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Robust Finite-Time Control for Guidance Law with Uncertainties in Missile Dynamics

Fazhan Tao[1,2], Jingtian Shi[1], Jie Zhang[1], Zhumu Fu[1,3,*], Song Gao[1]

1.Henan University of Science and Technology – College Information Engineering – Luoyang – Henan – China.
2.Longmen Laboratory – Luoyang – Henan – China.
3.Henan University of Science and Technology – Henan Key Laboratory of Robot and Intelligent Systems – Luoyang – Henan – China.

*Correspondence author: fuzhumu@haust.edu.cn

ABSTRACT

In this paper, the robust finite-time control for impact angle guidance of missile dynamic system with uncertainties is investigated by combining linear extended state observer (LESO) and adaptive non-singular fast terminal sliding mode method. Specially for dealing with existing uncertainties including time-varying parametric perturbation and nonparametric disturbances in high order line-of-sight rates and target acceleration, a robust LESO strategy is proposed for designing sliding mode-based impact angle guidance, which can guarantee that estimation error converges to the neighborhood of the origin in finite-time. Based on the proposed LESO framework, an adaptive non-singular fast terminal sliding mode guidance law is considered for realizing interception of maneuvering targets, which can guarantee asymptotically stability of the system. Simulation results are shown for confirming effectiveness of the proposed guidance strategy of this paper. Compared with former methods, accuracy of estimation is increased by nearly two times, and miss distance is reduced by nearly two times.

Keywords: Terminal guidance; Targets; Angle of attack; Missile control.

INTRODUCTION

Guidance law of missile plays an important role in intercepting targets. The performance of missile can be further refined when it can be insensitive and robust to target maneuvers and environmental disturbances. The well-known guidance laws design methods, line-of-sight (LOS) guidance, and proportional navigation guidance are widely employed due to their highly efficient and simple for practical applications (Lin *et al.* 1991). By utilizing proportional to the LOS angle rate to track targets in terminal phase, these approaches can effectively realize asymptotic or exponential stability against constant velocity targets (non-maneuvering targets) (Hou and Duan 2008; Ma *et al.* 2009). When dealing with cases where objects consist of uncertain maneuvers and disturbances of missile dynamics systems, the performances of the mentioned methods are limited. Therefore, robustness of engagement performance considering the uncertainties of the missile dynamics system is of importance for enhancing engagement performance during homing phase.

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In the past decades, in order to improve robustness of engagement performance of guidance laws, many advanced control techniques have been considered, such as nonlinear H∞ control terminal guidance law (Guo and Zhou 2009), optimal guidance law (Zhang *et al.* 2014), fixed-time tracking control (Cui *et al.* 2023), and finite-time convergence guidance law (Zhou *et al.* 2009, 2016). In practical engineering, the controlled systems are usually required to reach steady response from transient response quickly. In theory, the control schemes in the sense of infinity-time stabilization do not meet that requirement, since they usually lead to a long transient response of the closed-loop systems. In fact, the problem of finite-time control has been paid considerable attention. Some significant finite-time stability criteria have been proposed before. In Zhang *et al.* (2022), an adaptive finite-time tracking control scheme is developed for a category of uncertain nonlinear systems, with asymmetric time-varying full-state constraints and actuator failures. In Cui *et al.* (2021), a finite-time adaptive fault-tolerant control scheme is researched for a class of nonlinear systems with unmodeled dynamics, where the dynamic system is switched.

However, considering the advantages of fast response, insensitivity to parameter changes and disturbances, and no need for online system identification, sliding mode control (SMC) has broad prospects in practical missile interception targets. SMC (also known as variable structure control) is achieved by switching functions, switching the structure of the controller based on the degree to which the system state deviates from the sliding mode, thereby enabling the system to operate according to the prescribed laws of the sliding mode. The important advantage of SMC is robustness. SMC-based impact angle guidance law design is promising, achieving many insightful results (Lee *et al.* 2013). Considering head-on, tail-chase, and head pursuit engagements, SMC-based guidance laws is proposed, which can impose a predetermined interception angle relative to target's flight path angle in Tal (2007, 2011). An impact angle constrained guidance law based on SMC and linearization method is proposed to intercept stationary or slowly moving targets in Zhang *et al.* (2014). By combining power reaching law and exponential reaching law, a fast power reaching law is designed, reducing chattering of sliding mode and improving reaching speed (Zhou *et al.* 2009). In addition, by combining with other advanced control technique, integrated SMC-based strategies are also studied, such as fuzzy variable structure (Li *et al.* 2011) and radial basis function (RBF) neural network sliding mode variable structure (Wang *et al.* 2016). These methods increase the complexity of controller undoubtedly while suppressing system chattering. Liu *et al.* (2016) and Zhang *et al.* (2013) improved the sliding surface and the approach law, respectively, both of which achieved some results in suppressing system chattering. However, the above methods cannot guarantee finite-time convergent performance, and consist of symbol function term in controller.

As for dealing with the existing disturbances of missile dynamics, there are two main directions. One idea is that disturbances are regarded as the unknown parameter of system. For these cases, adaptive control is effective method, for example, in Cai and Xiang (2017) and Cai *et al.* (2017); adaptive finite-time controller and adaptive laws is constructed by adding a power integrator technique, and the stability of the corresponding closed-loop system is proved based on the finite-time Lyapunov theory. The other idea lies in estimating disturbances knowledge of maneuvering targets, using disturbance observer, Kalman filter techniques (Arasaratnam and Haykin 2009; Kandepu *et al.* 2008) technique.

In terms of the Kalman filter technique, its calculation process is complicated and algorithm is dependent on models of systems. If the proposed target model cannot match the actual motion, it may lead to divergence of filter, even failures on target tracking. In practical systems, the state variables are usually unmeasurable or just partly measurable, and some control plans may not be well implemented. The observer can estimate those unmeasurable state variables; it overcomes the difficulties caused by lack of accurate state information (Han 1995; Yao and Wang 2009). By using observer method, in Wang and Su (2013), a LESO is constructed using a linear function, meanwhile detailed selection procedure of the proposed LESO related parameters is given. In Fu *et al.* (2023), Ju *et al.* (2023), and Shao and Wang (2015), estimation error of the observer is proved to be bounded under unknown system conditions, and the quantitative relationship between observation error and expansion order is given. As for finite-time estimation of LESO, in Yang *et al.* (2015) and Yu *et al.* (2005), the relationship between observer parameters, speed error convergence, and steady-state error is considered. Moreover, the sufficient conditions and related proofs to ensure the boundedness of the observation error are given, which provides a theoretical basis for the design of observer parameters.

From the aforementioned analyses, to effectively estimate disturbances caused by maneuvering target and missile dynamic system for simplifying calculation process and realize finite-time convergence while suppressing chattering are important for intercepting targets. Therefore, in this paper, the LESO-based terminal finite-time SMC method is proposed to design an adaptive finite-time guidance law for impact angle of missile. The proposed guidance law can reduce difficulties of calculation in estimating uncertainties and improve tracking performance for maneuvering targets in finite-time.

The rest paper is organized as follows. In the next section, the missile-target relative motion model is introduced. Main results of this paper, including uncertainties estimation based on LESO method, terminal SMC-based guidance law, are developed in Main results section. Then, for confirming effectiveness of the proposed method, simulation results and comparison results are given, showing that the proposed method can improve high-precision guidance of missile. Finally, the conclusion of this paper is summarized in Conclusion section.

MISSILE-TARGET RELATIVE MOTION MODELING

Relative movement relationship between the missile and the target is introduced as shown in Fig. 1. The missile and the target are regarded as particles and are represented by M and T, respectively. In order to simplify the pursuit situation, it is assumed that the missile and the target are point masses moving in plane. According to the missile-target geometric diagram in Fig.1, motion equations of the missile-intercepting target can be obtained as follows (Eq. 1):

$$\begin{cases} \dot{r} = V_t \cos(q - \sigma_t) - V_m \cos(q - \sigma_m) \\ r\dot{q} = -V_t \sin(q - \sigma_t) + V_m \sin(q - \sigma_m) \end{cases} \tag{1}$$

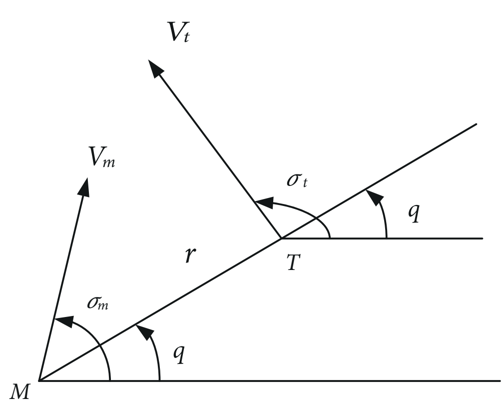

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Figure 1. Guidance geometry for missile-target engagement.

where $\dot{r}$ is the relative distance between missile and target, $\dot{r}$ is the derivative of r with respect to time, V_t, V_m, q, $\dot{q}$,σ_t, and σ_m are target rate, missile rate, line of sight angle, line of sight rate, speed direction angle of target, and missile, respectively.

Denoting the derivation of the relative motion states r and q as V_R and V_q (the relative speeds of the missile and the target in the line of sight and vertical line of sight), respectively, the transformation is shown in Eq. 2.

$$\begin{cases} V_R = \dot{r} \\ V_q = r\dot{q} \end{cases} \tag{2}$$

Differentiating Eq. 2 with respect to time, obtain Eq. 3:

$$\dot{V}_q = -\frac{V_R V_q}{r} + \omega - u + \varepsilon \tag{3}$$

where ω and u are the components of the target and missile acceleration in the LOS method, respectively, ε represents bounded uncertainties, including the unmodeled high-order part of the missile-target engagement model and disturbances of missile dynamic system.

By substituting Eq. 2 into Eq. 3, the model of missile-target relative movement can be derived as follow:

$$\ddot{q} = -\frac{2\dot{r}}{r}\dot{q} + \frac{1}{r}(\omega - u) + \frac{1}{r}\varepsilon \tag{4}$$

Based on extended state observer theory, the appearing uncertainties ε is extended to be a new state. Let $x_1 = \dot{q}$, $x_2 = -\frac{1}{r}(2\dot{r}\dot{q} - \omega - \varepsilon)$, combining the extended state with Eq. 4, the whole state expression of the missile-target model is described as follows (Eq. 5):

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + B_1\eta(t) \\ y(t) &= Cx(t) \end{aligned} \tag{5}$$

where $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -\frac{1}{r} & 0 \end{bmatrix}^T$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$, $x(t) = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$, and $\eta(t)$ are state variables and its derivative, respectively.

Based on Eq. 5, unknown target maneuver and disturbances are expanded to a new state of the missile-target engagement systems; then, in the next section, we will employ LESO to estimate impact of target maneuvering for designing terminal sliding mode guidance law.

MAIN RESULTS

Linear extended state observer LESO design

As for the time-varying missile-target systems with uncertainties as shown in Eq. 5, a LESO is established in Eq. 6:

$$\begin{aligned} \dot{z}(t) &= Az(t) + Bu(t) + B_1u_1(t) \\ y_1(t) &= Cz(t) \end{aligned} \tag{6}$$

where $z(t)$ is state variable and $u_1(t)$ and $y_1(t)$ are input and output of observer, respectively.

Considering Eqs. 5 and 6, the disturbance estimation error system can be obtained as follows (Eq. 7):

$$\begin{aligned} \dot{e}(t) &= Ae(t) + B_1(u_1(t) - \eta(t)) \\ y_2(t) &= Ce(t) \end{aligned} \tag{7}$$

where $e(t) = z(t) - x(t)$ and $y_2(t) = y_1(t) - y(t)$ are estimation error and output, respectively.

The aim of the proposed observer is to control the input $u_1(t)$ such that the estimation error $e(t)$ converges to 0. Due to the state variables of the original system cannot be directly measured and $\eta(t)$ is unknown, the state $e(t)$ in Eq. 7 cannot be obtained directly. Therefore, in this paper, an output feedback control scheme is constructed for realizing the estimation error can convergent in finite-time.

The output feedback control system is designed as follows (Eq. 8):

$$\begin{aligned} \dot{\zeta}(t) &= A_s\zeta(t) + B_sy_2(t) \\ u_1(t) &= C_s\zeta(t) \end{aligned} \tag{8}$$

where $\zeta(t) \in R^{2\times1}$ is the state variables of the output feedback controller, $A_s \in R^{2\times2}$, $B_s \in R^{2\times1}$, $C_s \in R^{1\times2}$ are the output feedback controller parameter matrix, respectively.

Considering Eqs. 6–8, the following closed-loop control system of observer can be obtained (Eq. 9):

$$\begin{aligned} \dot{z}(t) &= Az(t) + Bu(t) + B_1 C_s(\tau)\zeta(t) \\ \dot{\zeta}(t) &= A_s(\tau)\zeta(t) + B_s(\tau)Cz(t) - B_s(\tau)y(t) \end{aligned} \tag{9}$$

As for the feedback design system Eq. 9, suppose that there exists a real number $\tau > 1$ and the parameter matrices $A_s(\tau)$, $B_s(\tau)$, $C_s(\tau)$ satisfy $A_s(\tau) = A + B_1 C_s - B_s C$, $\lambda_i(A + B_1 C_s) = -\tau$, $\lambda_i(A - B_s C) = -\tau$, then the observer state $z(t)$ can convergence the system state $x(t)$, meanwhile such that the estimation error $e(t)$ can be satisfied with finite-time bounded, i.e., $x_0^T R x_0 \le c_1 \Rightarrow x^T(t) R x(t) \le c_2$, $\forall t \in [0 \quad T]$.

Due to space limitations, the proof of this lemma is omitted in this paper, and more details on them can be founded in theorem 2 of Yang *et al.* (2015). The reason lies in that the main purpose of this paper is to propose a simplified missile-target model for employing the LESO method into the considered systems.

After that, based on the obtained results on the feedback control design scheme, an improved sliding mode guidance law for high maneuvering targets will be discussed to improve high-precision interception effects in next section.

Guidance laws based on SMC method

The key of improving the guidance accuracy lies in how to control the LOS angular rate $\dot{q}$ by u, such that it approaches 0 to achieve quasi-parallel approach; in other word, to maintain a constant angle of sight between missile and target. Since the LOS angular rate cannot be obtained directly, the state observer system output y is selected as the sliding mode surface (Eq. 10):

$$s = y(t) = Cx(t) \tag{10}$$

Due to sign function in the traditional exponential reaching law, the system control quantity is frequently switched near the sliding surface in the process of intercepting the maneuvering target. It brings chattering to the system inevitably and weakens the interception effect of the system on high-speed maneuvering targets. In order to reduce the system chattering effectively and ensure the missile has a high accuracy of hitting, a new sliding-mode exponential reaching law based on the power function is designed (Eq. 11):

$$\dot{s} = -\frac{k|\dot{r}|}{r}s - \frac{\varepsilon}{r}fal(s,\alpha,\delta) \tag{11}$$

where $k > 0$, $\varepsilon > 0$, and $fal(s,\alpha,\delta)$ are the reaching law coefficient, the switching term coefficient and power function, respectively. The power function is given by (Eq. 12):

$$fal(s,\alpha,\delta) = \begin{cases} |s|^{\alpha}\, sign(s), & |s| > \delta \\ s/\delta^{1-\alpha}, & |s| \le \delta \end{cases} \tag{12}$$

where α and δ are values between 0 and 1.

This approach law chosen in this paper improves the system control effect from two aspects:

- Using the power function $fal(s,\alpha,\delta)$ instead of the symbol function term sgn(s), the chattering caused by the sign function in the traditional switching surface is reduced.
- The parameters of the reaching law are adjusted by using the missile-target relative distance r as a dynamic parameter. When r is larger, the approach speed is slower. On the contrary, the approach speed is increased rapidly when r is decreased, which ensures the rapid convergence of the LOS angular rate $\dot{q}$ and improves the missile hit accuracy effectively.

Considering the system modeling error and the unknown characteristic of the target maneuvering, Eq. 4 is replaced by the designed LESO Eq. 9. It is merely need to ensure that the error Eq. 7 converges to 0 in limited time. From the obtained Eq. 9, the sliding surface is represented as (Eq. 13):

$$s = y_1(t) = Cz(t) \tag{13}$$

Calculating the derivative of Eq. 13 and substituting Eqs. 9 and 11, the sliding mode orientation law is produced (Eq. 14):

$$u = -(CB)^{-1}CAz(t) - (CB)^{-1}CB_1C_s(\tau)\zeta(t) - (CB)^{-1}\left(\frac{k|\dot{r}|}{r}s_1 + \frac{\varepsilon}{r}fal(s_1,\alpha,\delta)\right) \tag{14}$$

Asymptotical stable means that as for the autonomous system $\dot{x} = f(x)$, suppose that $x = 0$ is equilibrium point of the autonomous system, if it is stale and there is $\delta > 0$, such that $\|x(0)\| < \delta \Rightarrow \lim_{t\to\infty} x(t) = 0$.

As for stability of the considered system, the detailed statement is shown in the following. Firstly, based on Lyapunov stability theorem, one Lyapunov function is chosen as (Eq. 15):

$$V(s) = \frac{1}{2}s^T s \tag{15}$$

From calculating the derivation on both sides of Eq. 15 and combining with Eqs. 9, 11–14, obtain

(1) when $|s| > \delta$, $\dot{s} = -\frac{k|\dot{r}|}{r}s - \frac{\varepsilon}{r}|s|^{\alpha} sign(s)$, then (Eq. 16):

$$\dot{V}(s) = s^T\dot{s} = s^T\left(-\frac{k|\dot{r}|}{r}s - \frac{\varepsilon}{r}|s|^{\alpha} sign(s)\right) = -\frac{\varepsilon}{r}\|s\|^{\alpha+1} - \frac{k|\dot{r}|}{r}\|s\|^2 < 0 \tag{16}$$

(2) when $|s| \le \delta$, $\dot{s} = -\frac{k|\dot{r}|}{r}s - \frac{\varepsilon}{r}(s/\delta^{1-\alpha})$, then (Eq. 17):

$$\dot{V}(s) = s^T\dot{s} = s^T\left(-\frac{k|\dot{r}|}{r}s - \frac{\varepsilon}{r\delta^{1-\alpha}}s\right) = -\left(\frac{k|\dot{r}|}{r} + \frac{\varepsilon}{r\delta^{1-\alpha}}\right)\|s\|^2 < 0 \tag{17}$$

From Eqs. 16 and 17, we can find both of them are negative definite, therefore, based on the Lyapunov stability theorem, the proposed design scheme for the considered missile systems is guaranteed to be asymptotically stable and meets the design requirements.

SIMULATION RESULTS AND ANALYSIS

For the systemic model Eq. 4 in the paper, this unknown information, such as the target maneuver and model uncertainties, are expanded to first-order variables to form a new system, as shown in Eq. 5. It can be seen, from Eq. 5, that the goal of the controller design in this paper is to make the LOS angular rate $\dot{q}$ close to 0 by solving the control quantity u, so the related items of $\dot{q}$ in $\eta(t)$ can be ignored.

Consequently, $\eta(t)$ can be described by $\eta(t) = \frac{\dot{\omega}+\dot{\varepsilon}}{r} - \frac{\dot{r}(\omega+\varepsilon)}{r^2}$. Due to the existence of relative distance r squared term, the second term in $\eta(t)$ is small, so $\eta(t) \le \frac{\dot{\omega}+\dot{\varepsilon}}{r}$, which is a bounded interval related to r. Assume that the target maneuver information is $\omega = 5g\sin(4\pi t)$, the model uncertainty is $\varepsilon = 0.5\sin(4\pi t)$, and $r = 0.5$, finally, then the system model uncertain term x_2 and its rate of change $\eta(t)$ is satisfied with the linearized condition, combining $g = 9.8$ m/s^2, get $d = 20.2$.

The values chosen for the observer parameter matrix are $B_s = [3\tau \quad 3\tau^2]^T$, $C_s = -[3\tau^2 \quad 3\tau]$, and $A_s(\tau) = A + B_1C_s - B_sC$, respectively, in which the range of observer parameter is $\tau \ge 8080$ when $\sigma = 2.5\times10^{-3}$ is chosen. The initial values of system Eq. 5 state, output feedback controller state, and observer state are selected as $x(0) = [0.4 \quad 0.4]^T$, $\zeta(t) = [0 \quad 0]^T$ and $z(t) = [0.4 \quad 0]^T$, respectively. The $5g\sin(4\pi t)$ sine signal and $5gsquare(2,t)$ square wave signal are used to simulate the target maneuvering conditions; for showing the effectiveness of the proposed method of this paper compared with (Tamhane *et al.* 2016), the simulations results on state estimation curve and state estimation accuracy curve are shown in Figs. 2 and 3, respectively.

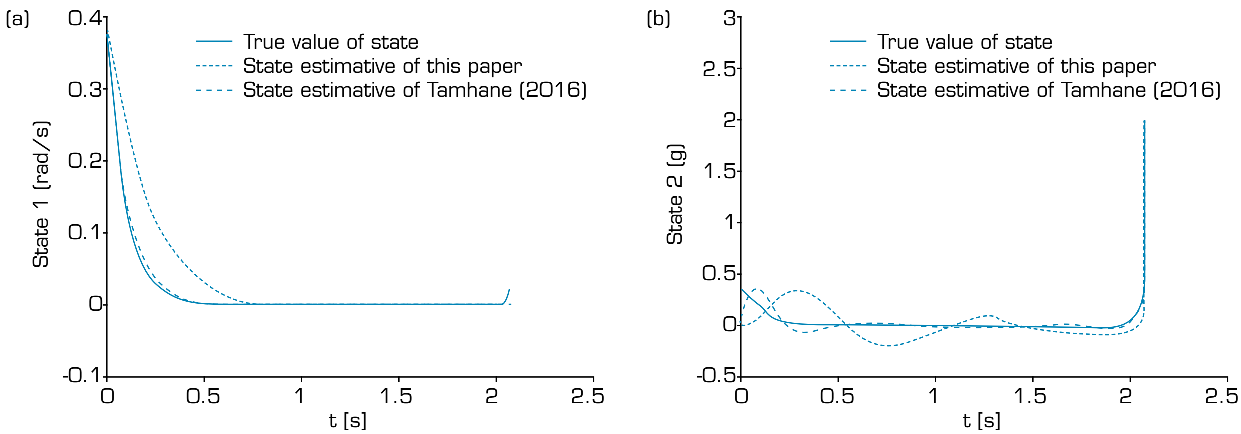

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Figure 2. System state estimation curve. (a) System state x_1 estimated curve; (b) System state x_2 estimated curve.

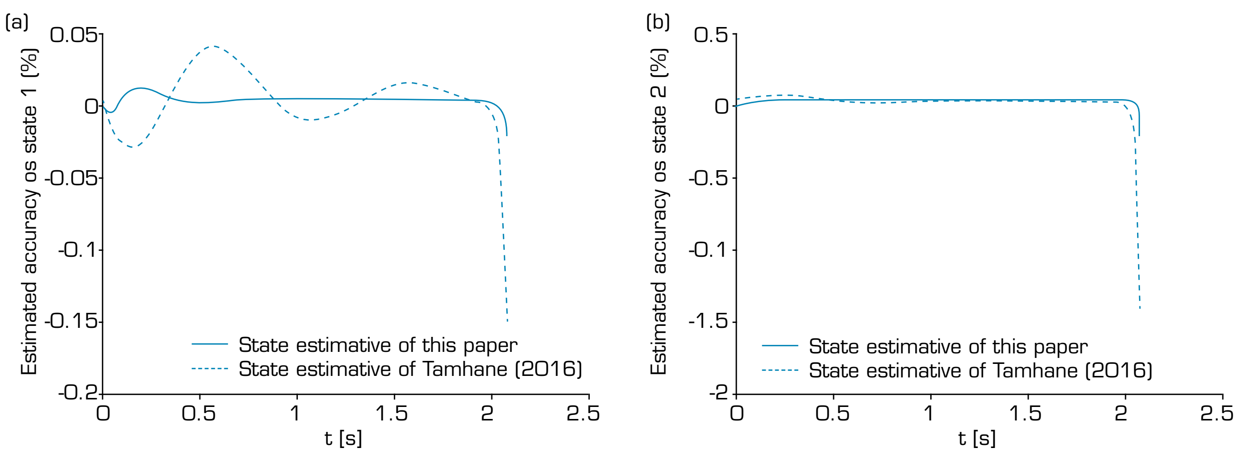

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Figure 3. System state estimation accuracy curve. (a) System state x_1 estimated accuracy curve; (b) System state x_2 estimated accuracy curve.

The comparison of the system state estimation is shown in Figs. 2 and 3, in which Figs. 2a and b are the tracking of state x_1 and state x_2, respectively. Figs. 3a and b show the estimation accuracy curve of state x_1 and statex_2, respectively. It can be seen from these figures that the LESO designed in this paper not only improves the system state estimation accuracy, but also reduces the time required for convergence to a stable state, which provides a guarantee for successful interception of high maneuvering targets. However, the estimation error of designed observer in Tamhane *et al.* (2016) has a sudden increase at the end of the guidance, which can be seen from Figs. 2b, 3a and b. This is due to the fact that there is a reciprocal of relative distance r in the system state x_2. Consequently, the system state approaches infinity and the estimation error of observer will increase suddenly when r close to 0. In contrast, the proposed design scheme of this paper has obvious improvement in suppressing the sudden increase of observation error at the end of the guidance. Meanwhile, it verifies the rationality of the controller design using the state estimation value instead of the state actual value.

Another simulation is the estimation of sine maneuvering and square wave maneuvers so as to analyze accuracy of designed observer on the given target acceleration information. The simulation results as shown in Figs. 4 and 5. It can be seen from results that the observation method proposed in this paper has a good estimation effect on the target of sine and square wave signals as maneuvering information, and the estimation accuracy is improved by nearly three times compared with Tamhane *et al.* (2016).

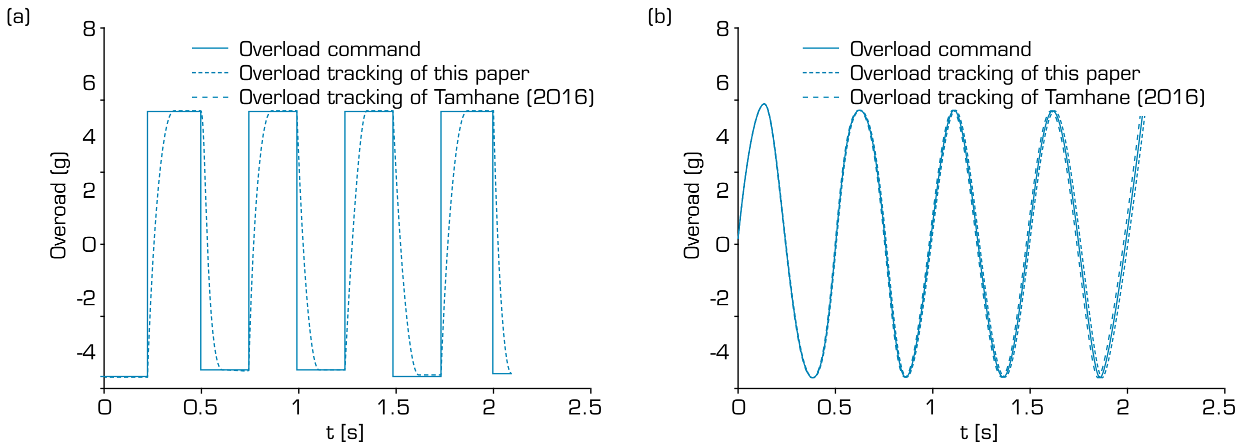

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Figure 4. Target maneuver overload estimation curve. (a) Square wave overload estimation; (b) Sine overload estimation.

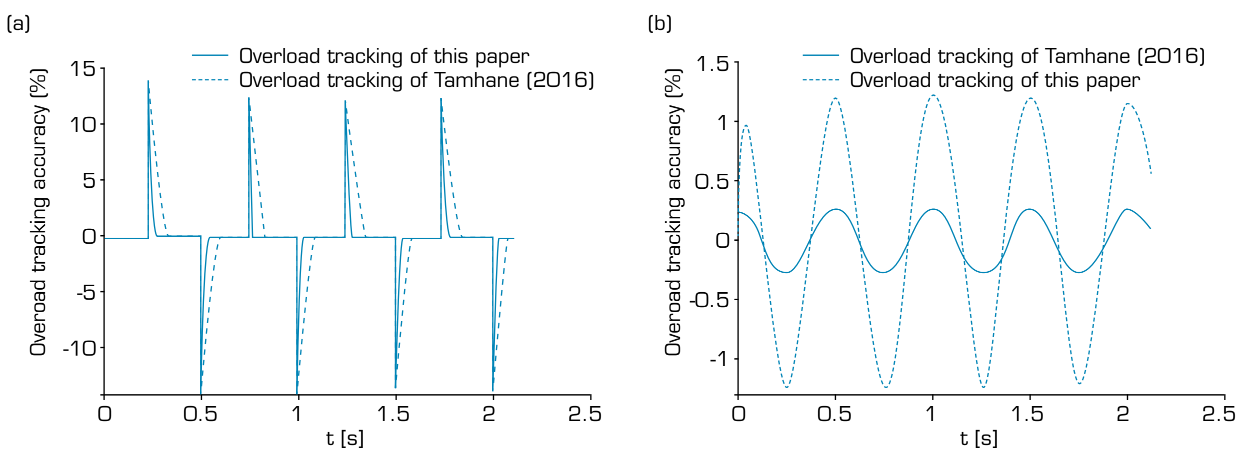

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Figure 5. Target maneuver overload estimation accuracy curve. (a) Square wave overload estimation accuracy; (b) Sine overload estimation accuracy.

In Zhou (2016) also uses the power law as the reaching law to design the terminal guidance law, the parameters of the SMC are selected as: reaching law coefficient is $k = 1.5$, coefficient of switch term is $\varepsilon = 1.2$, the power function parameters are $\alpha = 0.5$ and $\delta = 0.01$, respectively, missile initial position is (0,9000), flying speed is 600 m·s, and the speed direction angle is $\sigma_m = 0$. Due to space limitations, this paper only verifies the situation where the target uses $5g \sin(4\pi t)$ sine signal as maneuvering information. In order to verify the effectiveness of the interception strategy designed in this paper, three different interception methods of attack interception, pursuit interception, and forward interception are respectively compared with the finite-time sliding mode convergence strategy shown in Eq. 11 of Zhang *et al.* (2015).

- Attack interception: assume that the target flight speed is 400 m·s, the initial target speed direction angle is $\sigma_t = 165°$, and the target initial position is (2,000, 10,000). The result of simulation comparison is shown in Fig. 6a.
- Pursuit interception: assume that the target flight speed is 400 m·s, the initial target speed direction angle is $\sigma_t = 60°$, and the target initial position is (2,000, 10,000). The result of simulation comparison is shown in Fig. 6b.
- Forward interception: assume that the target flight speed is 700 m·s, the initial target speed direction angle is $\sigma_t = 165°$, and the target initial position is (500, 10,000). The result of simulation comparison is shown in Fig. 6c.

These results obtained are presented in Table 1.

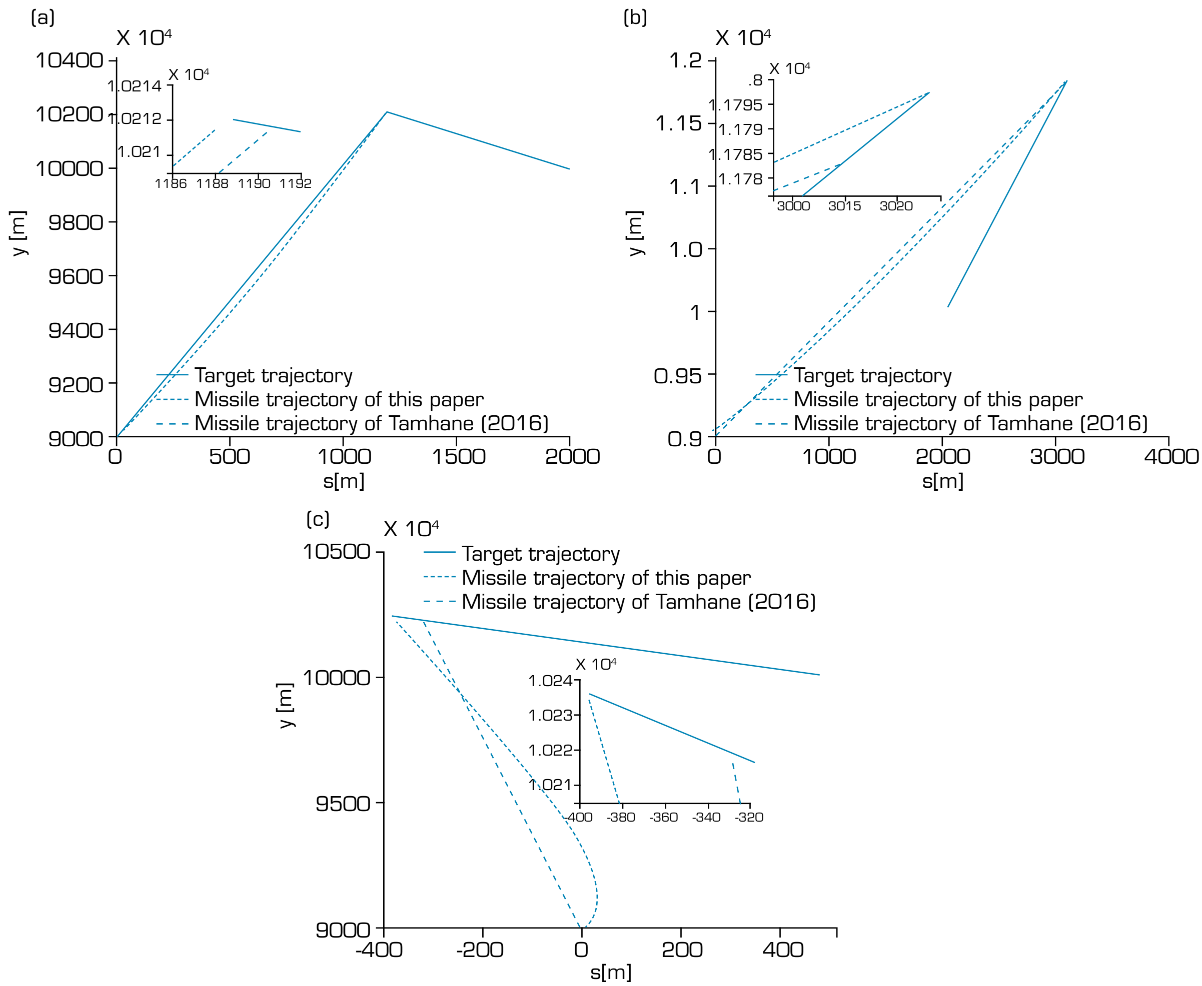

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Figure 6. Guidance interception curve. (a) Attack interception trajectory; (b) Pursuit interception trajectory; (c) Forward interception trajectory.

Table 1. Guided performance comparison.

Interception way	New sliding mode interception strategy		Finite-time sliding mode convergence strategy	
	Miss distance (m)	Flight time (s)	Miss distance (m)	Flight time (s)
Attack interception	0.489	2.09	1.047	2.11
Pursuit interception	0.536	5.12	1.664	5.27
forward interception	0.956	1.23	1.771	1.33

Source: Elaborated by the authors.

From the simulation results shown in Fig. 6 and Table 1, the proposed new sliding mode interception strategy designed in this paper has improved both in terms of flight time and miss distance. In particular, the size of miss distance can reflect the effectiveness of the interception strategy directly. In this respect, the effect achieved in this paper is nearly one time higher than that in Zhang *et al.* (2015).

Fig. 7 shows the comparison of the outputs of the two interception strategy controllers. It can be seen from the figure that the control requirements for both interception strategies is relatively large at the initial stage of movement, which is related to the selection of the initial value of system.

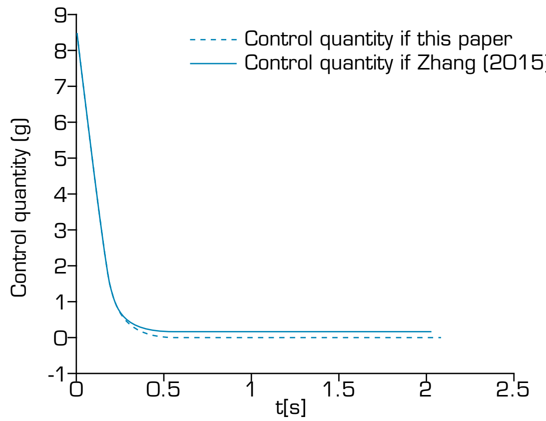

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Figure 7. Controller output comparison curve.

However, the required control of the interception strategy in this paper is smaller. The finite-time sliding mode convergence strategy described in Zhang *et al.* (2015) during the terminal guidance stage has a certain frequency of chattering. As a comparison of the strategies proposed in this paper, the performance of strategy in the terminal guidance stage is slightly improved, which makes the output of the controller smoother, thus the effectiveness of improvements made in selection of the approach law is verified.

CONCLUSION

In this paper, a new sliding mode guidance law based on LESO was designed for high maneuvering targets.

- Based on the proposed LESO method, an observer in order to guarantee finite-time convergence was developed to estimate the unknown information online for improving target maneuvers.
- Meanwhile, under the proposed LESO framework, the guidance law of SMC was designed, where the exponential reaching law was improved by the observer state and the relative distance. The extension state increase suddenly at the end of the guidance period by the common reaching law was reduced.
- In addition, the simulation results were given, which confirmed the proposed methods. The accuracy of state observation was increased by nearly two times and the miss distance in almost all cases was reduced by nearly two times compared with the former methods.

In this paper, since the LESO is used to estimate the unknown disturbances, the system is restrained to be linearized due to the LESO assumption. In the future work, a more comprehensive system model should be considered, meanwhile, a nonlinear observer is an important direction for improving higher precision interception.

AUTHORS' CONTRIBUTION

Conceptualization: Tao F, Shi J, Zhang J, Fu Z; **Methodology**: Tao F, Shi J, Zhang J, Fu Z; **Validation**: Tao F, Shi J, Song G; **Writing - Original Draft**: Tao F, Shi J, Zhang J; **Writing - Review & Editing**: Tao F, Fu Z; **Final approval**: Tao F.

CONFLICT OF INTEREST

Nothing to declare.

DATA AVAILABILTY STATEMENT

Data sharing is not applicable.

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