Latin American Journal of Solids and Structures
www.lajss.org

Assessment of Homotopy Perturbation Method for Study the Forced Nonlinear Vibration of Orthotropic Circular Plate on Elastic Foundation

Abstract

In this paper the nonlinear forced vibration of an orthotropic circular plate resting on Winkler, Pasternak and nonlinear Winkler foundation is investigated. Plates with edges elastically restrained against rotation and inplane displacement are analyzed and the Von-Karman geometric nonlinear equations are employed. In this study it is assumed that the plate can be subjected to any periodic distributed lateral loading with respect to time. The Galerkin method is used to obtain Duffing's equation for the central deflection. The Homotopy Perturbation Method was used to study the effects of various parameters including orthotropic parameter, elastic foundation parameters and initial deflection on frequency ratio. Highly accurate results were obtained by the application of the aforementioned method.

Keywords

Nonlinear forced Vibration; Circular Plate; Orthotropic; HPM.

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http://dx.doi.org/10.1590/1679-78252436

Received 06.09.2015
Accepted 21.10.2015
Available online 09.11.2015

1 INTRODUCTION

The increase in the use of thin composite structures, especially in aerospace engineering, leads to the rise of difficulties in nonlinear vibrations in various modern engineering challenges, which may results in the size of the vibration amplitude of these structures. This situation attains greater importance when the plate is subjected to the vibration amplitude of the same order of the plate thickness. There are a large number of publications concerning plate theories and plate dynamics. An extensive survey of the early investigations on the free vibration of the circular plates is given by Leissa (1969). Huang and Sandman (1979) used Kantorovich averaging method to investigate the nonlinear vibrations of a circular plate with a clamped and immovable boundary. The forced response of the plate under several different distributions of sinusoidal input was also investigated.

Nath and Alwar (1979) used the Chebyshev series to study the nonlinear dynamic response of orthotropic circular plates for both clamped and simply supported edge conditions. They considered the influence of orthotropic parameter under three types of dynamic loadings on the response of circular plates. Yamaki et al. (1981) investigated the axisymmetric nonlinear vibrations of a clamped isotropic circular plate under uniformly distributed lateral loading. In this study the effect of both initial deflection and initial edge displacement were considered. Additionally, Nath (1982) studied the effect of foundation parameters on the large amplitude response of orthotropic circular plates.

Dumir presented an approximation solution for the large deflection axisymmetric responses of isotropic (1986) and cylindrically orthotropic (1986) thin circular plates resting on nonlinear Winkler foundations. According to the results of these studies the buckling load and the linear frequency increased with the foundation parameters and the rotational stiffness of the edge support. Hadian and Nayfeh (1990) used the method of multiple scales to obtain the symmetric response of a circular plate to a harmonic external excitation. The results showed that internal resonance is responsible for the coupling of the modes involved and the excited mode is not necessarily the dominant one. Wang (2000) used the power series method in solving the nonlinear differential equations of the circular plates to obtain the exact axisymmetric post-buckling equilibrium.

Eihab et al. (2003) utilized the Von-Karman thin plate theory to account for large static deformations in axisymmetric annular plates. The natural frequencies and mode shapes were obtained numerically for a series of uniform loads. Shirong and Zhou (2003) investigated the axisymmetric nonlinear vibration and the thermal post-buckling of a heated polar orthotropic annular plate with both its inner and outer edges immovably hinged. Alipour et al. (2010) used the differential transform method to study free vibration of FG circular plates resting on two parameter elastic foundations. This study focused on the non-axisymmetric vibration and the modal stress analysis.

There exists a wide arsenal of analytic, semi-analytic and numerical tools for the nonlinear analysis of continuous systems. To investigate these problems, different methods such as the numerical methods (1995) (1998) and perturbation methods (1973) (1979) were employed although they have their own limitations as well. The perturbation methods on the other hand are also limited as that they can only be applied to the weakened nonlinear differential equations.

Researchers were prompted to find analytical solutions for nonlinear equations that did not contain the abovementioned limitations. Therefore techniques such as the variational iteration method (2008) homotopy analysis method (HAM) (1995) (2003) and homotopy perturbation method (HPM) (1999) (2003) were adopted. The present work uses Homotopy Perturbation Method (HPM) for the analysis of nonlinear forced vibration of orthotropic circular plate on elastic foundation. The basic structural model adopted in this study is the Von-Karman plate model. In most previous investigations study large amplitudes of circular plates for simplicity usually an assumed space or time mode were used. According to this a simple harmonic function in time was employed and based on Kantorovich averaging method it was eliminated from the equation of motion. In this study, this procedure ignoring and it is assumed that the time mode part can be any periodic function of time. The Galerkin method is used to obtain Duffing's equation for the central deflection. The results of this investigation demonstrated the applicability of HPM for analysis of circular plate for correct quantitative predictions and for qualitative description of operations.

2 GOVERNING EQUATIONS

The nonlinear Winkler foundation is adopted to model the elastic foundation. For the axisymmetric case, a distributed force on the circular plate is introduced as follow Dumir (1986):

$$-(k_L w - k_{NL} w^3 - g\nabla^2 w) \tag{1}$$

where w is the transverse deflection, k_L is the Winkler parameter, k_{NL} is the Winkler nonlinear parameter, and g is the shear parameter of the Pasternak Foundation. The governing equation for large axisymmetric deflection of an orthotropic circular plate in terms of w and stress function ψ is as follow Dumir (1986):

$$\begin{aligned} D\left[\frac{\partial^4 w^*}{\partial r^4} + \frac{2}{r}\frac{\partial^3 w^*}{\partial r^3} - \frac{\beta}{r}\frac{\partial^2 w^*}{\partial r^2} + \frac{\beta}{r^3}\frac{\partial w^*}{\partial r}\right] - \frac{1}{r}\frac{\partial}{\partial r}\left(\psi^* \frac{\partial w^*}{\partial r}\right) \\ = F - \gamma h \frac{\partial^2 w^*}{\partial t^2} - k_L w^* + k_{NL} w^{*3} + g\nabla^2 w^* \end{aligned} \tag{2}$$

$$\frac{\partial^2 \psi^*}{\partial r^2} + \frac{1}{r}\frac{\partial \psi^*}{\partial r} - \frac{\beta}{r^2}\psi^* + h\frac{E_\theta}{2r}\left(\frac{\partial w^*}{\partial r}\right)^2 = 0 \tag{3}$$

where $F(r,t) = q(r)\,f(t)$ can be any periodic distributed lateral loading function of time and

$$D = \frac{E_\theta h^3}{12(\beta - v_\theta^2)} \qquad \beta = \frac{E_\theta}{E_r} = \frac{v_\theta}{v_r} \tag{4}$$

In this study it is assumed that $q(r)$ is constant. In order to reduce the governing equation to dimensionless form the following dimensionless parameters are introduced:

$$w = \frac{w^*}{h} \qquad \psi = \frac{a}{D}\psi^* \qquad \rho = \frac{r}{a} \qquad \tau = \sqrt{\frac{D}{\gamma h a^4}}\,t \tag{5-a}$$

$$K_L = \frac{k_L a^4}{E_r h^3} \quad K_{NL} = \frac{k_{NL} a^4}{E_r h} \quad G = \frac{g a^2}{E_r h^3} \quad Q = \frac{q a^4}{E_r h^4} \tag{5-b}$$

Substituting Eq. (5) into Eqs. (2) and (3) yield Dumir (1986):

$$\begin{aligned} \frac{\partial^2 w}{\partial \tau^2} + \frac{\partial^4 w}{\partial \rho^4} + \frac{2}{\rho}\frac{\partial^3 w}{\partial \rho^3} - \frac{\beta}{\rho}\frac{\partial^2 w}{\partial \rho^2} + \frac{\beta}{\rho^3}\frac{\partial w}{\partial \rho} + \frac{12(\beta - v_\theta^2)}{\beta} \times \\ \left[K_L w - K_{NL} w^3 - \frac{G}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial w}{\partial \rho}\right)\right] = \frac{12(\beta - v_\theta^2)}{\beta} Q\,F(\tau) + \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\psi \frac{\partial w}{\partial \rho}\right) \end{aligned} \tag{6}$$

$$\rho^2 \frac{\partial^2 \psi}{\partial \rho^2} + \rho \frac{\partial \psi}{\partial \rho} - \beta\psi + 6(\beta - v_\theta^2)\rho\left(\frac{\partial w}{\partial \rho}\right)^2 = 0 \tag{7}$$

where a is the radius of circular plate.

3 METHOD OF SOLUTION

The plate deflection $w\ (\rho,\tau)$ is expressed as follow:

$$w\ (\rho,\tau) = (W_{max}/h)\left[1+\sum_{i=2,4,\ldots}^{\infty} C_i\ \rho^i\right]\phi(\tau) \approx (W_{max}/h)(1+C_2\ \rho^2+C_4\ \rho^4)\phi(\tau) \tag{8}$$

where W_{max} is the maximum deflection at the center of the plate and constants C_2 and C_4 are defined by the boundary conditions. The general solution of Eq. (7) is obtained by the solution of the substitution of Eq. (8) into Eq. (7):

$$\begin{gathered}\psi(\rho,\tau) = C_0\ \rho^\lambda + \frac{C_1}{\rho^\lambda} + k_1\rho^3 + k_2\rho^5 + k_3\ \rho^7 \\ \lambda = \beta^2\end{gathered} \tag{9}$$

The value of ψ is accordingly found to be finite at the origin, $C_1 = 0$. Additionally, C_0 the constant of integration is to be determined from inplane boundary conditions. The k_i coefficients are defined as:

$$k_1 = \frac{-24C_2^2(\beta - v_\theta^2)}{9-\lambda^2}\phi^2(\tau) \tag{10-a}$$

$$k_2 = \frac{-96C_2C_4(\beta - v_\theta^2)}{25-\lambda^2}\phi^2(\tau) \tag{10-b}$$

$$k_3 = \frac{-96C_4^2(\beta - v_\theta^2)}{49-\lambda^2}\phi^2(\tau) \tag{10-c}$$

The Substitution of the expressions for w and ψ given by Eqs. (8) and (9) respectively into Eq. (6) and the application of the Galerkin procedure in the nonlinear time differential equation obtained in the form:

$$\frac{d^2\phi}{d\tau^2} + \left(\frac{L_2}{L_1}\right)\phi + \left(\frac{L_3}{L_1}\right)\phi^3 = Q\left(1+\frac{C_2}{3}+\frac{C_4}{5}\right)f(\tau) \tag{11}$$

$$L_1 = 1 + \frac{2}{3}C_2 + \frac{2}{5}C_4 + \frac{2}{7}C_2C_4 + \frac{C_2^2}{5} + \frac{C_4^2}{9} \tag{12}$$

$$\begin{aligned}L_2 = &\left[8\ C_4(9-\beta) + K_L\frac{12(\beta - v_\theta^2)}{\beta} - 4G\frac{12(\beta - v_\theta^2)}{\beta}C_2\right]\left(1+\frac{C_2}{3}+\frac{C_4}{5}\right) \\ &+ \tfrac{12(\beta - v_\theta^2)}{\beta}[K_LC_2 - 16GC_4]\left(\tfrac{1}{3}+\tfrac{C_2}{5}+\tfrac{C_4}{7}\right) \\ &-K_LC_4\frac{12(\beta - v_\theta^2)}{\beta}\left(\frac{1}{5}+\frac{C_2}{7}+\frac{C_4}{9}\right)\end{aligned} \tag{13}$$

$$\begin{aligned}L_3 = &-K_{NL}\frac{12(\beta - v_\theta^2)}{\beta}\left(1+\frac{C_2}{3}+\frac{C_4}{5}\right) - \left(3K_{NL}C_2\frac{12(\beta - v_\theta^2)}{\beta} + \frac{32C_2^3}{9-\lambda^2}\right)\left(\frac{1}{3}+\frac{C_2}{5}+\frac{C_4}{7}\right) \\ &-\left[3C_2^2 + 3C_4 + 96C_2^2C_4\left(\tfrac{1}{9-\lambda^2}+\tfrac{2}{25-\lambda^2}\right)\right]\left(\tfrac{1}{5}+\tfrac{C_2}{7}+\tfrac{C_4}{9}\right)\end{aligned} \tag{14}$$

$$-\left[C_2^3 + 6\,C_2C_4 + 256\,C_2C_4\left(\frac{1}{49-\lambda^2}+\frac{2}{25-\lambda^2}\right)\right]\left(\frac{1}{7}+\frac{C_2}{9}+\frac{C_4}{11}\right)$$
$$-\left[3C_4^2 + 3C_2^2C_4 + \frac{640C_4^3}{49-\lambda^2}\right]\left(\frac{1}{9}+\frac{C_2}{11}+\frac{C_4}{13}\right) + 2C_0C_2(\lambda+1)\left(\frac{1}{\lambda+1}+\frac{C_2}{\lambda+3}+\frac{C_4}{\lambda+5}\right)$$
$$+4C_0C_4(\lambda+3)\left(\frac{1}{\lambda+3}+\frac{C_2}{\lambda+5}+\frac{C_4}{\lambda+7}\right)\Bigg]$$

As it can be seen coefficients L_1, L_2 and L_3 are function of $\beta, C_0, C_2,$ and C_4. These constants can be determined from boundary conditions. For a plate with an elastically restrained outer edge, with rotational and inplane stiffness K_b^* and K_i^*, subjected to applied inplane radial force resultant N^* at the outer edge the boundary conditions are:

$$r = a\colon\ M_r = K_b^*\frac{\partial w^*}{\partial r} \quad N_r = N^* - K_i^* u^* \tag{15}$$

where u^* is the radial displacement at midplane. Introduce dimensionless parameters K_b, K_i and N:

$$K_b = \frac{12K_b^* a}{E_r h^3} \qquad K_i = \frac{K_i^* a}{E_r h} \qquad N = \frac{N^*}{E_r h}\left(\frac{a}{h}\right)^2 \tag{16}$$

The boundary conditions at $\rho = 1$ take the following dimensionless form:

$$w = 0 \tag{17-a}$$

$$\left[\frac{(\beta - v_\theta^2)}{\beta}K_b + v_\theta\right]\frac{\partial w}{\partial \rho} + \frac{\partial^2 w}{\partial \rho^2} = 0 \tag{17-b}$$

$$K_i\left(\frac{\partial \psi}{\partial \rho} - v_\theta \psi\right) + \beta\psi = 12(\beta - v_\theta^2)N \tag{17-c}$$

Constants C_2 and C_4 can be found from the two first Eqs.(17-a) and (17-b) and C_0 is obtained from Eq. (17-c):

$$C_2 = -\frac{6+A}{5+A} \qquad C_4 = \frac{1}{5+A} \qquad A = \frac{\beta - v_\theta^2}{\beta}K_b + v_\theta \tag{18}$$

$$C_0 = \frac{12(\beta - v_\theta^2)N - K_i(3P_1 + 5P_2 + 7P_3) - (\beta - K_i v_\theta)(P_1 + P_2 + P_3)}{\beta + K_i(\lambda - v_\theta)}\phi^2(t) \tag{19}$$

$$P_1 = -24(\beta - v_\theta^2)\frac{C_2^2}{9-\lambda^2} \tag{20}$$

$$P_2 = -96(\beta - v_\theta^2)\frac{C_2C_4}{25-\lambda^2} \tag{21}$$

$$P_3 = -96(\beta - v_\theta^2)\frac{C_4^2}{49-\lambda^2} \tag{22}$$

In the next section the applicability of HPM to solve Eq. (11) is discussed in details.

4 HE'S HOMOTOPY PERTURBATION METHOD

The homotopy perturbation method (HPM) is a combination of the classical perturbation technique and homotopy concept. By the homotopy method, He constructed a homotopy $w(r,p):\Omega\times[0,1]\rightarrow R$ which satisfies He (2003):

$$H(w,p)=L(w)-L(u_0)+pL(u_0)+pN(w) \tag{23}$$

where p is an embedding parameter, L and N are linear and nonlinear parts of Eq. (11) and u_0 is initial approximation of Eq. (11) which satisfies the initial conditions. Assuming the solution of Eq. (11) can be expressed as a power series of p:

$$w=w_0+pw_1+p^2w_2+\cdots \tag{24}$$

In Eq. (24) when $p\rightarrow 1$ the approximate solution of Eq. (11) is obtained. In order to investigate primary resonance of the system and implementation of the HPM the following homotopy is constructed:

$$\ddot{\phi}+(L_2/L_1)\phi=p[f(\Omega\tau)-(L_3/L_1)\phi^3] \tag{25}$$

where $(\ddot{\ })$ is the second derivative of ϕ with respect to τ, and Ω is excited frequency. The solution of Eq. (25) and the square of natural frequency can be expressed by the parameter p as follow:

$$\phi=\phi_0+p\phi_1+p^2\phi_2+\cdots \tag{26}$$

$$\Omega^2=\omega_0^2(1+p\sigma_1+p^2\sigma_2+\cdots) \tag{27}$$

Introducing new variable $\tau^*=\Omega\tau$ then Eq. (25) can be written as:

$$\Omega^2\ddot{\phi}+(L_2/L_1)\phi=p[f(\tau^*)-(L_3/L_1)\phi^3] \tag{28}$$

Substituting Eqs. (26) and (27) into Eq. (28) and equating the terms with identical power of p, one can obtain:

$$\ddot{\phi}_0+\phi_0=0 \quad \phi_0(0)=W_{max},\ \dot{\phi}_0(0)=0 \tag{29}$$

$$\ddot{\phi}_1+\phi_1=(L_1/L_2)f(\tau^*)-(L_3/L_2)\phi^3-\sigma_1\ddot{\phi}_0 \quad \phi_1(0)=0,\ \dot{\phi}_1(0)=0 \tag{30}$$

If $f(\tau^*)$ is a periodic function of τ^* then it can be represented by infinite series of harmonic functions known as a Fourier series. Replacing $f(\tau^*)$ by its corresponding Fourier series one can obtain:

$$\begin{aligned}\ddot{\phi}_1+\phi_1=&\left(\frac{L_1}{L_2}\right)\left[Q\left(1+\frac{C_2}{3}+\frac{C_4}{5}\right)\right]\left[\frac{a_0}{2}+\sum_{i=1}^{\boxtimes}\left(a_n\cos\frac{2n\pi}{T}\tau^*+b_n\sin\frac{2n\pi}{T}\tau^*\right)\right]\\&-\left(\frac{L_3}{L_2}\right)\phi^3-\sigma_1\ddot{\phi}_0\end{aligned} \tag{31}$$

where T is the period of the function. If $\Omega_n = \frac{2n\pi}{T}$ then we have:

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} f(\tau^*)\, d\tau \tag{32-a}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(\tau^*) \cos \Omega_n \tau^*\, d\tau^* \tag{32-b}$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(\tau^*) \sin \Omega_n \tau^*\, d\tau^* \tag{32-c}$$

To satisfy the initial conditions, the initial guess of u_0 is chosen as follow:

$$w_0(\tau^*) = u_0(\tau^*) = W_{max} \cos\psi \qquad \psi = \tau^* + \theta \tag{33}$$

where θ is the phase angle. Substituting Eq. (33) into (31) results in:

$$\begin{aligned}\ddot{\phi}_1 + \phi_1 = X\left(\frac{L_1}{L_2}\right)(a_n \sin\theta + b_n cos)\sin\psi \\ + \left[\sigma_1 W_{max} - \frac{3}{4}\left(\frac{L_3}{L_2}\right)W_{max}^3 + X\left(\frac{L_1}{L_2}\right)(a_n\cos\theta - b_n\sin\theta)\right]\cos\psi \\ - \left(\frac{L_3}{L_2}\right)\frac{W_{max}^3}{4}\cos 3\psi + \left(\frac{L_1}{L_2}\right)\frac{Xa_0}{2}\end{aligned} \tag{34}$$

where $X = Q\left(1 + \frac{C_2}{3} + \frac{C_4}{5}\right)$. In order to identify the nonlinear frequency the secular term which may occur in the next iteration should be eliminated. By setting the coefficients of $\cos\psi$ and $\sin\psi$ to zero and ignoring the parameter ϑ the following equation is resulted:

W_{max}/h	Present study	Ramachandran (1977)	Haterbouch & Benamar (2003)	Yamaki (1961)	Huang Al-Khattat (1977)
0.2	1.007	1.0066	1.0072	1.007	1.0075
0.4	1.0238	1.263	1.0284	1.0278	1.0296
0.5	1.0414	1.0408	1.0439	1.0431	1.0459
0.6	1.0597	1.0583	1.0623	1.0614	1.0654
0.8	1.1049	1.1015	1.1073	1.1065	1.1135
1	1.1564	1.1547	1.1615	1.1617	1.1724
1.5	1.3262	1.3229	1.3255	1.3343	1.3568
2	1.5241	1.527	1.5147	1.5423	1.579

Table 1: Frequency ratio for an isotropic circular plate for different values of non-dimensional vibration amplitudes.

$$\left(\sigma_1 - \frac{3}{4}\left(\frac{L_3}{L_2}\right)W_{max}^2\right)^2 = \left(\frac{L_1}{L_2}\right)\left(\frac{X}{W_{max}}\right)^2 (a_n^2 + b_n^2) \tag{35}$$

If the first-order approximation is sufficient then we have:

$$\sigma_1 = \left(\frac{L_1}{L_2}\right)\Omega^2 - 1 \tag{36}$$

Substituting Eq. (36) into (35) one can obtain:

$$\left(1 - \left(\frac{L_1}{L_2}\right)\Omega^2 + \frac{3}{4}\left(\frac{L_3}{L_2}\right)W_{max}^2\right)^2 = \left(\frac{L_1}{L_2}\right)\left(\frac{X}{W_{max}}\right)^2 (a_n^2 + b_n^2) \tag{37}$$

5 RESULTS

In table 1 to verify the accuracy of the presented method, a comparison has been made between the results of previous works exist in the literature and the data given by Eq. (37) for non-resonance conditions (X=0). As it can be seen, discrepancy between numerical results obtained by HPM and previous works at large vibration amplitudes is about 3.6% at W_{max}/h=2.

In figures 1 and 2 the primary resonance response of an isotropic thin circular plate for two different cases; $f(t)=e^t$ and $f(t)=t$ with T=2π are shown, respectively. It should be noted that the predicted results are reliable since the vibration amplitude is smaller than the thickness of plate.

The influence of orthotropic parameter on the frequency ratio of the circular plate is indicated in Fig. 3. As it can be seen for β<10 the results show hardening type nonlinearity while for β≥10 turning to softening for vibration amplitude. Fig. 4 demonstrates the effect of Winkler linear foundation (K_L) and Winkler nonlinear foundation (K_{NL}) parameters for constant Pasternak foundation parameter (G=50) on primary resonance response for $f(t)=e^t$.

It can be concluded that the Winkler nonlinear foundation parameter (K_{NL}) may have a positive effect on the response behavior. The similar behavior for different values of K_{NL} on frequency ratio for free vibration can be seen in Fig. 5. According to this figure, the positive values of K_{NL} can have a positive effect on the hardening nonlinearity degree in comparison with negative values of K_{NL}.

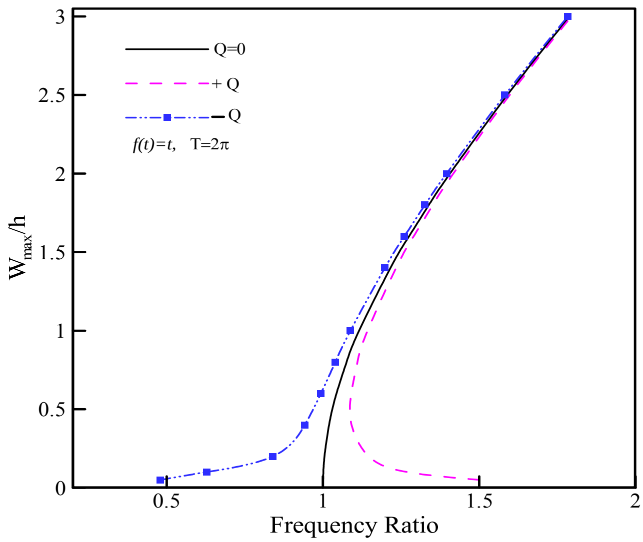

Figure 1: Effects of large vibration amplitudes on frequency ratio for $f(t)=t$ and T=2π.

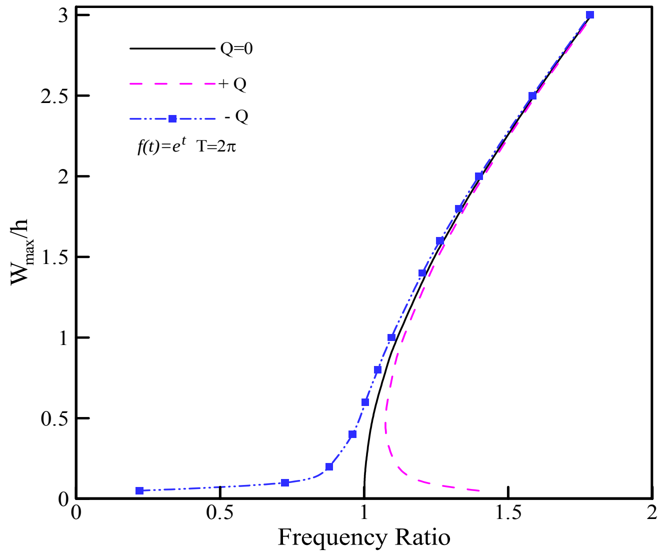

Figure 2: Effects of large vibration amplitudes on frequency ratio for $f(t)=e^t$ and T=2π.

Additionally, it is clear that for while vibration amplitude ($W_{\max}/h$) remains constant an increase in the value of K_{NL} results in the reduction of the frequency ratio. The variation of frequency ratio for various values of Pasternak foundation parameter (G) is depicted by Fig. 6. It demonstrates that all response curves exhibit initial softening trends and revert to hardening amplitudes at large amplitudes although the degree of hardening vary. It is interesting to note that for higher amplitude vibration ratios, increasing the Pasternak foundation parameter yields the discrepancy reduction between linear and nonlinear frequency. In other words, the Pasternak foundation parameter has the ability to limit the effect of large amplitude vibration on nonlinear response.

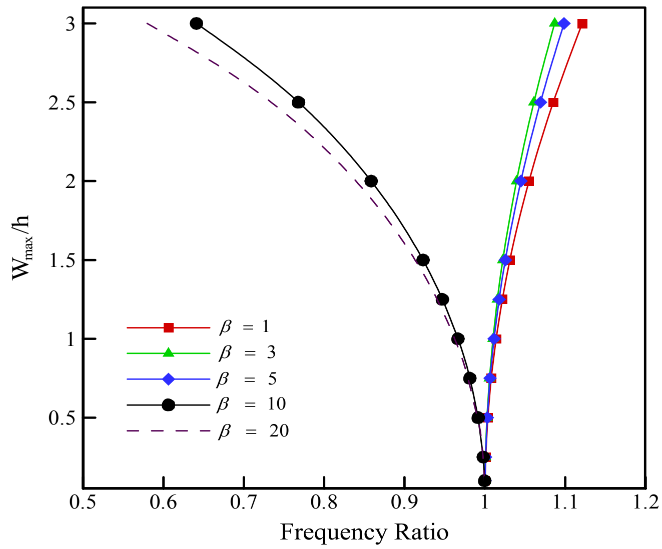

Figure 3: The influence of orthotropic parameter on the frequency ratio of the circular plate.

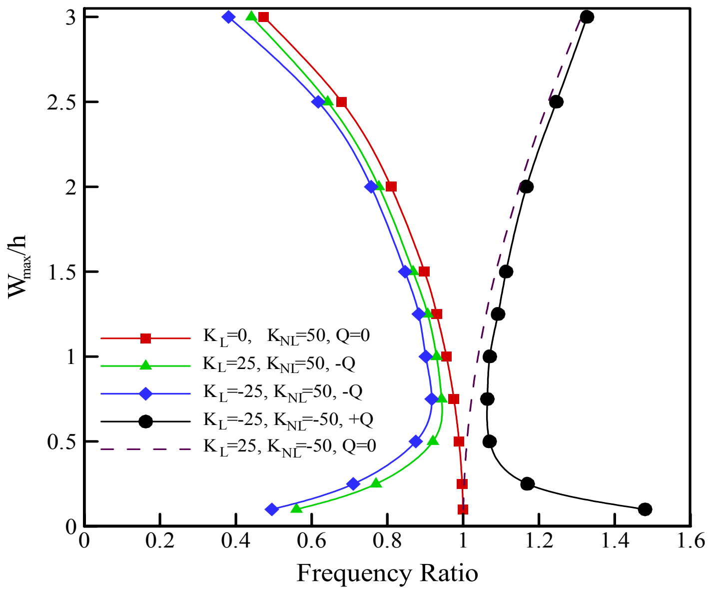

Figure 4: The effect of Winkler foundation parameters on the frequency ratio of the circular plate for $f(t)=e^t$ and T=2π.

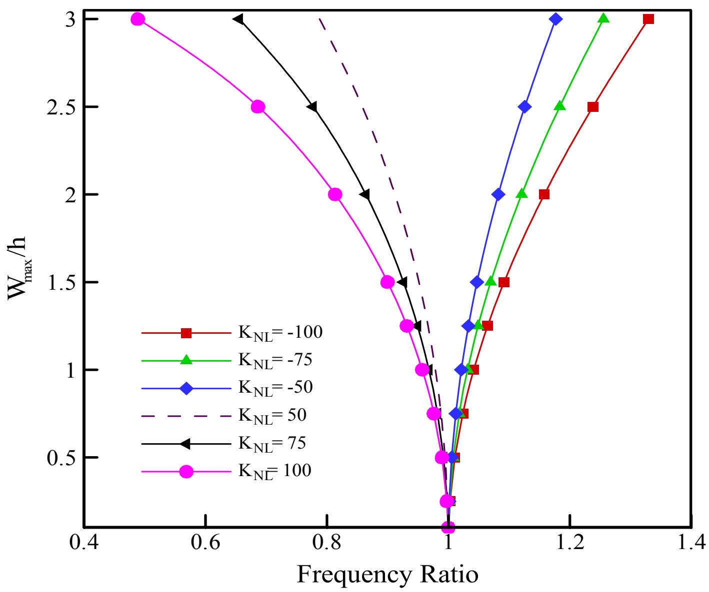

Figure 5: The effect of Winkler nonlinear foundation parameter on the frequency ratio.

Fig. 7 illustrates the effect of outer edge radial force resultant on frequency ratio. According to this figure the results are divided in two parts. The positive value of Winkler nonlinear foundation parameter increases the value of radial force resultant resulting in the rise of frequency ratio. While the negative values of K_{NL} resulting in the increase of N leads to the reduction of frequency ratio. This behavior can be attributed to the fact that the variation of N can affect the stiffness of orthotropic circular plate.

In Fig. 8 the influence of Pasternak foundation parameter and radial force resultant on frequency ratio are considered. As it is shown, a decrease in the value of Pasternak foundation parameter results in the rise of frequency ratio. In the other word the increase in the value of the Pasternak foundation parameter results the discrepancy reduction between linear and non-linear frequency. This fact is also emphasized in Fig. 6. Additionally similar conclusions can be reached from the decrease in the value of the resultant radial force.

As was indicated previously, the influences of Winkler nonlinear foundation and Pasternak foundation parameters in comparison with the other system properties on frequency ratio are more effective and considerable which is illustrated in Fig. 9. According to the results a combination of negative values of K_{NL}, with an increase in the value of the Pasternak foundation parameter leads to a reduction in the frequency ratio. Contrariwise, for $K_{NL}>0$, combined with an increase in the value of G can result in the achievement of higher values of the frequency ration. According to this figure, $K_{NL}=0$ is a turning point. Also, it should be noted that reduction of K_{NL} yields the growth of frequency ratio. Based on these results, the frequency ratio is deeply depending on K_{NL} and G.

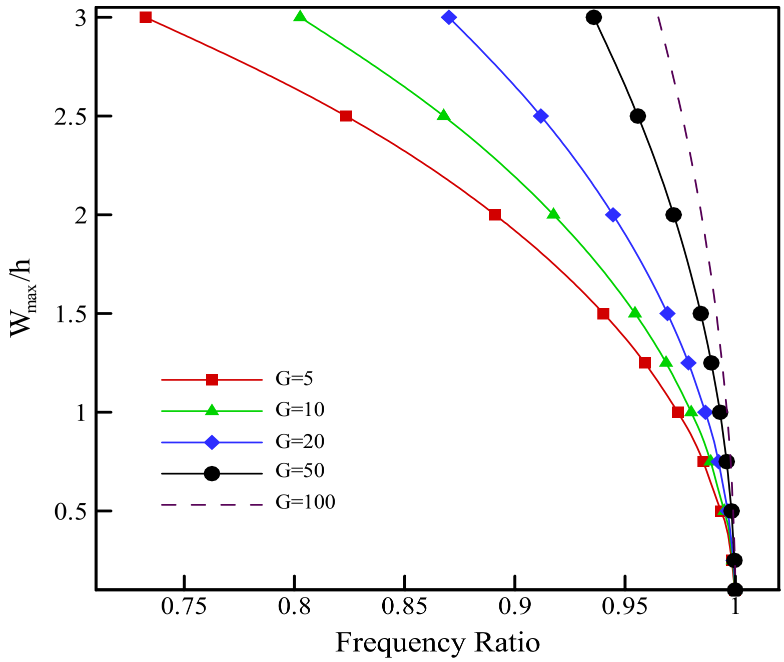

Figure 6: The effect of Pasternak foundation parameter on the frequency ratio.

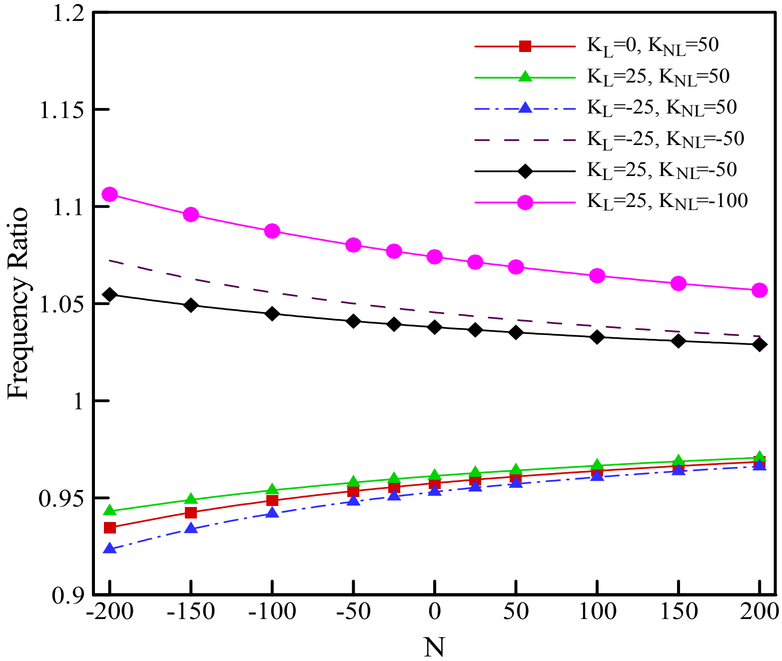

Figure 7: The effect of outer force radial resultant force on the frequency ratio.

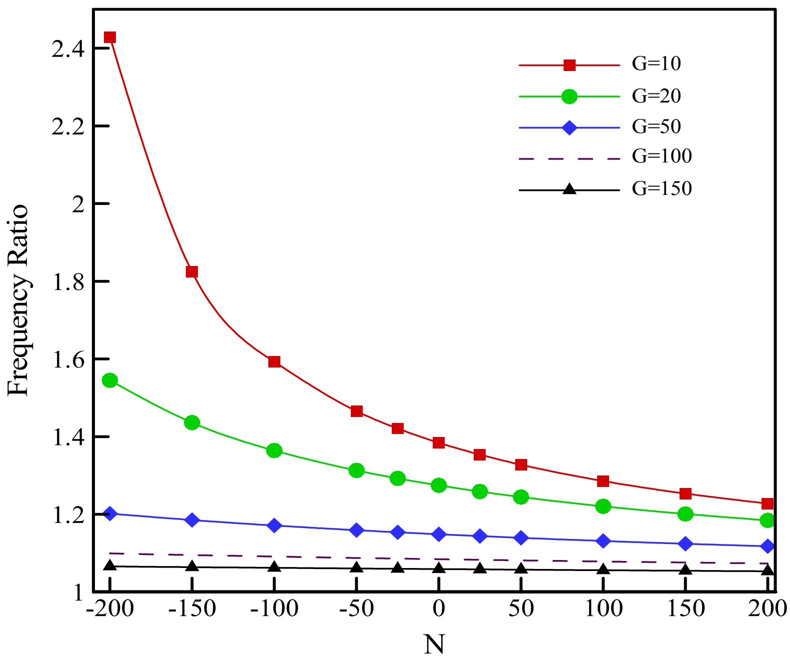

Figure 8: The influence of Pasternak foundation parameter and radial force resultant on frequency ratio.

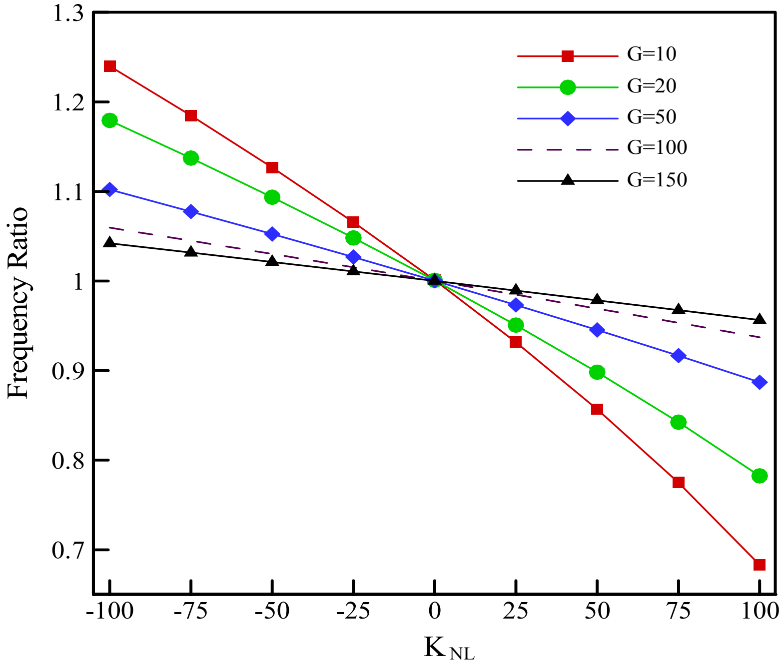

Figure 9: The influences of Winkler nonlinear foundation and Pasternak foundation parameters on frequency ratio of circular plate.

5 CONCLUSION

In this paper the nonlinear forced vibration of orthotropic circular plate resting on elastic foundation is investigated. The Galerkin method and homotopy perturbation method are both employed to study the nonlinear forced vibration of circular plate. The effect of different parameters such as elastic foundation parameters and orthotropic parameter are considered. According to the results, the effects of Winkler nonlinear foundation and Pasternak foundation parameters on frequency ratios are more considerable.

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