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Structural stability concepts in medieval and renaissance mechanics

Abstract

The identification of the origins of what we now call the theory of elastic stability is not an easy task. Most authors trace the origins to the pioneering work of Leonhard Euler in 1744, and some shift this origin to the experimental works of Petrus van Musschenbroek in 1729. However, other contemporary authors interested in the history of the discipline postulate that the works of Medieval and Renaissance scholars should be considered as the true sources of the buckling studies performed in the XVIII Century. This paper reports a historical research based on the original works of Al-Khazini, Jordanus de Nemore, Leonardo da Vinci, and Marini Merssene, in order to discuss what kind of knowledge they had about the topics of stability and lateral deflections of columns under axial loads. Our investigation shows that there were observations of the phenomenon considered, but those observations were not translated into a deeper understanding of the phenomenon, so that the causes of this effect or the role of strength on the response were not considered. Leonardo was closer than others in his understanding of the nature of the problem and produced some tentative rules of behavior; however, those were only documented in private writings and did not make an impact in his contemporaries or even 100 years later. We postulate that there was a continuity of problems between medieval authors and those who lived in the XVIII Century, rather than continuity in their concepts and approaches to solve those problems.

Keywords

buckling, columns, history of mechanics, Medieval Science of Weights.

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1 INTRODUCTION

During the last decade, there have been controversies regarding the origins of the field of elastic stability. Elishakoff [13, 14] has shown that most authors center the origin of this theory in the analytical work of Euler, dated 1744, whereas some authors consider the 1729 experimental work of Musschenbroek [16, 40] as a starting point. However, such claims have not been without disputes: Villaggio [42] argued that there were studies of elastic stability well before then, including works written during the Middle Ages.

To contest the claim that the studies of elastic stability started with Euler, Villaggio [42] writes: "The proper theory, as opposed to the notion, of stability commences long before Euler. Heron of Alexandria (c. 100 B.C.), in the course of a 'long dull work on statics' (see C. Truesdell) endeavoured to explain why the strength of a piece of wood reduces as its length increases. Leonardo da Vinci (1452–1519) provided two empirical rules for the strength of columns in compression. The Jesuit M. Mersenne (1588–1648), in his Reflections on the causes of resistance in solids, observed that 'iron, copper and other metals, even single bodies, when subject to a force or weight, curve and bend to the form of an arch before breaking'."

Texts on the history of science written by historians such as Sarton [38] or Bernal [3] are of limited use in this case, since they are too broad in their scope and do not address topics in detail. The first edition of the book by Mach [30] on the history of mechanics did not mention any contributions in the centuries that span from Archimedes to Leonardo da Vinci. However, at the end of the XIX and beginning of the XX Century, works of medieval authors were re-discovered, whereas others were translated [11, 24].

The identification of the origins of what we now call the theory of elastic stability of structures is a complex task, because of the difficulties to recover original manuscripts, to envisage what knowledge can be attributed with certainty to authors in this field before the XVII century, and up to what point they went beyond simple observations or marginal comments.

This paper reports historical research principally based on the works of Al-Khazini, Jordanus de Nemore, Leonardo da Vinci, and Marini Merssene, in order to identify what kind of knowledge they had about topics of stability and about lateral deflections of columns under axial loads. Some comments are of order regarding methodological aspects concerning the historical approach adopted in this work. In a previous article, the author discussed the historical perspectives adopted by various historians of the theory of elasticity [17], all of whom were active researchers in the field and wrote historical accounts of their subject of interest. The approach followed by those authors may fall under the category of internalist methodologies, in the sense that the historian attempts to write a rational reconstruction of events by looking at contributions within the framework of the discipline. Reference is not made to the context in which those contributions took place. Lakatos [27] was one of the philosophers of science who emphasized this approach; however, his approach seems to have been a reaction against those who proposed a strong externalist view of the history of science. In the externalist methodologies, special attention is given to factors outside the scientific field of research, such as those arising from the social, political, religious or economic context, and attempts are made to explain in what sense the scientific developments occurred in a way consistent with the historical/cultural context. This methodology is based on the assumption that the design of theories is a manifestation of such historical/cultural context.

This dichotomy between internalist and externalist methodologies does not exhaust the range of perspectives of study in the history of science. In a discussion of current approaches to historiography of science, Kragh [25] mentions that both methodologies may help understanding the science history in different ways: an externalist approach, which is usually followed by historians, may provide valuable information on changes in many fields of science in a given period. The treatise on the social history of science by Bernal [3] may fall in this category. The internalist methodology, on the other hand, is usually followed by scientists writing on the history of their discipline, as reported by Truesdell [40] and Godoy [17] among others.

This paper attempts to provide an account of a few contributions that may be related to the field of stability of structures, considering both rigid and flexible structures. The period covered spans from the IX Century in the Islamic world to the XVII Century in Europe. Because of the topical nature of this research, and because different periods, religions and cultures are considered in order to identify related contributions, an internalist approach is followed. However, the author acknowledges that the cultural, religious and political conditions at each time considered played an important role in terms of motivations, justifications, and achievements of each scientist.

2 THE GREEK LEGACY IN THE MIDDLE AGES

Early studies of the stability of rigid bodies were conducted for more than two thousand years as part of an effort to understand the behavior of the balance and the level arm, both relevant topics for practical purposes. Balances such as the Greek one shown in Figure 1, were used to weight goods and products and assign values to them. It is believed that such studies were also present in Asian countries and in the kingdoms of present-day India [37]. In the Roman Balance, the arms have different lengths in such a way that a lower weight placed at a distance from the center (called *fulcrum*) may be equilibrated by a larger weight placed closer to the center.

Figure 1 Early Greek balance with bronze scales and lead weights, from the *Vapheio Tholos* tomb, Lakonia, XV Century BC. (Archeological Museum, Athens, photograph by the author).

Our current historical knowledge about the stability of balances can be traced back to the Greek philosophers, whose works were recovered and translated into Arabic between the IX and XII Centuries by medieval Islamic authors; and those Arab texts were in turn translated into Latin in Toledo or Damascus.

The early documents about stability studies may be traced back to the works of Greek masters between 400 BC and 200 BC. The legacy that medieval Islamic and European authors received from the early Greeks on topics of statics and mechanics came from basically two different schools, which we now identify as the Aristotelian and the Alexandrian Schools.

The Aristotelian School emphasized qualitative research and focused on dynamics, whereas the School of Alexandria (which included Euclides, Ptolomeus, and Archimedes) based its findings in mathematics and geometry and gave special attention to statics. For example, Aristotle employed kinematic concepts to investigate changes in stationary systems; Archimedes (287 BC-212 BC), on the other hand, studied the stability of floating bodies using geometric techniques.

The Aristotelian tradition employed concepts of dynamics but did not have rigorous mathematics to carry out analysis and proofs. For the special sciences, such as Statics, Optics, and Astronomy, the medieval scholars referred to the works of the School of Alexandria, which was characterized by a mathematical perspective. Euclides provided a systematic approach to Geometry in 300 BC, but the most significant influence on medieval statics was that of Archimedes, with the consequence that the fundamental ideas in medieval Islamic and Christian countries were based on his axioms concerning the lever arm. Those studies included the concept of stability from the point of view of geometry, such as it would be needed to investigate the stability of rigid bodies.

3 ISLAMIC CONTRIBUTIONS TO THE "SCIENCE OF WEIGHTS"

An interest in learning about the sciences emerged in Islamic territories possibly before the IX Century. Al-Mamum created an institution called "The House of Wisdom" in Bagdad, in which the main activity was the acquisition of Greek texts and their translation into Arabic. Other centers were established in Damascus and in present day Iran, in which translations of scientific texts were also made. According to Lindberg [29], by the year 1000 all major Greek works on medicine, natural philosophy, and mathematics had been translated into Arabic.

This huge investment in translating and learning the Greek science was made because it was believed that they constituted valuable knowledge for the Islamic society. In some cases, such as medicine, mathematics, alchemy, astronomy or astrology, the immediate value did not require much justification, but the relevance of texts on natural history or mechanics were perhaps not so obvious.

The level of acceptance of the Greek knowledge by the traditional Islamic society has been the subject of some debate: According to some historians, science had only a marginal place in Islamic society and was tolerated by the religious establishment. Other authors state that Greek science was appropriated by Islamic countries and incorporated in various ways to satisfy societal needs [29].

The Greek discoveries were highly regarded by Islamic intellectuals, who adopted both their methodology and content; however, this was not done in a blind way and there were significant contributions made to improve the received knowledge. This interest in science lasted roughly between the IX and the XIII Centuries, but in the following centuries there was a decline associated to a general loss of power of the Islamic countries and a more strict control of all activities by the religious establishment. The tolerance under which Islamic science flourished during 400 years was lost after the XIII Century.

Some 30 important books on Mechanics by Islamic authors have been recovered during the XX Century. The contributions of medieval Islamic authors focused on more specialized topics than Greek authors, and this allowed them to gain deeper knowledge about problems related to balances and weights. The general framework for the analysis of problems in mechanics was taken from Aristotle.

Thabit ibn Qurra (836-901) wrote two important Arab texts on mechanics, one of which is devoted to the investigation of equilibrium conditions in balances. Thabit was born in Mesopotamia (present day Turkey) and died in Bagdad; his native language was Syrian but was also fluent in Greek and Arabic. He mastered most sciences and made significant contributions to mathematics, astronomy, mechanics, medicine and philosophy. In comparison with other Arab writers, Thabit seems to have been the most prominent and original scholar in mechanics.

Recent historical research and translation of medieval authors has identified that the expression "Scientia de ponderibus" (Science of Weights) is the Latin translation of a book by al-Farabi (also known as Alpharabius), who lived between 872 and 951 [1]. Al-Farabi established a separation between the science that considered machines and instruments from the more fundamental science of weights. This distinction was followed by other Islamic authors and was adopted in medieval Europe as "Scientia de ingeniis" (dealing with machines and instruments) and "Scientia de ponderibus" (dealing with weights).

Perhaps the most influential author to our understanding of equilibrium and stability during that period is al-Khazini, because his text not only reports his own work but also contains an excellent review of the works of his Islamic predecessors and contemporaries about theoretical and practical problems in statics. His statics is representative of Greek and Arab authors who preceded him, including short summaries of the teachings of Thabit ibn Qurra, al-Birruni (973-1048), and al-Afizari (1050-1110).

Abū-l-Fath 'Abd al-Rahmān al-Mansūr al-Khāzinī (who is known in the Western history of science as Al-Khazini) lived and was active in the city of Merv (or Marw) in a territory that is now Turkmenistan, towards the first half of the XII Century [20]. Little is known about his life: he was born as a slave of Byzantine origin and his master, who was a treasurer in the court of Merv, noticed the interest of his slave in philosophy and mathematics and gave him an excellent education in those disciplines[1].

Al-Khazini's books were written in Arabic (not in Persian); the most famous of them, titled *Kitab mizan al-hikma* or "The Book of Balance of Wisdom", was completed in 1121 or 1122

[1]The name al-Khazini means "related to the treasurer".

and explained the science of weights and the construction of a sophisticated balance that had previously been described by al-Afizari. According to Sarton [38] this is "one of the most remarkable books on mechanics, hydrostatics and physics of the Middle Ages". Rozhanskaya [37], mentions that the Russian orientalist Khanikoff [2] discovered a manuscript of "The Book of Balance of Wisdom" in Iran in the XIX Century, and two other manuscripts were found in India. The text is known to us thanks to a XIX Century translation [24].

In the opening pages of the text, al-Khazini states the idea that wisdom is composed of two parts, which are knowledge and action. Most of the introduction is devoted to explaining how the studies that follow fit into the religious beliefs of his time. The more technical part is concerned more with the description of several balances and their applications and less with the principles that form the basis of such knowledge.

The book is organized into eight lectures; each lecture is organized into several chapters and the chapters contain sections. In the first lecture, al-Khazini investigated the conditions under which a balance is in equilibrium. He considered a beam suspended at a point and found that the rotation around the center depends on the distance of the weights to the center and on the values of the weights. Special cases in which the weight of the balance could not be neglected, and problems in which the point of suspension was not coincident with the center of the beam were also studied by following the teachings of Euclides. According to al-Khazini, the center of gravity is the most noble and elevated concept in the exact sciences, which explains that the weight of a body varies with its distance to a point.

The author did not distinguish between forces and moments (as we do nowadays); however, his knowledge of stability criteria was clear and correct. The main focus of his work was on the description of the Balance of Wisdom; this is a hydrostatic balance of five dishes, spanning some two meters (Figure 2). Due to its length and design, this balance had good sensitivity and was also of great precision. Al-Khazini included a table showing the stability of the balance for different positions of the points of suspension. Because of the complexity of arms forming the balance, the author includes a table of actions that need to be taken by the user in order to get balance depending on the various positions in which the instrument may be in a given situation.

This sophisticated balance was improved by theoretical as well as by trial and error procedures. It was used not only to compare weights, but also to evaluate the contents of different metals (such as gold and silver) in a single object (such as a king's crown). Other applications included evaluating the specific weights of some 50 substances.

As said before, the contributions of al-Khazini to the science of weights do not seem to be due to his own original work, but mainly due to his capacity to review the work done by others. According to Hall [20], "As a student of statics and hydrostatics, even in their most practical aspects, he is heavily dependent upon earlier workers and borrows especially from al-Biruni and al-Asfizari; but his competence is not to be denied, and *Kitab mizan al-hikma* is of outstanding importance to the historian of mechanics, whatever its claims of originality or comprehensiveness may prove to be".

[2] N. Khanikoff was the General Consul of Russia in Tabriz, Persia, at the middle of the XIX Century.

The Science of Weights was no longer seen as a respectable field for serious scholars in Iran after al-Khazini's times, and it became more a concern of those fabricating or using balances.

We may conclude that the Islamic science of weight concerning stability was predominantly based on Greek thought and principles, but tackled new problems in a more systematic way than what had been done by the Greek philosophers. This allowed the development of new instruments to achieve greater precision based on traditional stability concepts. Finally, the Arab science was also enriched by knowledge brought from India thanks to expeditions made around the year 1000, in which an Arab scientist would spend several years in India to learn and teach discoveries in the two parts of the world.

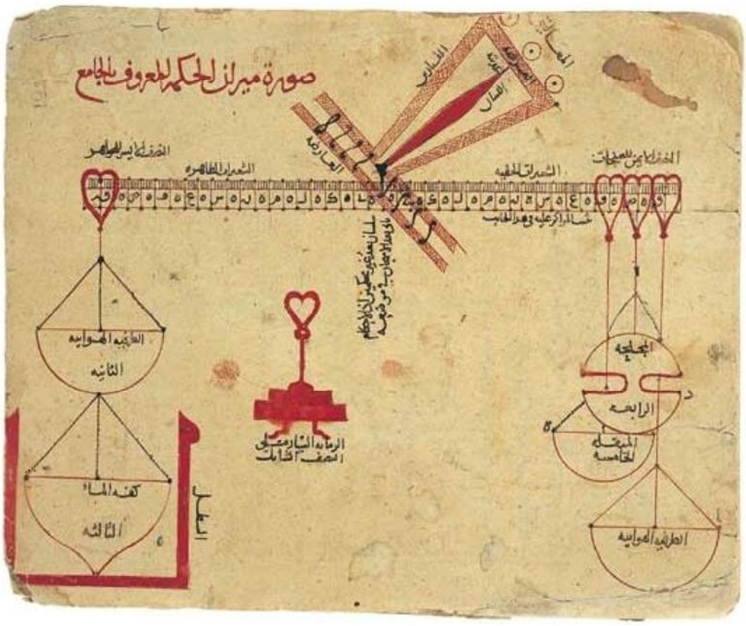

Figure 2 Balance of Wisdom [From: al-Khazini, The book of the balance of Wisdom].

4 EUROPEAN CONTRIBUTIONS TO THE "SCIENCE OF WEIGHTS"

The discipline known as "*Scientia de Ponderibus*", or the Medieval Science of Weights [5, 35], developed first in the Islamic countries and then was learned in Europe from translations.

The propositions of different medieval authors attempted to correctly predict the relations between weights and distances from the center of suspension of a balance arm (or *fulcrum*). One of the main concerns was to predict the consequences of slight modifications in a balance. The descriptions of those problems may be easily correlated with our modern concepts of stability: For example, Moody and Clagett [35] discuss the original medieval texts in terms of stable and unstable equilibrium. Similar discussions are used nowadays to introduce topics of structural stability for engineers, see for example Croll and Walker [6].

Mechanics research in the European Middle Ages was carried out by university professors who were concerned with establishing equilibrium conditions and movements of bodies. The professors were not limited to study the most obvious features of the Science of Weights, but were also interested in importing concepts from Physics to this field. Other non-academic scholars (not linked with universities) were also interested in the topic; they were geometers who used simpler methodologies in their work.

The main scholar in Mechanics during the Middle Ages in Europe was perhaps Jordanus Nemorarius (or Jordanus de Nemore), who was a well respected scholar in his own times, was forgotten for centuries, and was re-discovered towards the end of the XIX Century [11].

Nothing is known about the personal life of Jordanus, except that he lived and worked between the end of the XII Century and the beginning of the XIII Century; dating his birth and death has not been possible. According to Sarton [38], he was a German mathematician and physicist who joined the Dominican Order in Paris by 1220; however, this is based on the assumption that Jordanus Nemorarius was the same person as Jordanus Saxo, a claim that has been denied by many historians of science. Other historians [11, 19] state that he was born in France, and may have taught at the University of Tolouse (an institution that was created in 1229). Based on his writings, it seems that Jordanus was a gifted mathematician and physicist, who used knowledge from the Greek masters and went beyond them [19].

It has been argued that it is difficult to distinguish the writings of Jordanus from those of his disciples and commentators. The problem of authorship in the Middle Ages is complex because manuscripts were copied by hand and often the copyists altered the texts and introduced modifications according to their own believes. Again, much controversy exists among historians who attempt to emphasize a continuity between medieval and renaissance mechanics [11] and those who understand that there was a rupture in knowledge during the Renaissance [15].

It is now acknowledged that the book "*Elementa Jordanis super demonstrationem ponderum*", containing seven postulates and nine theorems with demonstrations, was written by Jordanus Nemorarius. Following the Archimedean tradition, Jordanus presented the laws of the lever, considering both the weights and the distances to the *fulcrum* as variables of the problem. He employed geometric demonstrations, in which the main concern was the localization of the center of gravity. According to Moody and Clagett [35], "Most medieval definitions are of a topological nature, i.e., they are of such a form as to show when two objects possess an equal or a greater or a lesser amount of some physical quantity. Thus they are of the form of proportions."

His other important book on the topic of this paper was "*Liber Jordani de ratione ponderi*" (Book of Jordanus on the Science of Weights), which was the title attributed to the book in the XIII Century. The book was translated into English by Moody and Clagett [35], and includes 45 theorems and demonstrations. Some of those theorems had already been included in *Elementa*, whereas others were corrections of previous versions that had some mistakes. The third part of this text contains six theorems about Statics and is of relevance for our historical reconstruction because Jordanus included a correct version of the conditions to reach stable equilibrium.

To understand the ideas of stability in Jordanus it is necessary to consider the concepts of decomposition of forces and of virtual displacements. Jordanus employs the concept "*gravitas secundum situm*", known as the principle of positional gravity, which means "the action of gravity depending on the position". Positional gravity was included in Postulates 4 and 5 in *Elementa*: A weight has larger positional gravity whenever the path that it follows is less oblique. Positional gravity is a fraction of the natural gravity of the weight, equal to the relation between the vertical projection (say v) and the path length (say l), so that with the relation v/l it is possible to evaluate a vertical projection. This concept is important to work with force components and is an original contribution that was not present in Greek or Islamic authors.

Next, consider the concept that we now call virtual displacement. Jordanus states that a body in equilibrium cannot have any real movement, so that to evaluate positional gravity it is necessary to open the possibility to introduce "virtual" displacements. For example, "What suffices to lift a weight w through a vertical distance h will suffice to lift a weight kw through a vertical distance h-k and it will suffice to lift a weight w-k through the vertical distance kh." [35]

The two ideas were put together to work and decide about the stability of given positions of a balance. Jordanus investigated the stability at a state by evaluating the theoretical response (i.e. its obliquity) to virtual changes in the position of the bodies that constitute a balance.

On the whole, the general concepts of stability are correctly applied. Jordanus correctly identifies an equilibrium position and imposes virtual displacements to understand how the balance behaves. To illustrate the use of stability concepts, we may refer to Theorem R1.02 in *Ratione Ponderi*:

> "When the beam of a balance of equal arms is in the horizontal position, then, if equal weights are suspended from its extremities, it will not leave the horizontal position; and if it is moved from the horizontal position, it will revert to it. But if unequal weights are suspended, the balance will fall on the side of the heavier weight until it reaches the vertical position." – Grant, 1974 [18], pp. 214

Jordanus introduced a mistake in this case when he assumes that equal weights acting with equal arms are in stable equilibrium. In fact, this would be a case of indifferent rather than stable equilibrium.

The introduction of potential gravity in the stability problem may be observed in this statement:

> "If two weights, suspended on opposite ends of a balance beam, are in equilibrium, this is because the positional gravities of the weights are equal, and equal in such manner that no advantage in positional gravity could be gained, by either weight, through any small displacement in either direction." – Moody and Clagett, 1952 [35], pp. 16

According to Jordanus, to achieve stability, a small displacement from the equilibrium position should result in an increase in potential gravity of the weight that was raised by this

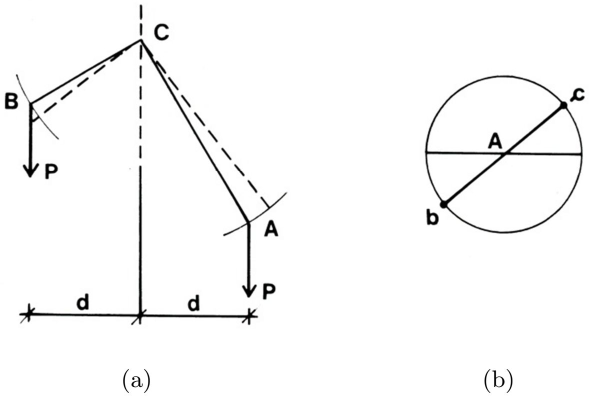

Figure 3 Illustrations in the book of Jordanus Nemorarius. (a) Reference for theorem R1.08, balance of unequal arms. (b) Reference for theorem R1.02, balance of equal arms.

displacement, in such way that the beam would return to its original equilibrium position. For the demonstration of this theorem, refer to Figure 3.b: it is assumed that c has more positional gravity than b because the movement of c to the horizontal position is less oblique than the movement of b to the vertical position.

Theorem R1.08 refers to a balance with unequal arms, which has an angle at the center of rotation (Figure 1.a). Then, "if the ends are at the same distance from the vertical line that passes through the support, equal weights suspended from the ends will have the same heaviness".

Our Figure 3.a (which is a simplified version of the drawing in Jordanus' book) is used to demonstrate this theorem. From the point of view of positional gravity, what matters is the obliquity of the displacement along the arc. To understand its stability, the balance is displaced a small angle but because the weights have different radius of rotation, then weight A has larger vertical and horizontal displacements than B. An out-of-balance moment occurs which tends to restore the balance to its original position.

> "The reason why a balance of bent arms will be in stable equilibrium where the weights are equidistant from the vertical passing through the axis of support, is because in any displacement from this position a weight would be raised some vertical distance, by an equal weight descending less than that distance." – Grant, 1974 [18], pp. 218

The knowledge reported by Jordanus was the basis of the statics taught at medieval universities and received in the Renaissance through Descartes and Torricelli. The great influence of Jordanus is evident from the inclusion of his theorems into Renaissance texts and the spread of his ideas (often without giving any acknowledgement to his work). Some stability topics reported by Jordanus were heavily criticized by Guido Ubaldo, Marquis del Monte (1545-1607), so that it is clear that Jordanus was still important in the XVI Century. Specifically, Ubaldo notices that Jordanus is not clear about an indifferent equilibrium in the balance of equal arms

[11]. According to Moody and Clagett [35], Roger Bacon had already noticed this error in 1266 in his *Opus Maius*.

An assessment of the contributions of Jordanus attempts to explain why he did not produce a tradition of work in the Middle Ages: "During the Middle Ages the scientific phase of the Greek heritage was maintained, but it was maintained at a low level of social interest. A writer like Jordanus, who possessed a great deal of originality, managed to incorporate, along with erroneous ideas inevitable in the atmosphere of his time, several original demonstrations which were in advance of the achievements of the Greeks in that field. This work failed to germinate and bear fruit in the Middle Ages because not enough persons were interested in that line of investigation" (Ginzburg, 1936 [15], pp. 361).

Stability was an established concept in the texts by Jordanus and was part of the basic knowledge inherited by the Renaissance scholars. The introduction of "*gravitas secundum positio*" allowed having a criterion regarding what a hypothetical displacement would do to the system, which is what we now call stability criterion. Finally, the discussions only addressed problems relating rigid links and masses, whereas the idea of a strange behavior of columns was not present during medieval times.

5 THE RENAISSANCE ENGINEERS: LEONARDO DA VINCI

Bernal writes that "The professions of artist, architect, and engineer were not separated in the Renaissance. The artist might be called on by his town or prince, or might offer himself, to cast a statue, build a cathedral, drain a swamp, or besiege a town. The master craftsman always had to know the properties of materials and the means of handling them. The artist of the Renaissance had to know all that and much more: he had to instill into his work geometry and mechanics in the conscious imitation of antiquity. It was in this field that Leonardo da Vinci…showed his greatest ability" (Bernal, 1965 [3], pp. 395)

Leonardo (1452-1519) was valued as an artist both in his own times and during the following centuries; however, it seems that his own interests were more closely related with engineering, mechanics, physics, anatomy and mathematics than with art. He was a self-educated person in those topics, and recorded his readings and thoughts in unpublished notebooks.

What Leonardo read about mechanics has been a topic of research and discussion among science historians [11, 12, 21, 22]. It is now acknowledged that Leonardo was familiar with the works of Archimedes on equilibrium of balances and levers, and that he also read Jordanus Nemorarius on the Medieval Science of Weights. His readings were oriented at understanding the topics in order to do something with them, such as applying the ideas to solve practical problems.

Leonardo never published science/engineering books or pamphlets during his lifetime, and all his written work is now stored into ten codices containing some 7000 pages (perhaps half of what Leonardo originally wrote since 1480). More than one hundred years after his death, Leonardo's writings were re-organized according to topics of interest, such as painting, architecture, elements of machines, and anatomy.

As said before, Leonardo was mainly concerned with what he could do with knowledge to solve problems, rather than with the prestige that authoring books could give him. After all, it was his ability to understand and solve problems that gave him work opportunities in the territories that are now Italy and France. Further, Leonardo lived during a transition in written communication with the newly invented printing press playing an important role. This was a changing environment with evolving forms of communication and information technologies.

As said before, Leonardo did not have a formal education nor did he learn Latin at school (which was the language for technical writing in his own times), so that part of the knowledge that he acquired was from conversations with travelers and colleagues, and partly from his own speculations and (sometimes) from observations and experiments that he thought or conducted. In his writings, he remarks the need to repeat experiments to be certain about the outcome. Whenever he conducted experiments, he made a series of them in which some parameter was changed in a gradual form to understand its influence. His aim was to derive quantitative rules from experiments; and if a new experiment contradicted a rule, then perhaps it was time to abandon the old rule. This is important in the present context, because he would attempt to derive rules for the lateral displacements of columns.

It is possible that he wrote in an encrypted form to protect his writings; but because his notebooks were not prepared to be published it is more difficult for us to understand his ideas on mechanics and statics. They are presented in the form of short statements derived from observations, and they sometimes contradict other statements on the same subject written a few pages before.

The notebooks that survived were mutilated by their temporary owners who attempted to give them a new organization, with the consequence that writings about the same topics are scattered in various notebooks.

His annotations on mechanics deal with movement, weight, force and percussion. There are several references about the strength of columns under compression, observing the lateral deflection of such columns. Those are his private thoughts and speculations on what we now call buckling problems. Specifically, there are annotations on these topics in *Codex Atlantis* (with texts written between 1480 and 1518) and in *Codex of the Institute of France*, which includes texts written between 1492 and 1516. There are also studies on mechanics in Codex Madrid I [9], possibly written between 1490 and 1496, which were re-discovered in 1966 [36]. The discovery of *Codex Madrid I* was made after Truesdell [39] and Hart [21] wrote on the topic and much later than the writings of Duhem [12].

A dominant form of expression in the notebooks is the drawing, whereas the text is inserted to explain information that was shown in the graphics. Those are carefully drawn figures with text filling the spaces between them. Some drawings show schematics of deflections of columns. Projects are developed to measure quantities from experiments, but there is no evidence that he actually performed such experiments. According to Truesdell [39], whenever Leonardo writes about experiments he does that in future tense, showing that those were plans of things to be done to find something. On the other hand, it seems clear that he was constantly observing from nature as a source of evidence provided by the physical world, and it was from observations that he could draw conclusions and write tentative laws. This was the case for columns under compression.

Codex Madrid I is devoted to mechanics and statics, partly on fundamentals and in part on applications. At various places there are entries on pillars supporting weight in the axial direction; his annotations in this Codex are of a qualitative nature (rules on pillars are given in *Codex Institute of France* [10]). In page 135 of *Codex Madrid I* [9] we read:

> "A support in which the center of weight that loads is not perpendicular on the support center will have little stability".

The influence of column length is considered in page 155, showing that Leonardo was aware that length has a negative influence on the column capacity to bear loads:

> "Supports bearing loads along a perpendicular line will be weaker the longer the distance they are from their upper end. This is understood in cases of frame columns made of timber, columns, pillars and things like that and the rule may be checked as we did in the case of a cord, because one could imagine a pillar so high that its own weight would reduce its base to thin powder. The same thing would happen if it were loaded by superimposed excessive weight".

Here Leonardo argues using an analogy between the pillar and a cord, probably because at that time he had no opportunity to develop a solid argument for the pillar.

The most important part is found at the reverse of page 177, with reference to Figure 4:

> "The loaded support necessarily bends or compresses in that part which is under the center of the weight that loads it. And whatever the position in which the center of weight is placed on the support, this will always bend in such a way that at mid-height it will deflect towards the inside. Any part of the support placed below the center of weight will be subjected to this torsion, except for the center of support. Whenever this [center of support] is placed below the center of weight, it will not be possible that the support bends to anywhere. For any other place in which the centers are located, they will make that the support will twist or bend to the opposite side, as shown here below".

We may infer from these annotations that Leonardo qualitatively understood the phenomenon of lateral bending due to a vertical load acting on a column. Finally, the reverse of page 189 contains writings on the strength of pillars:

> "If twelve supports weight separately one ounce each, with a length of 1/2 code and bear one pound, once they are grouped together they will bear 4 pounds".

Next, he repeats the idea of centering the loads:

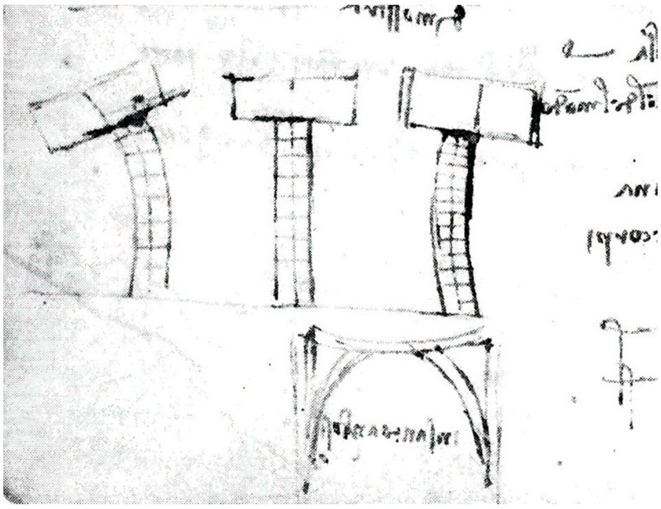

Figure 4 Leonardo da Vinci: drawing to illustrate the bending of a column under a vertical weight [from *Codex Madrid I* [9], page 117].

> "A support in which the center of weight coincides with the center line of the support will take full advantage of its power".

There are many entries in the notebooks in which Leonardo drew columns supported at the base under an axial load. We find especially interesting those annotations in which he goes beyond observations and advances rules that he had found. Figure 5, from Codex Atlanticus [8], shows two compressed columns of the same cross section and different lengths, indicating the relative loads that each can take. For the short column (with unit length) a load of 1000 is indicated, whereas a load of 100 is written in connection with the long column (with length equal to 10). According to this, one sees an inverse relation between column length and strength.

In *Codex Institute of France* [10], Leonardo writes that if several pillars are tied so that they resist together, the strength of the group will be larger than one single equivalent pillar:

"Many little supports held together, i.e. in a bundle, are capable of bearing a greater load than if they are separated from each other. Of 1,000 such rushes of the same thickness and length which are separated from one another, each one will bend if you stick it upright and load it with a common weight. And if you bind them together with cords so that they touch each other, they will be able to carry a weight such that each single rush is in the position of supporting twelve times more weight than formerly" [21]. In the same page, he considers the relative load capacity of two bars with equal length but different cross section:

> "A support with twice the diameter will carry eight times as much weight as another, both having the same height".

Hart [21] considers that the proof is not convincing. According to Leonardo, the load capacity is directly proportional to the area of the cross section and inversely proportional to

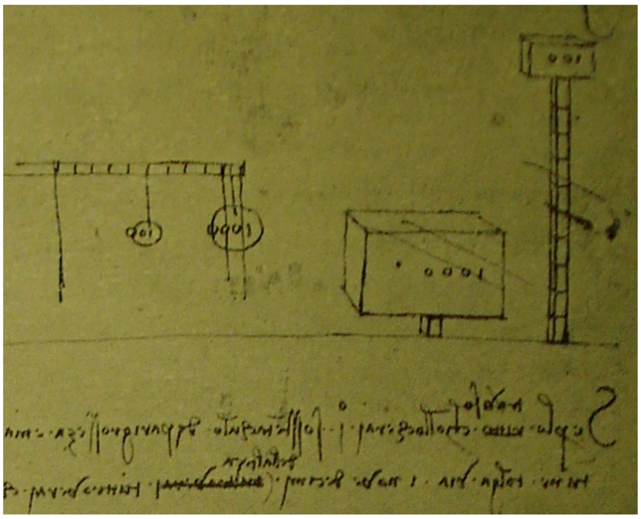

Figure 5 Drawing by Leonardo da Vinci [from *Codex Atlanticus* [8]]. The figure shows that two columns with the same cross section and loaded by vertical weights can resist different loads on an inverse relation to their length.

the ratio between height and thickness. Such rule should apply to the case of Figure 6: the column on the left has thickness $d = 2$, area $A = 4$, and length $L = 8$, with a ratio $L/d = 1$. The strength should be $S1 = 1$. For the column on the right, the numbers are $d = 1$, $A = 1$, $L = 8$, $L/d = 8$ and $S2 = A/(L/d) = 1/8$. The ratio $S1/S2 = 8$ is the number written by Leonardo in Figure 6.

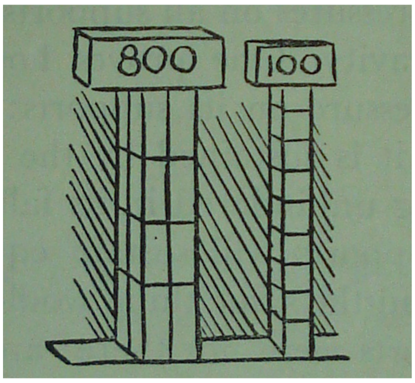

Figure 6 Drawing by Leonardo da Vinci: [from Manuscript A, Collection of Institute of France]. The figure shows that two columns of the same length but different cross sections can carry loads proportional to their area.

Finally, a comparison is made between the loads in pillars of the same cross section and different lengths, to conclude that they are inversely proportional to their lengths.

Truesdell translates Leonardo's estimates on vertical pillars from *Codex Institute of France A* as follows:

> "If you load a pillar erected vertically in such a way that the center of the pillar is beneath the center of the weight, it will compress rather than bend..." – Truesdell, 1960 [39].

Truesdell has identified two rules on the strength of pillars loaded by P:

$$P_c \propto \frac{\sqrt{A}}{\ell}$$
$$P_c \propto d^3$$

where A is the cross section of the pillar, d the diameter, l the length. "His rules, while showing that he was an acute observer of experience, seem to arise from a kind of plausible rhetoric in a background of deep attachment to simple proportion" (Truesdell, 1960 [39], pp. 20).

Truesdell acknowledges the difficulties in interpreting those rules, on account that Leonardo did not provide definitions and did not specify what he kept constant in each case and what he changed.

> "If we regard the second rule as a correction to the first when l = constant, then it may follow that Leonardo's final rule is $P_c \propto \frac{d^3}{\ell}$, but this is far from certain". – Truesdell, 1960 [39], pp. 20

In summary, we observe that the annotations in *Codex Madrid I* are of a qualitative nature and help to assure that Leonardo had clear ideas about the behavior of compressed pillars. He made significant contributions to understanding the conditions under which columns develop bending. From the writings of *Codex Institute of France*, Truesdell [39] has ventured that Leonardo understood that the strength of pillars was a function of the cube of the thickness and the inverse of the length.

There are at least six areas of difficulties to fully understand the contributions of the notebooks to our field. (1) His writings on any topic are extremely short. (2) The communication style of Leonardo emphasized graphics over text, in contrast with his contemporaries who emphasized text and seldom employed graphics. (3) Leonardo did not keep a strict order in his annotations, so that the same topic may be found at various places and times, and they sometimes contradict each other. (4) The ideas of Leonardo changed over time, leading to contradictory statements and rules. (5) Perhaps half of the original notebooks have been lost. (6) Following Leonardos death, the notebooks were dismembered and shuffled like in a card game.

The impact of his ideas is also difficult to assess. On any given topic we have no direct means of assessing what was his original idea and what was a recollection of the work of others with whom he was in contact or whom he read. In the first case, due to the private nature of the notebooks, the impact would have been negligible on Renaissance scholars and after, but in the second case we would be referring to public domain knowledge that should show in the writings of the XVI and XVII Centuries. Thus, an indirect means of assessing the private-public nature of his knowledge is with reference of the work of others in the years to follow. This is the topic of the next section.

Leonardo left extremely interesting short notes in his voluminous notebooks on the bending of clamped columns under compression. He also produced quantitative information on the maximum load that the column can resist. Had this been part of an open chain of public literature, perhaps others would have taken his observations and refine them; however, the manuscripts were never in the hands of scientifically oriented persons and were only available when they were of historical rather than scientific interest.

6 THE EXPERIMENTAL PHILOSOPHERS IN THE XVII CENTURY

One hundred years after the death of Leonardo, the load that could be taken by a column was considered by Frere Marini Mersenne (1588-1648), a priest of the order of Minimis in France [7]. "The new experimental philosophers... no longer formed part of the intense city life of the Renaissance; they appeared more as individual members of the new bourgeoisie" (Bernal, 1965 [3], pp. 418).

Mersenne (shown in Figure 7) was better known for his role in establishing what became known as the Republic of Letters for savants, a system of communication between scientists of different parts of Europe. "He was an indefatigable correspondent, acting as a kind of general post office for all scientists in Europe from Galileo to Hobbes" (Bernal, 1965 [3]). Thus, Mersenne may be considered as an extremely well informed scholar in his own time, thanks to his contacts with other French and European researchers. If any new contribution was made to the scholarly field of mechanics, he would be one of the first to know.

Figure 7 Marini Mersenne, painting by his contemporary Philippe de Champaigne.

There are two places to read Mersenne about the strength of pillars: his translation of the work of Galileo [31] and his own treatise of mechanics [33]. In the first case, it is not easy to recognize the work of Galileo in the translation by Mersenne: Galileo presented his Dialogues on Two New Sciences in the form of a theatrical dialogue between three characters, but Mersenne eliminated the dialogues and converted everything into impersonal statements. Lenoble [28] compared the contents of both versions (original and translation) to find that the translation was much shorter.

Although Galileo's work was exceptional in developing strength of materials, there was no discussion on compressed columns, in part because apparently he did not distinguish between tension and compression.

The treatise of mechanics is the third part of the book *Cogitata* by Mersenne [32], containing a summary of what was known in his times. The mechanics start with topics: "*de balanza, de trochleis, & ad vectem referee; planique inclinati mechanicum auxilium investigaie*". There are no equations employed to represent knowledge, but there are basically propositions and arguments on various topics.

Consider his Proposition XIX, in which he lectures on the strength of cylinders by discussing the influence of length and thickness on strength. With reference to Figure 8, Mersenne presents four lines of cylinders. First, he takes those identified as AC and DF, with equal length but different diameters (to be more precise, Mersenne refers not to diameters but to thickness-*crassitudine*). He states that the strength of DF depends on the triple relation (cube) of the strength of AB, and to illustrate his point he provides numbers: if the diameter of DE is three times that of AB, then the strength of DE is 27 times that of AB.

Next, he illustrates the incidence of length by comparing a long cylinder KP with a short one (HI). If the length of KP is twice that of HI and has twice the diameter, then there will be a cubic relation due to diameters plus a linear relation due to length, leading to a quadruple relation. Clearly, he believed that the strength increases with length, having a positive influence. Notice that Mersenne postulates relations between strengths but provides no indication of the way the cylinders are loaded.

His narrative with reference to Figure 8 does not provide experimental information, but at some point he discusses the response of metal columns made of different materials as they stand compression. He acknowledges that a perplexing behavior occurs under compression:

> "Among all the difficulties, the most serious is that almost all bodies are distinguished in a peculiar way because of the different arrangements of the different fibers: where some happen to have much laminae, as for instance an oak, others present few or none, e.g., iron, marble and glass. It may be added that iron, copper and other metals, even single bodies, subject to force of weight, curve and bend to the form of an arch before breaking. This produces a new difficulty which escaped even Galileos notice. If there is a physico-mathematician who is capable of finding a solution, he will deserve more credit than the inventor of quadrature." – Mersenne, 1647 [34], pp. 150

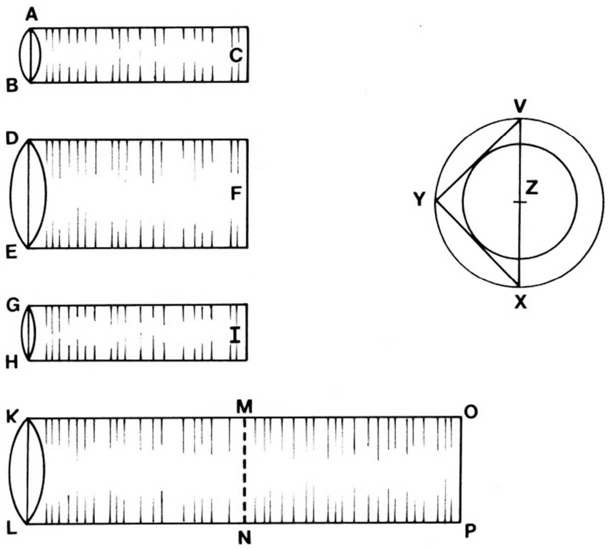

Figure 8 Figure adapted from the drawing presented by Mersenne in his treatise on Mechanics [33], pp. 68.

This illustrates the astonishment of Mersenne as he faced the phenomenon of lateral displacement of a compressed member before breaking. Mersenne also speculates that this effect will surprise many readers, but the sole observation did not advance to the comprehension about causes or parameters that would be involved. In his times, this was one of several topics for which no answers were available. Notice that this is a significant step backwards with respect to the writings of Leonardo, who was not astonished and even provided quantitative estimates of the response under compression.

Benvenuto states that: "Mersenne and his contemporaries had no idea what the root cause of resistance might be. The physical reasons for resistance were still wrapped in the deepest mystery. But this did not exclude the possibility of gathering useful information about the measure of resistance (a purely phenomenological description, but hardly a trivial one). Mersenne is explicit on that point, and promises to return to the subject if the Lord gives him strength." (Benvenuto, 1991 [2], pp. 205)

This astonishment in Mersenne, rather than showing a search direction to his contemporaries, illustrates his vision of frontiers that cannot be reached by the use of reason (as Galileo attempted in other topics) and belonged to what was only accessible through observation and experimentation.

The limitations in the understanding of Mersenne show that the knowledge of Leonardo did not permeate to Mersenne and his contemporaries, thus serving as an indirect proof that Leonardos thoughts were only private in nature. Even 100 years later, Petrus van Musschenbroek [41] carefully cited Mersenne with great respect but did not mention Leonardo. His own work was based on a similar philosophy, i.e. that the results had to be derived from experiments because the doctrine of resistance is always too problematic for the look of mathematicians".

Duhem, on the other hand, had less favorable comments on the work of Mersenne, stating that his Tractatus Mechanicus was only a compilation of work done by others on statics [11].

7 CONCLUSIONS

As said before, it is not simple to make a fair assessment of the knowledge that researchers had before the XVI Century. Part of the difficulties are the limited early original manuscripts that survived up to our times, language problems (texts being written in Arabic or Latin), but there are also problems with the translations of concepts, especially with those that were employed during medieval times (as mentioned by Kuhn [26]).

The symbolic value of balances throughout the centuries is a fascinating topic. In Greece, balances were used not just to weight goods, but also for the weighing of the souls in the afterlife (an activity known as psychostasia). In the medieval Islamic world, the balance was associated to moral and justice: a good precision was required to assure honesty in the transactions [1]. However, to understand the importance of the study of a balance in the medieval world it is crucial to understand not just the cultural context or the practical applications of the studies but also their theoretical importance within the discipline: The methodology to treat Statics problems of any kind in the Science of Weights was to reduce each problem to the case of a lever or a balance. Thus, the study of the balance played a role similar to the use of equilibrium equations in the mechanics of the XIX Century.

One key point about the mind frame in which authors developed their ideas is that following the scientific revolution, a sense of knowledge accumulation existed and this led to a sense of progress. Each contribution was seen as an advance in the frontier of knowledge towards some form of "truth". Herbert Butterfield argued that such ideas of progress did not exist during medieval and renaissance times:

> "On this view of the universe, time and the course of history were not considered to be generative of anything... Men assumed rather the existence of a closed culture, assumed that there were limits to human achievement, the horizon reaching only to the design of recapturing the wisdom of antiquity, as though one could do no more than hope to be as wise as the Greeks or as politic as the Romans. On the same view, the notion of something like a "Renaissance" was a comprehensible thing." –Butterfield, 1957 [4], pp. 212

Thus, it seems that during the Renaissance it made no sense to believe in what we now call progress or to work in that direction. The crisis of this paradigm occurred sometime during the XVI Century leading to the controversy between Ancients and Moderns [23]. This may help to explain why the medieval and renaissance scholars only tackled the then classical topic of stability of rigid bodies and did not approach problems related to the stability of flexible bodies. It took an outsider to the scholarly system, like Leonardo, to start from his own observations rather than relying mainly on established views on the subject.

This review has shown that the focus of stability studies was on the behavior of rigid bodies, such as the balance. We now know that in the next two centuries the emphasis would be placed on the bending behavior under compression of columns. But the connection between stability concepts (such as those developed in the Science of Weights) and lateral deflections under compression was never made during medieval and renaissance times. Thus, the main works of the XVI Century can hardly be seen as fundamentals for the work on lateral deflections of pillars under compression. In contrast, the work of Petrus van Musschenbroek considered the maximum loads in columns without recurring to geometric ideas (as in the stability of rigid bodies), whereas the mathematical work of Euler had to do with the findings of his contemporaries on the bending of beams.

Finally, we may consider if there was continuity in terms of (1) problems and (2) concepts and approaches between the periods considered in this work. We postulate that there was a continuity in problems of stability of rigid links between medieval authors and those who lived in the XVI and XVII Centuries, and also a continuity of problems of compressed columns between some authors in the XVI and XVII Centuries and those of the XVIII Century; however, in the latter case there was a discontinuity between the concepts and approaches employed to solve those problems.

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