology, Rhadamanthus, Minos and Eacos judged the souls that came to hell; in Carroll's work, these characters judge students and authors of Euclid's rival manuals, which Carroll believes to be an infernal task. The character of Minos argues with Euclid up to the end of the book.

³ Charles Lutwidge Dodgson is Lewis Carroll's real name, with which he preferred to sign his nonfiction works.

all succumb to logic and higher arrangement in *Euclid's elements*" (Cohen, 1998, p. 452). This surreal situation allows the reader to follow Minos and Euclid's judgments of the works of Legendre, Cooley, Cuthbertson, Henrici, Wilson, Pierce, Willock, Chauvenet, Loomis, Morell, Reynolds and Wright, and, by doing so, familiarize him with the different ways through which these authors proposed their systematization of Geometry.

Throughout the book, Carroll emphasizes the importance of Euclidean geometry as the basis for our perception of the world, in which parallel lines never meet. The literary narrative serves as a support to and aims at understanding mathematical ideas. The author also interferes in the characters' attitudes by not neglecting a style that he defines a scenic structure, and by depicting his characters so as to warn the reader of how pathetic some of the arguments of the modern rivals and the demonstrations proposed are.

According to Cohen (1998), the result was as expected: the dramatic narrative attracted and held the reader. The April 12, 1879 issue of *Vanity Fair* deemed it quite nice and defined it as a wonderful book by the results presented and the Infectious humor with which the seriousness of the matter was addressed. The *English mechanic* May 2 issue of the same year stated that the author had triumphed in the attempt to prove that, so far, no other published book could be compared to the immortal Euclid's *Elements* as an introduction to geometry for beginners⁴. The first edition sold out in six weeks and the second, which was the basis for the translation and supports and motivates this text, was published six years later, to which text Carroll added comments he considered important.

However, considering the positive criticism to the text, its relative oblivion in the course of time seems strange. There are at least two possible answers to this: the first considers the hypothesis that *Euclid and his modern rivals* has been compared to the *Alice* books (Cohen, 1998), a misunderstanding of some critics who did not understand that, although the author is the same, the purpose and target audience of the book were distinct. Another possible reason for the relative neglect of the work is the fact that Carroll was too dependent on the scientific knowledge developed and brought by the reader, without realizing that not everyone had the same ability and affinity with the subject matter he had decided to discuss and had helped to develop over the years. In fact, some sections of *Euclid and his modern rivals* show an "absurdly difficult subject for most people - but which, in spite of not being easy, is at least intelligent and fun" (Cohen, 1998, p. 451).

These are, in short, the main features of Carroll's book, whose translation was used as a starting point and provided continuous support to the hermeneutic of this study. A hermeneutic examination requires not only that internal webs of the text - its composition, the structure of its chapters, contents, ways these contents are dealt with - be analyzed, but also that aspects of the production and circulation of the book - public welcome, the

⁴ These perceptions on *Euclid and his modern rivals* have been cut out from Morton Cohen and Anita Gandolfo's notes in *Lewis Carroll and the House of Macmillan* (p. 154), a book containing the correspondence exchanged between Carroll and his publisher.

space and time in which it was elaborated - be investigated. John Thompson, an English sociologist whose work on culture and mass media communication has its roots in Paul Ricoeur's philosophy, points to some routes for the interpretation of symbolic forms, among which books, which have been used in mathematics education and which he calls depth hermeneutics⁵.

Depth hermeneutics comes into play due to one of its guidelines: in order to analyze a piece of literary work hermeneutically, knowledge of its production and circulation surroundings is required. These surroundings, in the case of *Euclid and his modern rivals*, are set in Victorian England. An understanding of England in the second half of the 19th century, its educational policies, its book trade and schools, for instance, provides us a background that feeds the interpretation of the effort made by Carroll in favor of Euclid's work; it has also encouraged us to translate the text into Portuguese.

This article, therefore, aims to investigate the context in which *Euclid and his modern rivals* is set, highlighting, in particular, education, the teaching of mathematics and the teaching of geometry in England in the second half of the 19th century.

England: Lewis Carroll's time and space

To understand Carroll's *Euclid and his modern rivals*, attach significance to it, and interpret it, we deemed it necessary to examine Carroll's stand as a teacher and writer of several books, including math textbooks, and seek clues on what might have led him to write them. A survey of some aspects of the Victorian Age allowed an approximation to the reasons of the author's motivations, his intentions, directions, lifestyle, time and surroundings.

The Victorian era is the period between 1837 and 1901 when queen Victoria ruled over England; this period was characterized by euphoria arising from the industrial growth that changed the English lifestyle, hitherto based on agriculture, into a modern urban economy based on trade and industry (Morais, 2004). In the period following the Industrial Revolution, England became the most industrialized nation in the world, being known as the world's workshop; among many inventions of the time - the steamboat, telegraph, automobile, electricity - railroads affected everyday life and the English scene most intensely (Flores; Vasconcelos, 2000). All these changes contaminated the literary output

⁵ Depth hermeneutics is a theoretical and methodological reference frame modeled on the concept of symbolic forms whose guidelines were given by Thompson (1995), notably in *Ideology and modern culture*. It is a critical social theory in the era of mass media. In short, depth hermeneutics is a way of analyzing, interpreting and understanding symbolic forms that involve text and context hermeneutics through a feedback process. The text, according to Paul Ricoeur, is everything which has been written, but this is a rather limited understanding, since it refers only to written texts. In a more general grasp, the text is comprehension that results from reading. Context will be taken here as everything that has to do with the text, such as the dynamics of its development and adequacy, and the spaces and agents involved in its production and circulation.

of this period. The train was responsible for an increase in the circulation of publications, whose issues also grew accordingly. The 19th century

was characterized by the rapid development of the sciences: physics led to the apogee of the mechanistic (Cartesian) view of the universe; Biology, in its evolutionary course, proposed important problems to philosophical thought, as Charles Darwin (1809-1882), with his treatise on the origin of the species, challenged the idea of man that had been accepted for centuries. Genetics was first introduced by Gregor Mendel (1822-1884) through his laws of segregation and the independence of hereditary characteristics. (Morais, 2004, p. 10)

Family evenings were ideal for reading moments which had a strong moral appeal, as “when they [the English] needed advice, they resorted to literature” (Morais, 2004, p. 36). These evenings frequently included the reading of the Bible, and Genesis passages were thought of as true events. No wonder the *Origin of Species* caused so many conflicts and tensions in this scenario, once what he was being stated by scientific authority was against the Scriptures. Interestingly, this view that contradicted Church dogmas does not seem to have shaken Carroll's beliefs: he expanded his library by adding 19 works of Darwin and his critics, created a kind of contrary wise Darwinism in the fifth chapter of *Sylvie and Bruno* and devised a game called Lanrick, where the winner was the most evolved being of his species (Cohen, 1998). Carroll was an atypical anglican reverend⁶ who did not bother with the theories of Darwin, played with and entertained children, and advocated the theater.

The Victorian era was, therefore, a period of great changes spread by an ever increasing number of publications and readers. Towards what they considered modernity, which had its apogee with the Great Exhibition of 1851⁷, Victorians experienced a conflicting coexistence between the proposed changes and the preservation of traditions. This dichotomy was reflected in education, originating several changes, among which those related to the teaching of geometry.

Education in Victorian England

In England, differently from what had been happening in France since the Revolution, the idea of education as being a civil right and an obligation of the State was totally alien. Chastenet (undated) reported that the English family was responsible for offering their children an education which would suit their gender and social class, i.e., the education of

⁶ Carroll was ordained a deacon in the Anglican Church in 1861, with plans to become a priest, like his father.

⁷ The characteristic spirit of confusion and exaggeration of the 19th century is well represented by the Great Exhibition of London, opened on May 1, 1851, organized by Prince Albert, Queen Victoria's consort. Several exhibitions had already been held in various European countries, especially France, but these had only had a national character, with limited goals. The Great Exhibition in London was the first for which all the powers of the world were invited to show all sorts of products.

the bourgeoisie and upper classes was supported by parental money. One could only resort to charitable initiatives in the face of a lack of financial resources. This state of affairs contributed to turning England into the country with the worst education in Europe (Howson, 2010). Moreover, it seemed rather pointless to the English that the so-called lower classes should acquire knowledge that they judged unnecessary to this segment of the population and which could give them ideas that would strengthen their perception of social class differences, a viewpoint which was fueled by the very fear that some members of the wealthiest classes had for French Jacobinism.

In a context which denied knowledge and any schooling to many, Carroll was privileged: the Dodgsons epitomized that part of Victorian society known as upper middle class which, “in the absence of aristocratic titles, inheritances, land or other property, could only aspire some sort of rise by developing the spirit - and so they did” (Cohen, 1998, p. 24).

After the Industrial Revolution, the increase in population, especially in the English capital, the change of classes brought about by those who were beginning to open their own businesses, the need for a workforce to maintain machines, and the belief in progress, began to gradually impose changes in the English school system. The first governmental investment in public education occurred in 1833, when the Parliament passed a £ 20,000 loan for the construction of school buildings amounting to £ 30,000 in 1839. However, these data, which allegedly claimed that a major change was under way, are relative: Chastenet (undated) states that in the same year, the same amount was spent on expanding the stables at Buckingham Palace.

At the time, education was restricted to the teaching of reading and writing, with some rare schools where basic arithmetic was taught. In 1858, the commission assembled under the supervision of the duke of Newcastle to analyze the steps that could be taken to expand cheap elementary education⁸ to all classes found out that, of the 1,824 weekly public schools, only 69.3% taught arithmetic, 0.6% taught mechanics, 0.8% taught algebra and 0.8% taught Euclid. Consequently, a system of awards by results was presented to Parliament in 1862, through which each school would receive up to four shillings per student and an additional eight shillings if the student passed his examinations in reading, writing and arithmetic.

Some changes, which were implemented gradually, could be perceived in the school curricula. According to Howsam, Stray et al (2007), the progress of subjects was parallel to social status, starting with books to educate young gentlemen in classical languages and mathematics, and later addressing issues of science, also including middle class students as their target audience. Books dedicated to the education of workers' children would only

⁸ The word education was not used to refer to schooling in the 18th and 19th centuries. The term instruction appears instead, as e.g. in the instruction reform in revolutionary France, where all educational strategies were broad and aimed to form a full citizen, regardless of social class.

appear after the 1870's and 1880's. History and Geography, as well as foreign languages and literature, were added to the curriculum as a result of the proliferation of topics to be addressed in teaching at the end of the 19th century. Around 1871, the math curriculum was divided into six levels containing no more than four operations, including only basic divisions, the monetary system, common weights and measures, ratios and decimal measures, which were supposed to meet the worker's needs (Howson, 2010).

The textbook⁹ as it is known today - including questions, answers, appropriate vocabulary, notes and motivations - first appeared in England around 1830 (Howsam, Stray et al, 2007). The distribution of textbooks increased both in quantity and speed around 1840, due to postal services¹⁰ and railroads which reached some schools in rural areas:

Earlier innovations had included lithography, which spread rapidly in the first two decades of the century, and facilitated the flexible printing of 'exotic' (i.e. non-roman, including Greek and Hebrew) and mathematical texts. It also made possible cheap, short-run, in-house printing of teaching material. (Howsam; Stray et al, 2007, p. 3)

During the 19th century there were many changes in the textbooks and school setting of England. Chastenet (undated) describes this process: the country had a disorganized primary instruction system, which was ensured for poorer classes by two associations - the National Schools, supported by the anglican church, and the British Schools, of non-conformist orientation¹¹; together, the two associations attended to more than 18,000 students. There were also schools that were subsidized by parishes or large landowners, whose wives accumulated various occupations and only taught in their spare time. In major cities charities organized schools that became known as ragamuffin schools (Howsam, Stray et al, 2007), which struggled to attract children who lived on the streets in poor neighborhoods. In the countryside, the wives or daughters of squires¹² and ministers took the role of teachers in schools which were so poor that often there were no seats, although biblical teachings were always present. Generally speaking, teaching was almost

⁹ The authors state that prior to the textbook there used to be the so-called text book, which usually consisted of a book containing an important text, often of religious character, to be discussed in the context of education. The word text-book only appeared around 1900, which would eventually evolve to textbook, showing an expansion and accommodation of gender.

¹⁰ The authors refer to the penny post system introduced by the British Post Office, through which normal letters could be posted for one penny.

¹¹ During the Protestant Reformation the Anabaptists - the name means to be rebaptized in Greek and characterized a more radical group who believed that baptism was only validated when performed in adults with full acknowledgement of the individual - advocated the separation between church and state, that is, their goal was to achieve independence from any worldly power including, of course, the power of the king, to depend only on Christ. Thereafter England became a land where numerous religious groups faced the State and its determination to establish a church that was under its control. These groups were called non-conformist.

¹² The word squire, which in the Middle Ages referred to a skilled nobleman, was later associated with the leader of an English village, often a justice of the peace or a member of parliament.

never free, since parents had to contribute in theory with a third of the costs and, as attendance to classes was not mandatory, many children left school benches in search of work. According to Chastenet (undated), 33.7% of men and 49.5% of married women were unable to sign their names in 1839.

Between 1780 and 1870, elementary schools were maintained by religious or individual donations, without any government interference. Only in 1867 did the Reform Act, supported by politicians such as Robert Lowe, begin to gradually change this situation. The Education Act of 1870 made education compulsory, though not necessarily free, for children up to 11 years of age. But the government neglected its enforcement with respect to poor children - a very small number of them was assisted by this law - as well as many other children living in areas unaware of the enactment of this law. This mandatory act would be implemented at a slow pace, inasmuch as the government did not provide free education for all until 1918, and any child above 11 years of age, upon having reached some standards, could be excused from school (Howson, 2010).

A significant change had its origins in the experiences of Andrew Bell (1753-1832) who, having worked as a chaplain of the Anglican Church for the Indian Army, got to know the work of the Madras Orphan Asylum, an institution founded by the East India Company to cater to soldiers' children which hired tutors to care for small groups of children. From this model Bell created the mutual instruction method (Cambi, 1999), subsequently improved by Joseph Lancaster¹³ (1778-1838), an educator and a member of the Quaker movement, which became known as monitorial system.

The monitorial system consisted in grouping students by skill level: the teacher taught only the most advanced children, and later some of the students from that group would be selected to monitor the others. The monitors were responsible for the education of their younger or less advanced¹⁴ classmates, whereas the teacher acted as a supervisor, evaluator and disciplinarian (Lesage, 1999). Actually, this was a cheap system since each group could include up to 500 students under the control of a single teacher.

There was a different scenario for wealthy families, whose children studied with tutors, and for the middle class, who sent their children to day schools. Secondary studies

¹³ Lancaster, who opened the first fully free school in England, was responsible for other significant changes in education: he proposed a basic mathematics curriculum containing the four operations, integers and fractions, and the use of imperial units and proportion. He also started training teachers in their schools in London and Somerset in the 1830's. The National Society, which opposed the teacher training system, soon acknowledged its need and devised its own program for this purpose (Howson, 2010).

¹⁴ The system became widespread in England, France, Switzerland, Italy, Spain, Russia and the USA, where Lancaster himself introduced it in 1818 (Cambi, 1999). This system was also quite known by Brazilian schools and was used until the establishment of the school groups, initially adopted by the State of São Paulo in the late 19th century." The monitorial system was a response to the Industrial Revolution: like the mechanistic model of industrial output, it allowed maximum use of a scarce resource at that time: the teacher" (Morais, 2004, p. 56).

were done at a Grammar School, a Private School and a Public School¹⁵ under the supervision of a preceptor, according to parents' will or resources. Carroll studied at two Grammar Schools, first in Richmond, then Rugby. At the former he had classes in Latin, Greek, Religion, Mathematics, English Literature and French; at the latter, classes in Classical Literature, History, the Sacred Scripture, French and Mathematics¹⁶ prevailed.

In the 1820s, due to the scarcity of good Grammar Schools and the limited capacity of public schools, Proprietary Schools, organized and maintained by a committee of owners, were created. These schools, aimed at the children of the growing middle class, responded to the demand of the Armed Forces, which at the time wanted their newly integrated officers to have knowledge of mathematics, as well as master commercial and industrial techniques (Howson, 2010).

After the end of secondary education¹⁷, some young men entered the armed forces, and others the university. In England there were only two universities which a gentleman could join: Oxford and Cambridge (Chastenet, [undated]). It is worth remembering that Carroll, having been educated at a Grammar School, went on to Oxford, an institution in which students always wore caps and gowns, and where those who came from affluent families were differentiated from others by wearing golden tassels. Carroll, who belonged to the other group, wore a black one (Cohen, 1998).

Both universities had not changed much since the Middle Ages and maintained a certain status. Cardinal Wolsey, King Henry VIII and the founders of Christ Church had spent some time at Christ Church; Queen Elizabeth I had stayed there, and king Charles I had taken refuge there during the Civil War; also, the English Parliament had met there in the 1644 crisis. In the rows of portraits of those who preceded him, Carroll “would identify several eminent people, including priests, viceroys, ministers and leaders from different spheres of society” (Cohen, 1998, p. 55).

Once admitted, the young men were given a bedroom and a workroom of their own. The only three obligations - come in every evening before eleven o'clock, not to miss the

¹⁵ Grammar Schools, or Endowed Grammar Schools, were usually maintained by a particular group of people, almost always linked to the Church, offering educational programs directed to specific interests. Howson (2010) reports that most of these schools followed a classical curriculum, with little room for mathematics or modern languages. Private schools were run by ministers or clergymen. Howson (2010) states that these schools were of very variable quality. Chastenet (undated) mentions the fact that students would leave school with some knowledge of Latin, French, History and Mathematics. In the early 19th century there were only nine Public Schools in England. According to Howson (2010) these schools, which were originally intended for the education of poor students, soon began to be almost entirely attended by the children of the wealthy. The syllabuses were based on Latin, Greek and mathematics, and the schools also maintained ties with the Church, and their principal was usually a doctor in Theology or Canon Law (Chastenet, [undated]).

¹⁶ Carroll's weekly routine included attending approximately twenty lessons per week, each lasting from one to two hours. Of these twenty classes, sixteen covered the classics, the Sacred Scripture and History, two the French language, and the remaining two, Mathematics (Cohen, 1998).

¹⁷ It is noteworthy that the words primary and secondary were not used in Victorian England to refer to education but, according to Howson (2010), are useful for a retrospective analysis.

Sunday religious ceremonies held in the chapel, and have dinner several times a week in the hall - were not followed by all Oxford gentlemen: festivals, hunts, dances and gambling were common. Cohen (1998) states that most of the students - the descendants of noblemen who had been raised practicing horse riding, shooting and hunting - went to Oxford just to kill time.

Young men with intellectual interests and ambitions made up a minority who knew how to take advantage of the extraordinary amenities that the colleges offered: the possibility of isolation, extensive libraries and competent professors. Both at Oxford and Cambridge, most students were content with the minimum syllabus, which included Latin, Greek and History¹⁸, but some developed a deeper knowledge of special subjects and acquired great literary culture. "In fact, within the two old universities, not just scholars, jurists, economists, theologians and politicians, but also poets, artists and philosophers were shaped" (Chastenet, [undated], p. 155)¹⁹.

Studies lasted three years interspersed with long vacations at the end of which students received a bachelor's degree in Arts. After getting their degrees, some students still remained at the institution for another two years, without any further examinations, but at a considerably higher price, and were then awarded a Master of Arts degree. Still others who, like Carroll, chose to stay there for their whole lives, were called fellows and needed to remain celibate, in accordance with a rule established prior to the Reformation (Cohen, 1998).

English education, in addition to class distinctions discussed here, also showed gender differences. In the 19th century, due to a belief in the intellectual inferiority of women, these were only fit to a kind of education that reinforced their fragile nature, with emphasis on embroidery and activities in home organization (Mitchell, 2004). Girls studied until around sixteen and their education was mainly intended to turn them into good wives and housekeepers. Because of this, the Benevolent Institution for Housekeepers was founded in 1843 (Morais, 2004), which opened Queens College in London for girls over 12 years of age with an innovative curriculum: students could choose between talks and lectures on Modern Languages, Mechanics, Geography, Geology, English Grammar, English Literature, Latin, Botany, Chemistry, Philosophy and Political Economy. In 1910 there were about "1,000 women occupying university chairs at Oxford and Cambridge, though they were not entitled to getting any degrees" (Morais, 2004, p. 66).

¹⁸ The Cambridge curriculum also included the study of Mathematical Sciences (Chastenet, [undated]).

¹⁹ Howson (2010) states that, in the 1830s, more than half of Oxford and Cambridge graduates became clergymen due to the connection of these universities with the Church of England. Upon comparing this text with that of Chastenet, which covers the period between 1837 and 1851, one may conclude that this situation changed along the years.

Carroll and the teaching of Mathematics and Geometry in the Victorian era

Carroll, a middle-class student, was educated at good schools at a time when England did not even have schools for all. His attraction and penchant for mathematics was noticed and encouraged by some of his teachers²⁰.

Euclidean geometry had long been regarded as the ideal tool to teach reasoning and logical thinking. "As a science supported by absolute truths" (Howsam, Stray et al, 2007, p. 4), the study of geometry was in agreement with classical teaching and offered adequate preparation for students who pretended to Oxford or Cambridge. Augustus De Morgan²¹, like many of his contemporaries, saw mathematics as a means of developing the faculty of reasoning and a method by which the student could, having acquired basic knowledge on it, arrive at conclusions through logical arguments (Howson, 2010). It is worth mentioning, however, that most students did not go beyond the first and second books of *Euclid's elements* (Price, 1994). Those who sought a career in public administration or the army also could not avoid studying geometry, because the belief that its study helped to develop logical and organized thought made it indispensable.

Conversely, others opposed to the efforts required to learn Geometry by the Euclidean method. They questioned its rigorous logic and whether it was the most suitable book for beginners. Mathematician James Joseph Sylvester, a professor at the Royal Military Academy at Woolwich, declared in his inaugural address as president of the British Association for the Advancement of Science (Baas): "The early study of Euclid made me a hater of geometry" (Sylvester apud Price, 1994, p. 23).

In the 1860s England, two committees, similar to that of 1858²², were created to investigate the quality of English education in secondary studies: the Clarendon Commission report (1864) on Public Schools, which showed a disastrous scenario in which the Mathematics taught was not good enough to enable students to enter the Royal Military Academy at Woolwich, and the Taunton Commission (1868), whose report showed significant differences in the quantity and quality of mathematics taught at different Endowed Schools (Howson, 2010). However, these reports were less impacting than the poor mathematical knowledge results of a competition for students aged between 18 and 23 years held in Exeter in 1856. Because of the disillusionment with the poor knowledge of the students who had entered the competition, Exeter schools created

²⁰ It is known, for instance, that James Tate, headmaster of a school in which Carroll studied as a child, sent a letter to the boy's father praising the results of his mathematics studies and his logical reasoning (Cohen, 1998).

²¹ De Morgan's views on geometry cannot be overlooked in this context, inasmuch as Carroll himself considers them important, and even quotes a text of the former in appendix 2 of *Euclid and his modern rivals*.

²² This committee, to which we have previously referred, was supervised by the duke of Newcastle, and analyzed the possibility of extending elementary education to all classes of society.

examinations which would eventually be supervised by professors from Oxford and Cambridge (Howson, 2010).

The use of exams for administrative purposes of certification and selection of students and as indicators of what should either be kept or changed in terms of educational standards developed rapidly in the mid-19th century. In higher education, exams, especially the Mathematical Tripos²³, helped to revise privileges and were considered beneficial because they promoted competition; their popularity might also have been partly related to the British interest in sports (Price, 1994).

Meanwhile, in other countries, such as Switzerland, Germany and France, Euclid was thought to be outdated. In France, Legendre, Jacques Demogeot and Henry Montuucci's book *Éléments of géométrie* had been adopted and these authors were sent to Britain in 1866 in order to analyze the local system of education. When their comparative research²⁴ was published, the authors pointed out that the use of Euclid as a textbook exercised only memory, not intelligence, and that, in spite of its robust logic, its approach was tedious. These claims would be later advocated on English soil by Thomas Arnold, principal of the traditional Rugby School and a respected educator and man of letters who, however, knew little of mathematics, but who, for taking part in Her Majesty's Inspectorate²⁵, was an accredited man (Price, 1994). Other voices also rose against Euclid: Frederick Temple, one of the members of the Taunton Commission, said that Euclid had been used as a textbook for a rather long time, and that it was necessary to investigate whether there was another easier and less abstract book to teach geometry for beginners; JM Wilson, who worked with Temple in Rugby, said students knew Euclid, but they knew nothing of the spirit, the method or the results of geometry and, because of this, a new method should be sought. Wilson would eventually become a founding member of the Association for the Improvement of Geometrical Teaching - AIGT - and published his own book, *Elementary geometry*²⁶, to reverse the state of affairs which he criticized. On the other hand, De Morgan argued that Euclid, "in spite of some minor logical imperfections, was still the best treatment of geometry largely because of its difficulty" (Price, 1994, p. 23).

Several books were published intending to teach geometry in a faster, easier and more modern way. A highly recommended book at the time was *A treatise on geometry* by

²³ According to the University of Cambridge website, an examination including Mathematics and Philosophy questions was proposed *circa* 1725, and was used for the purpose of listing the best students. The Cambridge Mathematical Tripos was initially oral, and later had its questions dictated, but the answers were written. Questions to examinees began to be printed around the year 1790. The exam, which gave notoriety and respect to those who obtained the top positions, lasted eight days at the time, with five and a half hours of daily tests. The result was listed according to the score obtained. The applicant who obtained the best score earned the title of senior wrangler, followed by the second wrangler.

²⁴ These publications are criticized by Carroll in Appendix 1 of *Euclid and his modern rivals*.

²⁵ Her Majesty's Inspectorate was an official institution responsible for the inspection of English schools.

²⁶ The book is criticized for Carroll in act 2, scene 6, § 1 of *Euclid and his modern rivals*.

Robert Wallace, which was published in 1831, and in which “the connection of theory with practice is never omitted where it can be introduced” (Howsam, Stray et al, 2007, p. 9). Another book - *Principles of geometry* (1848) - by Thomas Tate, proposed to teach the content of Euclid by making use of references to everyday objects and situations. However, when Tate’s manual was being considered as a new textbook for all schools, he was attacked by the *Civil Engineer and Architect’s Journal*, “the true spirit of geometry will be lost in England as it is elsewhere, if Euclid shall cease to be our text-book” (Howsam; Stray et al, 2007, p. 9).

Several manuals intended to replace *Euclid’s elements* were published, initially in very small number and then like a flood: in the second half of the 19th century there were no fewer than 73 distinct publications for teaching Geometry (Price, 1994). “Winds of change were blowing. The middle class demanded a more practical approach to mathematics, while traditional and classical education was put on the back burner” (Wilson, 2009, p. 114). On the other hand, Euclid was a classic text, the pinnacle of the Greek culture, which was supposed to bring out moral and spiritual benefits. The use of Euclid fitted the education of the dominant class perfectly, and could be defended as the basis of its humanistic value.

When he began to work as a teacher in 1855, Carroll came across a scenario characterized by the emergence of popular schools with scarce contents, poorly prepared teachers and badly trained students who were beginning to join in the defense of a teaching of geometry that was not so hard, parameterized by Euclid’s book and which, in addition, could also be more practical. After the Great Exhibition of 1851, the government established the Department of Science and Art, which was allotted large sums of money, being also responsible for supplying museums, which was supposed to work so that there was a science education which “assist the industrial classes” (Howson, 2010, p. 28), resulting in several curricular experiences²⁷. Baas established a committee which was joined by Sylvester and Wilson, and also by other professors from Cambridge - De Morgan friends - who supported his view, to investigate other methods of teaching Geometry, approaching it in association with the Natural Sciences.

Wilson stated in the preface to his book that “the manuals of geometry will not differ from one another nearly so widely as the manuals of algebra or chemistry” (Wilson apud Carroll, 2012, p. 238). Taking this for granted, Geometry examinations at schools and universities no longer needed to be based on *Euclid’s elements* only, since several other

²⁷ The teaching manual idealized by John Perry (1850-1920) was only used in 1899, and included topics such as the use of the slide rule, Simpson’s rules to estimate the area of an irregular figure and the volume of three-dimensional bodies, practical methods to calculate areas and volumes using graph paper, interpolation, Cartesian or polar coordinates, solving equations, the calculation of minimum and maximum values, practical geometry - the angle between a straight line and a plane, between two straight lines, projection - vectors and differentiation (Howson, 2010).

options were available at the time. Carroll opposed to the increasingly popular idea of the possibility of performing tests and evaluating them using various books.

The Association for the Improvement of Geometrical Teaching - AIGT -, established in 1871, was concerned about the proliferation of textbooks and the impartiality of examinations. How could an explanation or enumeration of propositions be evaluated fairly if the examinee had studied other systems and books?²⁸ A major concern of AIGT was the preparation of a new Geometry textbook. Thus, the Victorian educational scenario would go back to the early 19th century, when only one book - Simson's edition²⁹ -, was used (Howson, Stray et al, 2007). The first AIGT meeting, held on January 17, was summoned by a memo sent to the various principals and teachers revealing what the Association intended to propose with regard to textbooks in item 5:

(6) That this meeting is of opinion that in any new text book (a) the following principles, only partially or not at all recognized by Euclid, should be adopted: (i) Hypothetical constructions; (ii) The arithmetical definition of proportion; (iii) Superposition; (iv) The conception of a moving point, and of a revolving line. (b) the following limitations should be removed: (i) the restriction of the number of axioms to those only which admit of no proof; (ii) the restriction which excludes all angles not less than two right angles. (c) modern terms, such as locus, projection, etc, should be introduced. (Price, 1994, p. 25)

Of the twenty-eight members included in the first official AIGT list published in October of that year, only two were college professors. In its original organization, AIGT chose Thomas Hirst (1830-1892) as its first president, Wilson as one of two vice-presidents and R. Wormell as its treasurer. Wilson and Wormell published new geometry books, thus meeting the conditions described in the previous list, which were thoroughly considered and analyzed by Carroll in *Euclid and his modern rivals*.

²⁸ In line with AIGT, Carroll thought it more appropriate to have a single textbook for the evaluation of Geometry, with which one could confront the students' performance in each question and the numbered references of propositions. The difference between them was that AIGT intended to develop a new textbook, whereas Carroll saw no need to abandon Euclid.

²⁹ The best edition of Euclid, according to Carroll (Wilson, 2009), was published by Robert Potts, entitled *The school edition, Euclid's elements of geometry, the first six books, chiefly from the text of Dr. Simson, with explanatory notes*. This book, published in 1845, is still for sale. This edition is based on Simson's first edition in English. *Euclid's Elements, viz: the first six books together with the eleventh and twelfth*, published in 1756, in which he claims to have corrected some errors that Théon and other translators had made over the centuries, restoring some Euclid demonstrations (Heath, 1956). A Portuguese translation of the first six books, the eleventh and twelfth of *Euclid's Elements* is also available, from the Latin version by Frederick Commandino, published by the University of Coimbra. The edition of this translation, which belongs to the collection of the Group of Oral History and Mathematics Education, is from 1855. Simson's edition was translated into English from the Latin translation of Frederick Commandino which, in turn, was the translation of the Greek edition of Théon. Here there is a small problem: the edition of Théon was contaminated by insertions which he had found necessary to the original work, and includes some differences from the original, such as the use of the term "axiom" instead of "common notions", a reduction in the number of postulates to three instead of five, an increase in the number of definitions (in Théon's edition, they are twenty-five instead of twenty-three), and changes in proposition numbering. These differences can be observed in several sections of *Euclid and his modern rivals*.

The cover page of the first paperback published by AIGT was a letter from secretaries R. Levett and EF McCarthy criticizing the use of Euclid's book, and attributing the chaotic scenario of geometry teaching to it. *Euclid's elements*, they said, had fossilized; content and style needed to be simplified and modernized, useless propositions should be eliminated, the book did not fit the formation of mathematicians or the needs of a mechanized society anymore (Price, 1994). Also quoting a report by the French authors Demogeot and Montucci, the letter commented on the "universal rejection" (Price, 1994, p. 27) of Euclid in countries of the continent, considering this to be further evidence of its inadequacy.

AIGT's priority was to prepare a manual to be recognized as valid and high-quality by Baas, which occurred in 1873. The first manual contained books one to four of Euclid, and was supported by Baas. The latter, however, demanded that the AIGT produced a text that contained up to the sixth book, what would only happen in 1876, when Baas began to advise institutions to adopt it. However, prominent Cambridge and Oxford professors - Price (1994) quotes Carroll among them verbatim - still opposed the reforms and defended Euclid, as the examination system was particularly complicated, comprising a large number of independent examining boards; also, as there were no mechanisms to standardize Geometry tests, most examiners remained faithful to Euclid since he was best known.

AIGT did not back down, and in less than ten years had established subcommittees whose work extended beyond the sixth book of Euclid, addressing the geometry of solid, plane geometry and the conics. The Association continued to insist that examining boards should at least accept *The elements of plane geometry* as an alternative textbook to Euclid. The discussion went on for quite a long time, leaving behind even the origins of the Association itself: in 1897, AIGT changed its name to the Mathematical Association, and started holding meetings with teachers and students of Mathematics who continued their efforts to displace Euclid in the classroom.

Carroll remained staunch to the tradition and memory of Euclid. If, on the one hand, his dedication to writing *Euclid and his modern rivals* was not enough to give back to Euclid the outstanding position he had had for such a long time, on the other hand it made it clear that one must be careful in developing teaching materials, as many of the books that intended to replace *Euclid's Elements* showed shortcomings with respect to concepts and organization which had gone unnoticed both by teachers or by eminent mathematicians.

Carroll criticized repetitive teaching³⁰; nevertheless, it is known that he supported the teaching of Geometry by Euclid's book, whose evaluation consisted in redoing - or, in other words, repeating - demonstrations by following, whenever possible, the sequence and numbering of *Euclid's elements* propositions. This leads us to believe that, with reference to Euclid's systematized science, Carroll was faithful to his origins, to classicism, and to the assumption that it would shape scholarly, higher thinking: his time and his story had presented him this Geometry and, because of this, he perceived it this way, the reason why his repetition criticisms referred only to other contents. Everything about teaching could change - methods, evaluations, teaching practices - except changes referring to geometry, which should continue to be taught using *Euclid's elements* as a textbook, as had been done for two thousand years.

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³⁰ According to Cohen (1998) and Gattegno (1990), professor Mein Herr, character from *Sylvie and Bruno* (1889) and *Sylvie and Bruno concluded* (1893), is a manifestation of Carroll's personality, through which he criticizes learning by repetition.

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