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Design of tensegrity structures by minimizing static compliance

Abstract
This paper presents a new approach to simultaneously obtain the optimal design parameters of tensegrity structures, including pre-stress force and sectional area of elements. The proposed method is based on minimizing the static compliance in terms of some linear and nonlinear constraints using a genetic algorithm tool. This work is devoted to optimize the concurrent cross sectional area of element and pre-stress of a tensegrity system which works under the applied external loading along with a fixed topology and shape. An increase in the rigidity of systems is one of the outcomes of modifying of tensegrity structures according to optimal design parameters. The validation, efficiency and characteristics of proposed algorithm to find the optimum values for design variables of tensegrity structures is tested through three known tensegrity structures.

Keywords
Design tensegrity Structures, Pre-stress, Static compliance, Genetic algorithm

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1 INTRODUCTION

Tensegrities are structures whose integrity is based on a balance between tension and compression. They are a kind of spatial structural system composed of cable (in tension) and strut (in compression) components with reticulated connections, and assembled in a self-balanced fashion, (Barbarigos et al. 2010). Several descriptions of tensegrity structures have been recommended so far depending on their intended focus. A recent and the most accepted definition was suggested by Motro (2003). Optimal design of tensegrity structures is relatively complex, mainly because they have a non-linear behaviour.

The design of tensegrity structures can be categorized into two sections. The first one, which is called a form-finding step, focuses on finding a super stable geometry and normalized force density coefficients corresponding to a proper topology of the structure. In the second step, the optimal self-stress (pre-stress) force and cross-sectional area of elements are obtained based on minimizing an objective function such as static compliance work or dynamic behaviour etc.

In the literature, the first section has been broadly investigated by different approaches such as nonlinear programming (Pellegrino, 1986), dynamic relaxations (Barnes, 1999), force density (Vassart and Motro, 1999), stochastic search (Paul et al., 2005), numerical analysis (Tran and Lee 2010-a, Tran and Lee 2010-b) and more recently by finite-element approach (Pagitz and Tur, 2009) and genetic algorithm (Xu and Luo, 2010-b). The form-finding is usually seen like a distinct stage in the structural design process of tensegrity, yet it can also be considered as a part of the static analysis (Dalilsafaei et al. 2011). There are many contributions on finding initial element forces for a given geometry is known as pre-stress design, (Dalilsafaei et al. 2011). The pre-stress not only plays a key role to prevent slackening of cable elements, but also it provides the structural stability of the system. Pellegrino and Calladine (1986) proposed a method to obtain the force density coefficient from the null space of the equilibrium matrix. Moreover, Tran and Lee (2010-b) provided an approach for finding a single force density coefficient by considering the unilateral properties and the structure stability. More detailed information about finding the optimal design pre-stress force can be found in (Xu and Luo, 2010-a). More recently, Dalilsafaei et al. (2011) suggested a method for finding the least value of the pre-stress corresponding to the value of the direction and position of external loads. They used the flexibility-based in order to finding the least magnitude pre-stress for a given tensegrity.

In spite of a vast literature on the form finding step, and pre-stress, few contributions have focused on designing the tensegrity structures. Kebiche et al. (1999) stated that the level of the pre-stress and rigidity ratio between strut and cables are important to design tensegrity structures. Zhang et al. (2006) suggested a direct approach to design the geometry of tensegrity structures. Smaili and Motro (2007) showed a design analysis of curved tensegrity systems and investigated the deployment procedure requirements of such systems. Despite of numerous contributions used on the form finding and determination of the pre-stress of tensegrity structures, there is no general investigation to obtain the optimal design parameters of tensegrity structures including, the pre-stress force and cross-sectional area of elements, simultaneously.

The main contribution of this work is summarized as obtaining the optimal of pre-stress and cross-sectional area of the tensegrity systems simultaneously in such a way that their static compliance in terms of some physical constraints is minimized. The physical constraints include no slacking for cable element, no buckle for strut element and yielding stress capacities of cable and strut elements. Besides, the upper bound of the axial stresses at each member is smaller than the ultimate stresses in both loaded and un-loaded equilibrium configurations. Here, genetic algorithm is used for solving nonlinear constraints' minimizations. It also should be noted that the lower and upper bound of design variables should be defined. Modifying of the tensegrity structure according to the obtained design variables increases the rigidity of the given tensegrity structure, because all cable elements are in tension, and all compressive strut elements are not subjected to buckle. In this study, it is assumed that the form finding step has been done. It means that a super stable pre-stress configuration of the tensegrity and the normalized force density coefficients are known. In order to finding the unilateral force density coefficients, a method suggested by Tran and Lee (2010-b) is applied.

The rest of the paper is organized as follows: In Section 2, the analytical formulation used for solving the nonlinear static response of the structure is described and optimization parameters, nonlinear constraints are explained. Section 3 describes the solution procedure for solving the nonlinear constrained minimization. Furthermore, numerical examples are given in Section 4. Finally, in Section 5, some conclusions are drawn.

2. Analytical formulation

2-1 Static formulation of the problem

The pre-stress state between tensioned cables and compressed struts makes a tensegrity structure stable. The feasible node coordinates (nodal coordinate), element pre-stressed lengths, l_i, and normalized force density coefficients q_i of tensegrity systems are known.

To describe the static behaviour of a tensegrity structure around an equilibrium configuration, a static model can be used. The matrix form of the equilibrium equation at a pre-stressed configuration along with external force in the global coordinates is as follows:

$$\mathbf{K}_{\mathrm{T}}^{\mathrm{g}}\,\mathbf{u} = \mathbf{F}^{\mathrm{ext}} \tag{1}$$

where $\mathbf{K}_T^g$ and $\mathbf{F}^{ext}$ denote respectively the tangent stiffness matrix and external load vector. Also, $\mathbf{u}$ is the vector of nodal displacement. It is worth mentioning that, an incremental- iterative solution scheme such as modified Newton- Raphson method is employed to obtain the solution of nonlinear Eq. 1. The components of the tangent stiffness matrix are the material stiffness matrix $\mathbf{K}_E^g$, normally used for small-deformation truss, and the geometrical stiffness matrix $\mathbf{K}_G^g$ caused by pre-stresses, (Guest, 2006).

$$\mathbf{K}_T^g = \mathbf{K}_E^g + \mathbf{K}_G^g \tag{2}$$

A finite-element model of a tensegrity system can be developed in local coordinates by the following stiffness matrices for each member in the structure, (Ali and Smith, 2010).

$$\begin{gathered}\mathbf{k}_E = \frac{E_i A_i}{l_{0i}}\begin{bmatrix}\mathbf{I}_0 & -\mathbf{I}_0 \\ -\mathbf{I}_0 & \mathbf{I}_0\end{bmatrix}, \mathbf{I}_0 = \begin{bmatrix}1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{bmatrix} \\ \mathbf{k}_G = \frac{F_i}{l_i}\begin{bmatrix}\mathbf{I}_3 & -\mathbf{I}_3 \\ -\mathbf{I}_3 & \mathbf{I}_3\end{bmatrix}, \mathbf{I}_3 = \begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{bmatrix}, i = 1, \ldots, b\end{gathered} \tag{3}$$

where, E_i, A_i and b express the elastic modulus, cross-sectional area of elements and the number of sum cable and strut elements, respectively. Furthermore, l_{0i} indicates the rest length of the member computed using the following Eq. 4-a:

$$l_{0i} = \frac{E_i A_i l_i}{F_i + E_i A_i} \tag{4-a}$$

where F_i expresses the pre-tension force, and it is calculated as follows:

$$F_i = \alpha q_i l_i \tag{4-b}$$

where α is the coefficient of pre-stress force. The main goal of this study is to find the optimal coefficients of pre-stress force and cross-sectional area of strut and cable elements when the tensegrity structure is subjected to various external forces. Generally, in tensegrity structures, the design task for the element optimization is to find out such an optimal design variables that produce the structure with the high rigidity. If design variables are not optimally selected, it will be possible that one or more cable elements might go to slack, and/or internal member stresses get bigger than the ultimate stresses. Or, one or more strut members might go to bucking. Therefore, the rigidity of the tensegrity structures reduces, and then the risk of the structure failure increases. To overcome this problem, the following nonlinear constraint of the minimization is taken into account to design tensegrity structures.

2.2. Defining the objective function and design variables

The rigidity of the tensegrity structure can be measured by different criteria. One possible evaluate is the work done by the external loads in deforming the tensegrity structure from the prestressed configuration to the new equilibrium configuration. Therefore, the inner product of the external forces and deformation of the structure called the static compliance and then it is considered as the objective function.

$$\text{Fit} = \mathbf{f}_{\text{ext}}^{\text{T}}\mathbf{u} \tag{5}$$

It is obvious from Eqs. 1-4 that the static responses of the system are depended to the coefficient of the pre-stress, cross-sectional area of members, pre-stressed length member ($l_{\text{ci}}, l_{\text{si}}$), normalized force density coefficients (q_{ci}, q_{si}) and the mechanical properties. It is mentioned previously that, in this study, the normalized force density coefficients and pre-stressed length of members (which are computed from the form finding stage) along with the mechanical properties are known as a given data. Therefore, the design variables are the coefficient of pre-stress force (α) and cross-sectional area of strut (A_{si}) and cable (A_{ci}) elements. As a result, the minimization of the static compliance of tensegrity structures in terms of some constraints is mathematically formulated as the following:

$$\begin{aligned}
&\text{Given data}: \text{E}_{\text{ci}}, \text{E}_{\text{si}}, q_{ci}, q_{si}, l_{ci}, l_{si} \\
&\min_{\alpha, \text{A}_{\text{ci}}, \text{A}_{\text{si}}} \ \mathbf{f}_{\text{ext}}^{\text{T}}\mathbf{u} \\
&\text{S.t.} \\
&1: \frac{\text{F}_{\text{ci}}}{\text{A}_{\text{ci}}} = \frac{\alpha q_{ci} l_{ci}}{\text{A}_{\text{ci}}} \le \text{S}_{\text{cable}}^{\text{ub}} \\
&2: \frac{\text{F}_{\text{si}}}{\text{A}_{\text{si}}} = \frac{\alpha q_{si} l_{si}}{\text{A}_{\text{si}}} \le \text{S}_{\text{strut}}^{\text{ub}} \\
&3: -\left(l_{nci} - l_{ci0}\right) < 0 \\
&4: \left(\frac{\text{E}_{\text{ci}}\left(l_{nci} - l_{ci0}\right)}{\text{l}_{\text{ci0}}}\right) - \text{S}_{\text{cable}}^{\text{ub}} \le 0 \\
&5: \left(\frac{\text{E}_{\text{si}}\text{A}_{\text{si}}\left(l_{nsi} - l_{si0}\right)}{\text{l}_{\text{si0}}}\right) - \text{S}_{\text{strut}}^{\text{ub}} \le 0 \\
&6: \left(\frac{\text{E}_{\text{si}}\text{A}_{\text{si}}\left(l_{nsi} - l_{si0}\right)}{\text{l}_{\text{si0}}}\right) - \frac{\pi^2 \text{EI}_{\text{mini}}}{\text{l}_{\text{si0}}^2} < 0
\end{aligned} \tag{6}$$

where, $\text{E}_{\text{ci}}, \text{A}_{\text{ci}}$ and $\text{E}_{\text{si}}, \text{A}_{\text{si}}$ are the modulus of elasticity, cross-sectional area of cable and strut elements, respectively. l_{ci0} and l_{si0} denote the rest lengths of the cable and strut elements, respectively which is computed from Eq. 4-a. l_{nsi} and l_{nci} are the lengths of the strut and cable elements at new equilibrium configuration. The system is moved from pre-stress configuration to a new equilibrium configuration when it is subjected to external loading. It should be noted that these lengths are calculated by adding the pre-stressed length plus the vector of nodal displacement $\mathbf{u}$. $\text{S}_{\text{strut}}^{\text{ub}}$ and $\text{S}_{\text{cable}}^{\text{ub}}$ denote the upper bound constraints on the member stress in the strut and cable elements. I_{mini} shows the minimum moment of the inertia of strut elements. The linear and nonlinear constraints of the optimization problem for tensegrity structures are described as below.

The first and second constraints express that the internal stress capacity in cable and strut elements at pre-stressed configuration should be smaller than the upper bound for ultimate stresses in cable and strut elements.

The third constraint is only applied for string elements. By satisfying this constraint, it is guaranteed that the cable elements do not prone to slacking, and all of them are in tension so the rigidity of the structure is higher than those cases in which one or more cables go to slacking.
The fourth constraint states that the member stress capacity in string elements must be less than the upper bound ultimate stress for a member after applying the external load. Thus, the tensegrity structure will go to yielding if this constraint is not satisfied.
The fifth constraint dictate that the member stresses capacity in strut elements must be less than the upper ultimate stress on member after applying the external load. The tensegrity structure again goes to yielding when this constraint is not satisfied.
The last constraint is only applied to the strut elements owing to this fact that the strut elements are the only members that should be compressed. Satisfying of this constraint causes that the strut elements are prevented of buckling mode in tensegrity structure.

3. Genetic algorithm

In the literature there are many options for static compliance optimization of regular structures. One of the popular methods is classical deterministic models. In classical deterministic models, the end result is a function of the starting point or starting interval. Another disadvantage of deterministic methods is the computational difficulty involved in the calculation of derivatives and hessians. Genetic algorithms, on the other hand, are simple and competence to execute and include evaluations of only the objective function and the use of certain genetic operators to investigate the design space. Moreover, a population of optimum points is obtained that will allow the designer to select a design that satisfies all subjective constraints as well. These characteristics make this approach well suited for finding optimal solutions to the non-linear tensegrity problems. Genetic algorithm (GA) is a stochastic global search technique corresponding on Darwin's evolution theorem of 'survival of fittest' (Goldberg, 1989; Michalewicz, 1992). GA is a powerful method for different types of optimization problems. This approach was first inspired by Holland (Holland, 1975) and used by many others as one of the most common and practical meta-heuristic methods. The algorithm starts from an initial set of random feasible solutions which is known as population. The individuals of the population called 'chromosomes' are estimated according to a predefined fitness function. The chromosomes develop through successive iterations based on some genetic operators (selection, crossover, mutation, etc.). This process is called 'generations'. At each generation, a new population is generated by applying the crossover and mutation operators on old population. The procedure is repeated until a particular stopping criterion is reached (El-Baz, 2004). Here, GA is used as a global search to find optimal solution of the attempted problem. The procedure of the genetic search algorithm used for our desired problems is described in the following steps.
First of all, an initial population of individuals is randomly generated. Secondly, the nodal of the system are calculated based on the initial population. Using the obtained nodal displacement, the other constraints given in Eq. 6 are checked. If all constraints are satisfied this set of population will be feasible, otherwise, if one or more of these constraints are not satisfied this set of population will be removed from the initial population. Among all initials population, the value of the objective function for all extracted feasible solutions is computed, and is ordered from minimum value to maximum one. This ordering is necessary for starting new selections. In each iteration, the minimum objective function is picked which is then called elite. Next, in order to select the parents, the roulette wheel technique is used. The selection chance of each chromosome is relative to its fitness value, the selection chance of chromosome i denoted by $p_{selection}(i)$ is stated by the Eq. 7 in which $Ft(i)$ is the fitness of chromosome, i, (Lee et al., 2003).

$$p_{selection}(i) = \frac{1/Ft(i)}{\sum_j 1/Ft(j)} \tag{7}$$

It is worth noting that the objective function value associated with a chromosome is considered as the fitness value of that chromosome. Since the objective function of the formulated model is minimized, we consider the reverse value of the fitness in Eq. 7 to enhance the selection probability of the chromosome with lower fitness. Furthermore, in order to generate offspring (children) from parents the cross over operator is employed in such a way that the two genes are chosen randomly between two parents and their contents are swapped. Therefore, the first parent and the second parent construct the first child code and vice versa for the second child. Then, Mutation is mainly used to discover new areas in the investigation space and used to avoid being trapped in local optimum. The mutation used in this study is called swap mutation. First a random number, says R, is generated between zero and three, because there are, here, three design variables. If the random number is $0 \le R < 1$, $1 \le R < 2$ and $2 \le R \le 3$, the first, second and third columns of the population are respectively selected and their contents are swapped by a random value in the range of their design parameters. The new generation is then chosen between the old generation and newly generated children. Here, in order to select the new generation two mechanism, namely the roulette wheel and elitism policy are taken into consideration. This procedure is repeated until the number of iterations is reached to the specified stopping criterion. The flow chart of the GA procedure is shown in Fig.1.

4. Numerical examples

It is worth mentioning that static compliance optimization of regularly truss structure is a rather well-investigated problem. But, this work is one of the pioneers study dealing with simultaneous cross-sectional area of elements and pre-stress force optimization of tensegrity. The lack of researches in the discipline leads to the problem of result comparison in this study. In order to remove this obstacle another truss numerical example is investigated to prove the formulation accuracy.

4.1 A 72 bar double layered grid

Tensegrities are considered as a particular case of spatial trusses, and, therefore, an application of space lattice systems in which mainly axial efforts are observed (Kahla and Moussa, 2002). Therefor, the following numerical example is studied for ability and validation of proposed algorithm.
A 72 bar double layerd grid from Levy and Hanaor (1992) is considered. The truss is prestressed with configuration shown in Figs.2-a, b. The system is composed 72 elements having equal lengths of 54 in. An external load of 13,534 lb acts on the center node in the vertical direction.

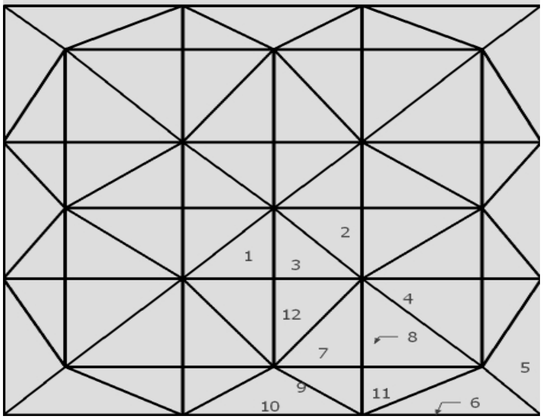

Figure 2-a. A top view of three-dimensional bar grid

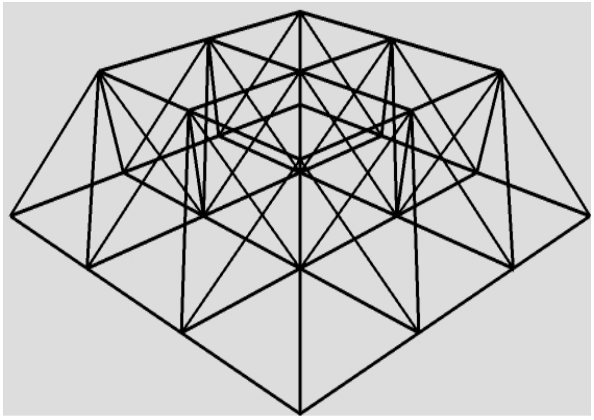
Figure 2-b. A perspective of three-dimensional bar grid

All edge nodes are constrainted in vertical direction while the center node is free to move in the vertical direction. Additionally, one corner node is restrainted in all directions. Material properties are specified as follows: Young modulus E= 16e6 ksi, the allowable stress in tension 40000psi and for compression 20000 psi. Due to symmetry, one eighth of the structre is numberd as shown in Fig. 2-a. In this system, the memebrs 5 and 9 is prestressed and the value of prestress is -0.1205 and -0.3085 lb respectively. Levy and Hanaor (1992) studied the effect of prestress on the minimum wieghit design of singly loaded trusses. The optimal force and cross sectional area of members are tabulated in Table 1. Also, the results obtained from proposed algorithm are tabulated in Table 1. It is worth mentioned that number of generations and number of individuals in each generation are 30 and 1000.

Table 1. Optimal design of a 72 bar double layerd grid

	Optimal force (lb)	Optimal force (lb)	Optimal force (lb)	Optimal force (lb)	Cross sectional area (in^2)
	Bar 1	Bar 3	Bar 5	Bar 6	
Levy (1991)	5583	-2791	1595	-798	0.1395
Proposed algorithm	5405	-2700	1570	-782	0.1363

According to Table 1, the results from Levy's approach (1992) and proposed algorithm are in a good agreements.

In continue, three numerical examples of tensegrity optimozation are provided. Due to the aim of the minimization of static compliance, the original shape, topology, the pre-stressed length, normalized force density coefficient, mechanical constraints and external loading conditions are known. Here, the design parameters are only the coefficient of the pre-stress force and cross-sectional area of strut and cable elements. The main idea of this study is that design parameters are selected in lower and upper bounds of them in such a way that tensegrity structures subjected to various external loads have the minimum deformation.

4.2 Two- module Snelson's X tensegrity

A two-module tensegrity grid truss assembled from basic Snelson's X is shown in Fig. 3. It is composed of six nodes, four struts and nine cables.

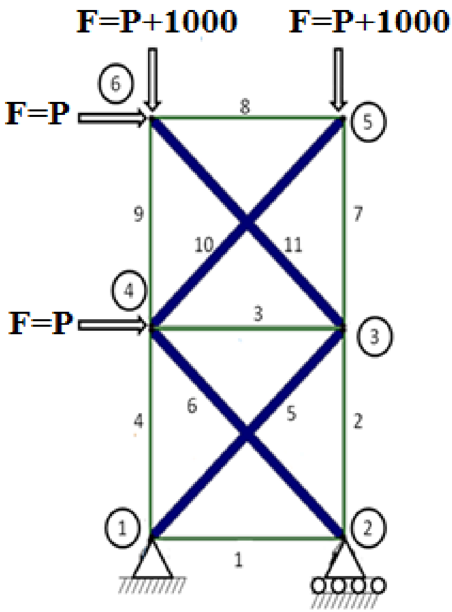

Figure 3. Two module Snelson's X tensegrity truss

Using the form finding procedure, the pre-stressed lengths of cable components are 3 m. A normalized force density coefficient (q_i) for the two module of Snelson's X tensegrity structure are found based on Tran and Lee (2010-b) approach as $q = [1,1,2,1,-1,-1,1,1,1,-1,-1]^T$. To obtain a more rigid structure, however, minimizing static compliance has to be taken into account in the design process. For this example, the mechanical properties are considered as follows: Young's modulus and yield strength of cable and strut elements are 206 GPa and 690 MPa, respectively. Three degrees of displacements for the bottom nodes are fixed in order to avoid rigid body motions. The nodes (4, 5 and 6) are loaded by external loads as shown in Fig.3. The upper bound for radius of strut and cable are 25 mm and 10 mm, respectively, and for the coefficient member force is 4 kN. Number of generations and number of individuals in each generation are 25 and 1000.

Table 2. Optimal design parameters two-module tensegrity grid

	Radius of cable m	Radius of strut m	Pre-stress coefficient kN	Generation number	The value of fitness
Proposed algorithm	**0.0080**	**0.0248**	**2.0154**	**25**	**1.3145**

Employing these data into proposed algorithm, the optimum coefficient of pre-stress, cross-section area of cable and strut are easily obtained as reported in Table 2. The history of optimization of two-module tensegrity structure is shown in Fig. 4.

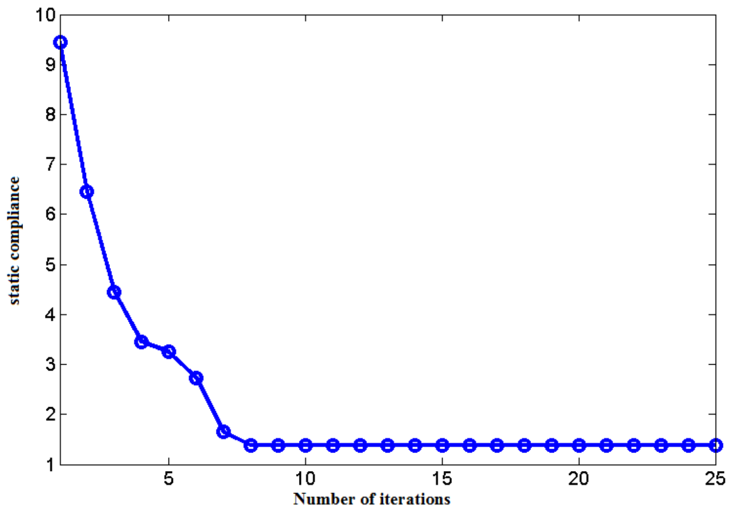

Figure 4: Two- module Snelson's X tensegrity optimal history

Tables 3 and 4 present internal force members after applying design parameters into tensegrity structures for cable and strut elements. In other words, the internal force members given in Tables 3 and 4 are referred to the state after the application of the load.

Table 3. Internal forces of cable elements after optimal designing

elements	1	2	3	4	7	8	9
Internal force (N)	8471	1471	12272	7472	4348	6848	5849

Table 4. Internal forces of strut elements after optimal designing

Elements	5	6	10	11
Internal force (N)	-7738	-11980	-9684	-11807

The benefit of using the results given in Table 2 is that modifying the tensegrity systems according to these data causes the system has not tend to collapse under external loads. Because all cable and strut elements in the structure have no slacking and buckling, respectively, and also yielding load capacity is satisfied in both elements. The vertical displacement of nodes 5 and 6 versus the applied external load (F) after implementing the optimal design variables is shown in Fig.5. Here, the value of P varies from 0 to 1500 N.

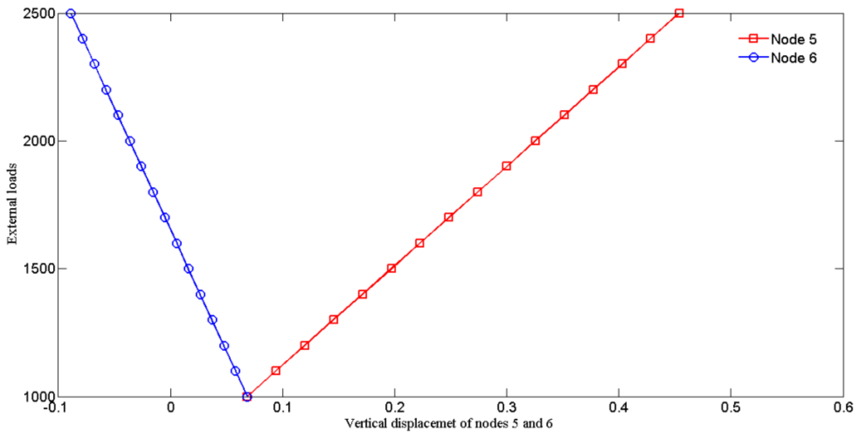

Figure 5: Behaviour of the two-module tensegrity grid truss under external loads

4.3 A quadruplex unit

A three-dimensional tensegrity structure is considered as Fig. 6, which consists of sixteen members (12 strings and 4 struts) and eight nodes.

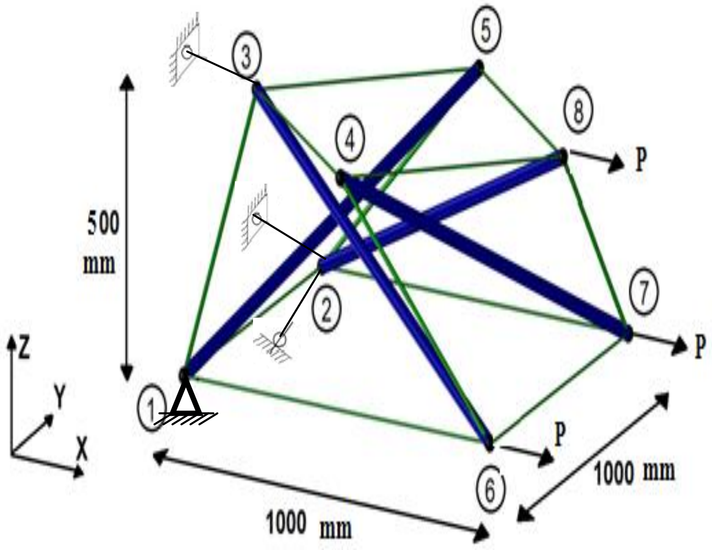

Figure 6. A unit Perspective quadruplex module

The boundary conditions are implemented as following: Node 1 is completely fixed, node 2 is fixed in the X and Y directions, and node 3 is fixed only in X directions. Before applying the external load, the method presented by Tran and Lee (2010-b) is employed to obtain the normalized force density coefficient. Mechanical properties are chosen as follows: Young's modulus of cable and strut elements are 40 GP and 206GPa, respectively and also yield strength of cable and strut 134 MPa and 690 MPa respectively. As illustrated in Fig.6, the loads are applied on nodes 6, 7 and 8 along x direction. The quadruplex unit is subjected to a tension external load of F= 2 kN. The upper bound for design parameters are taken into account as: radius of strut and cable are 30 mm

and 10 mm, respectively, and the coefficient member force is 20 kN. Individulas in each generation is 1000 and maximum genereation is 30.
Applying this information into proposed algorithm, the optimum values for design variables are obtained as tabulated in Table 5.

Table 5. Optimal design parameters for quadruplex unit tensegrity

	radius of cable m	radius of strut m	Pre-stress coefficient kN	Generation number	The value of fitness
Proposed algorithm	**0.0047**	**0.0281**	**12.397**	**30**	**22.2307**

The history of optimization of the two-module quadruplex unit tensegrity structure is shown in Fig. 7.

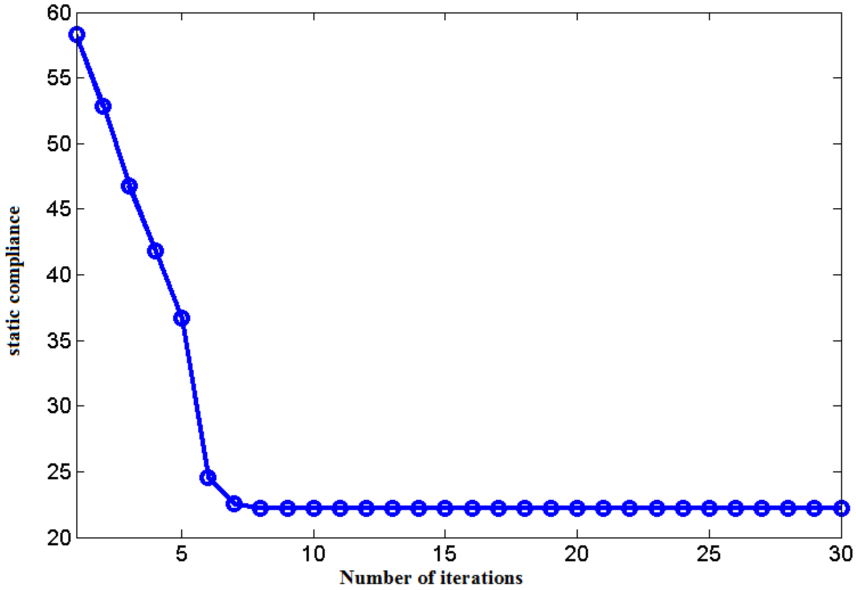

Figure 7: A quadruplex unit optimal history

Tables 6 and 7 present internal force members after applying design parameters into tensegrity structures for cable and strut elements. These internal force members given in Tables 6 and 7 are referred to the state after the application of the load.

Table 6. Internal forces of cable elements after optimal designing

elements	1-2	2-7	3-5	5-8	7-8	4-6
Internal force (N)	4207	4951	6251	5906	6012	5707

Table 7. Internal forces of strut elements after optimal designing

elements	1-5	4-7	8-2	3-6
Internal force (N)	-2132.9	-2151.2	-1930	-1991.1

Results shown in Table 5 evince that all cable elements are in tension and no strut elements are in bucking mode. Therefore, modifying the system based on results given in Table 5 cause that the system becomes more rigid. In other words, the super stability of the system is guaranteed when tensegrity structure is modified by using the optimal design parameters. The behaviour of the quadruplex module versus external tension load after implemnting the new design parameters in the structure is shown in Fig. 8. Here the tension external load value of each load varies from 0 to 2 kN.

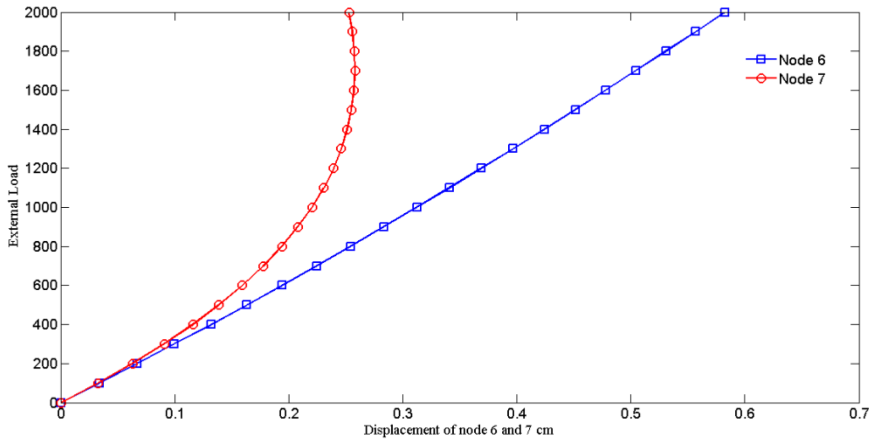

Figure 8: Behaviour of the quadruplex module under tensile load

4.4 Three dimension tensegrity structure

Consider a double layer quadruplex tensegrity, which is assembled from 20 quadruplex units. The foundation constraints of the 20 quadruplex module which is built of 79 nodes, 209 strings and 80 struts as depicted in Figs. 9-a and 9-b.

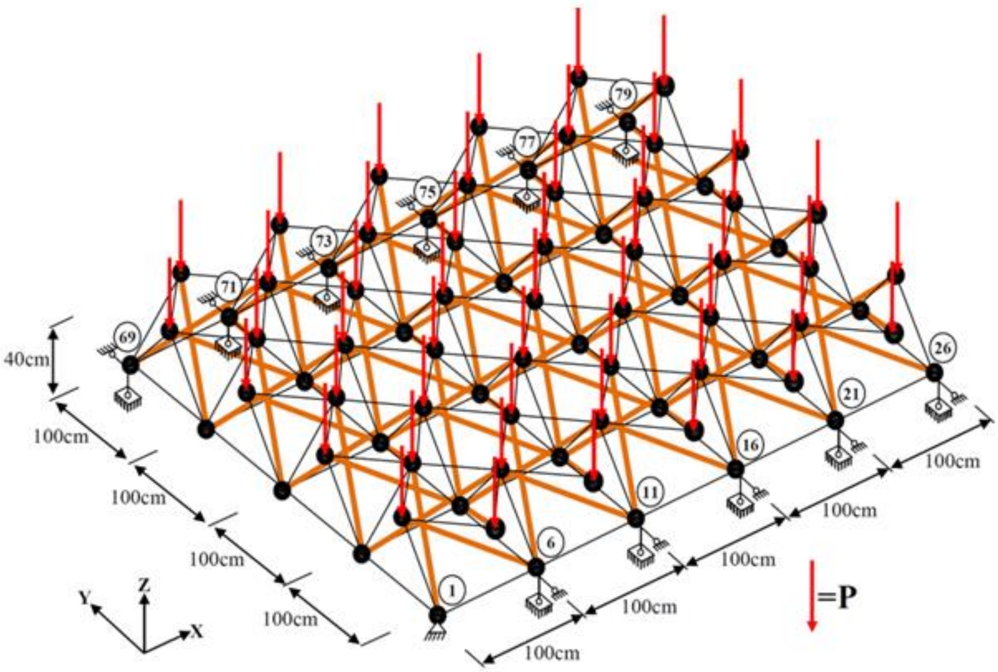

Figure 9-a: A perspective of three-dimensional tensegrity grid formed of 9(3×3) quadruplex modules

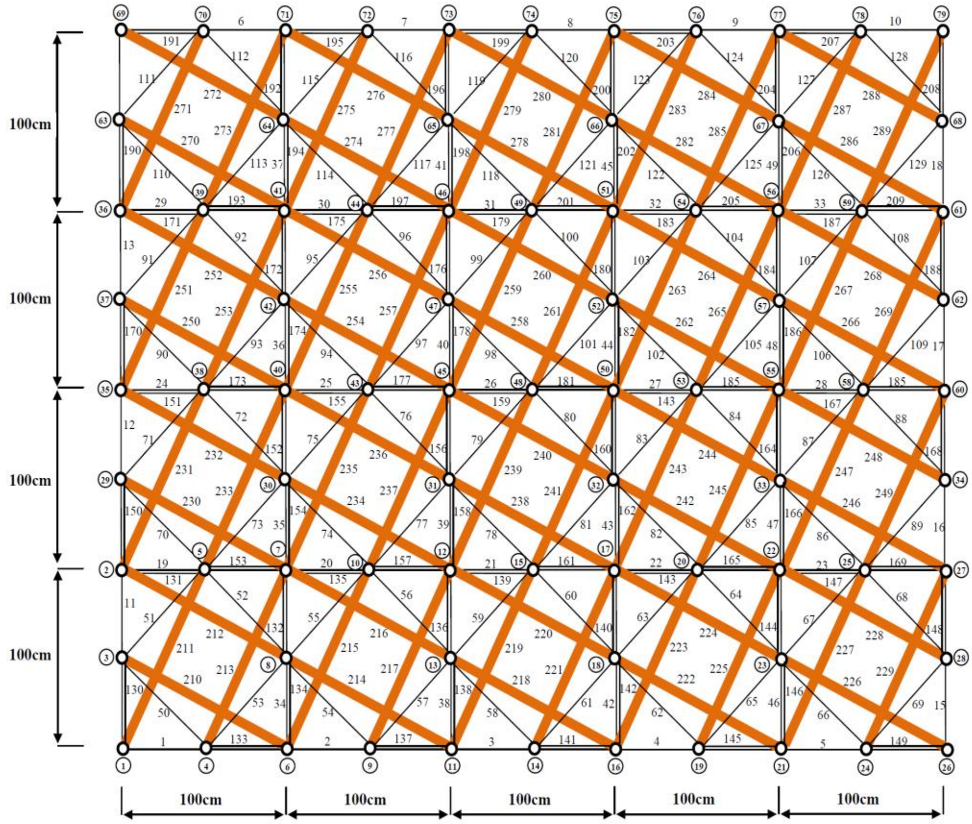

Figure 9-b. A top view of three-dimensional quadruplex modules tensegrity

Mechanical properties of the double layer quadruplex tensegrity are the same with the example 2. Using the Tran and Lee's method (2010-b), the force density coefficient is chosen as follows: for string and strut elements of 1-10, 11-18, 19-33, 34-49, 50-129, 130-209, and 210-289 are $q=[1,17,2,18,2,2,-2]$, respectively. In this numerical investigation, external loads are applied on the top nodes in vertical directions and take values of 900N. The same upper bound of the previous examples is considered for design parameters in this problem. Population in each generation is 500 and maximum genereation is 20.

Using the proposed method for this rather complicated example, the optimized design parameters is calculated as reported in Table 8.

Table 8. Optimal design parameters for three dimension double layer quadruplex tensegrity

	Radius of cable m	Radius of strut m	Pre-stress coefficient kN	Generation number	The value of fitness
Proposed algorithm	**0.0078**	**0.0148**	**3.6565**	**20**	**266.8295**

The history of optimization of the double layer quadruplex tensegrity structure is shown in Fig. 10. Tables 9 and 10 present some internal force members after applying design parameters into tensegrity structures for cable and strut elements. These internal force members given in Tables 8 and 9 are referred to the state after the application of the load.

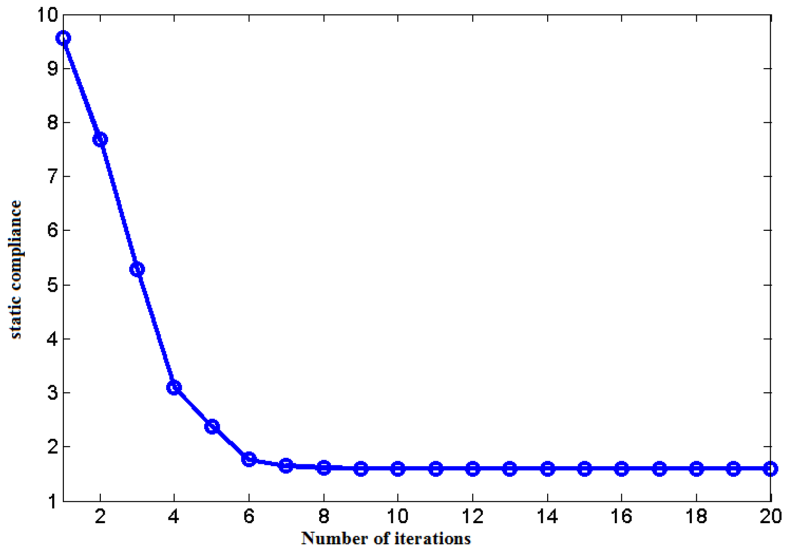

Figure 10: a double layer quadruplex tensegrity optimal history

Table 9. Internal forces of cable elements after optimal designing

elements	1-6	1-2	2-7	6-7	4-3	1-3	2-5
Internal force (N)	3816	61901	7186	65898	5070	5003	4497

Table10. Internal forces of strut elements after optimal designing

elements	3-6	1-5	2-8	5-7
Internal force (N)	-14299	-683	-8932	-9910

The outlined results given in Table 8 show that all cable elements are in tension and strut elements are not in bucking mode. Therefore, modifying the system according to these optimal results cause the rigidity of the structure becomes much higher and it has not tend to collapse under these external loading. In other words, one can conclude the super stability of the structure is guaranteed when the structure is re-designed by using the optimal results given in Table 8. The vertical displacements of top and bottom nodes such as 15, 49, 48 and 17, 51, 50 versus the external load after applying the new design variables are shown in Figs.11 and 12 respectively. Here, the value of each load varies from 0 to 900 N. the global external load-displacement behaviour of the system is nearly linear. The structure is softened as the external loads increase. A similar trend has been exhibited in previous study on nonlinear analysis tensegrity structures (Tran, Lee, 2011). Due to the symmetry of the structure, the displacement of nodes 15, 17 and 49, 51 are similar respectively as depicted in Figs. 11 and 12.

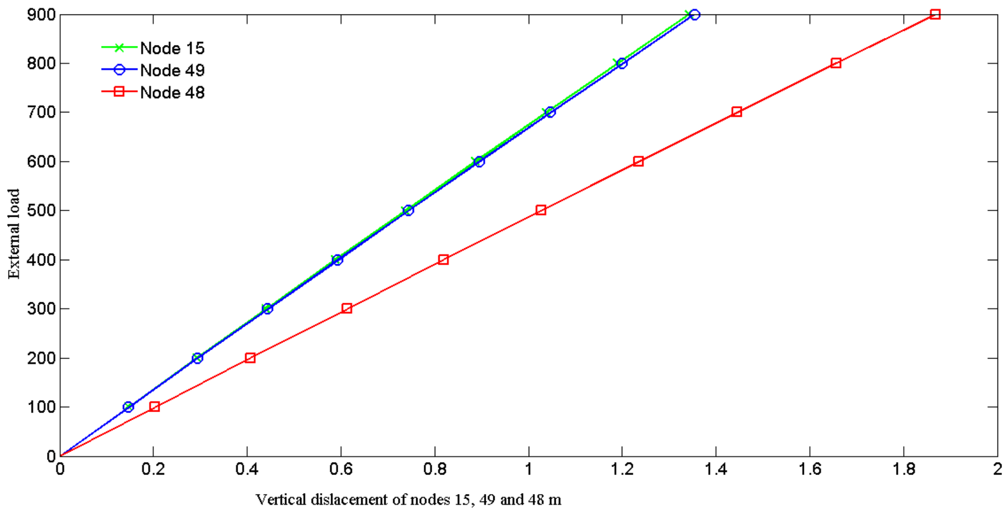

Figure 11: Vertical displacement versus external loads for nodes 15,49 and 48

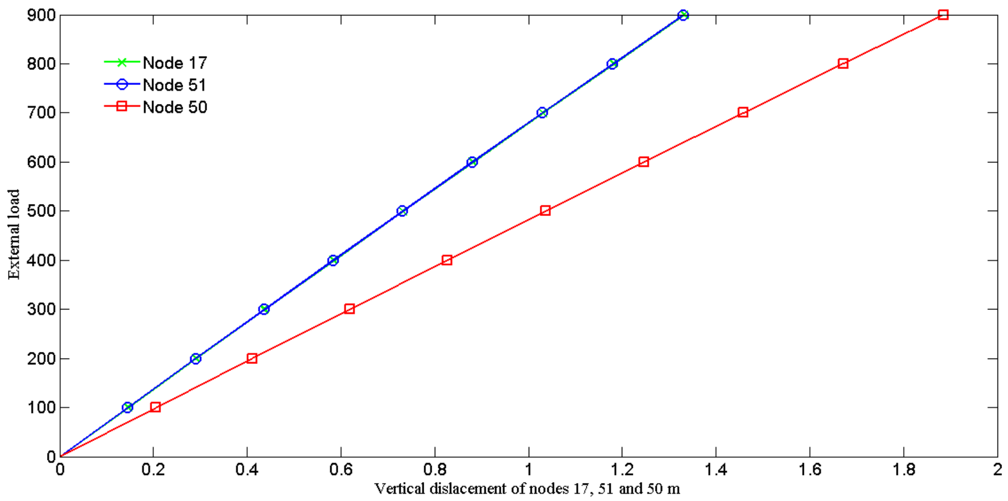

Figure 12: Vertical displacement versus external loads for nodes 17,51 and 50

5 Conclusions

This work has taken a new look on the influence of the pre-stress and cross-sectional area of element simultaneously on tensegrity structure design. In the literature, most contributions have focused on finding either the normalized of the pre-stress or the optimum pre-stress. However, the aim of this study was to find the optimal pre-stress force and cross-sectional area of elements by minimizing the compliance work in terms of some physical constraints until the rigidity of the structure becomes much higher. Genetic algorithm as a global search has been taken into consideration for solving the nonlinear minimization constraints. The proposed algorithm starts with an initial randomly population along with given data such as nodal coordinates, normalized force density coefficients as well as the mechanical properties of the structures. Afterward, the objective function values for all extracted feasible solutions are computed, and using these values the new generations are created by some genetic operators such as selection, crossover, mutation, etc. The procedure is repeated until a specific stopping criterion is met. Using this algorithm, the design variables are determined by considering both yielding load capacity and no slacking of the cable members and both yielding load capacity and no buckling of strut elements. Modifying the tensegrity structure based on these optimal values of design parameters causes that the structure becomes far more rigid and has no risk to collapse. In addition, the super stability of the structure at new equilibrium configuration is guaranteed when the system is re-designed using the obtained optimal design parameters. The efficiency and applicability of the algorithm in the tensegrity design is then shown through three numerical examples.

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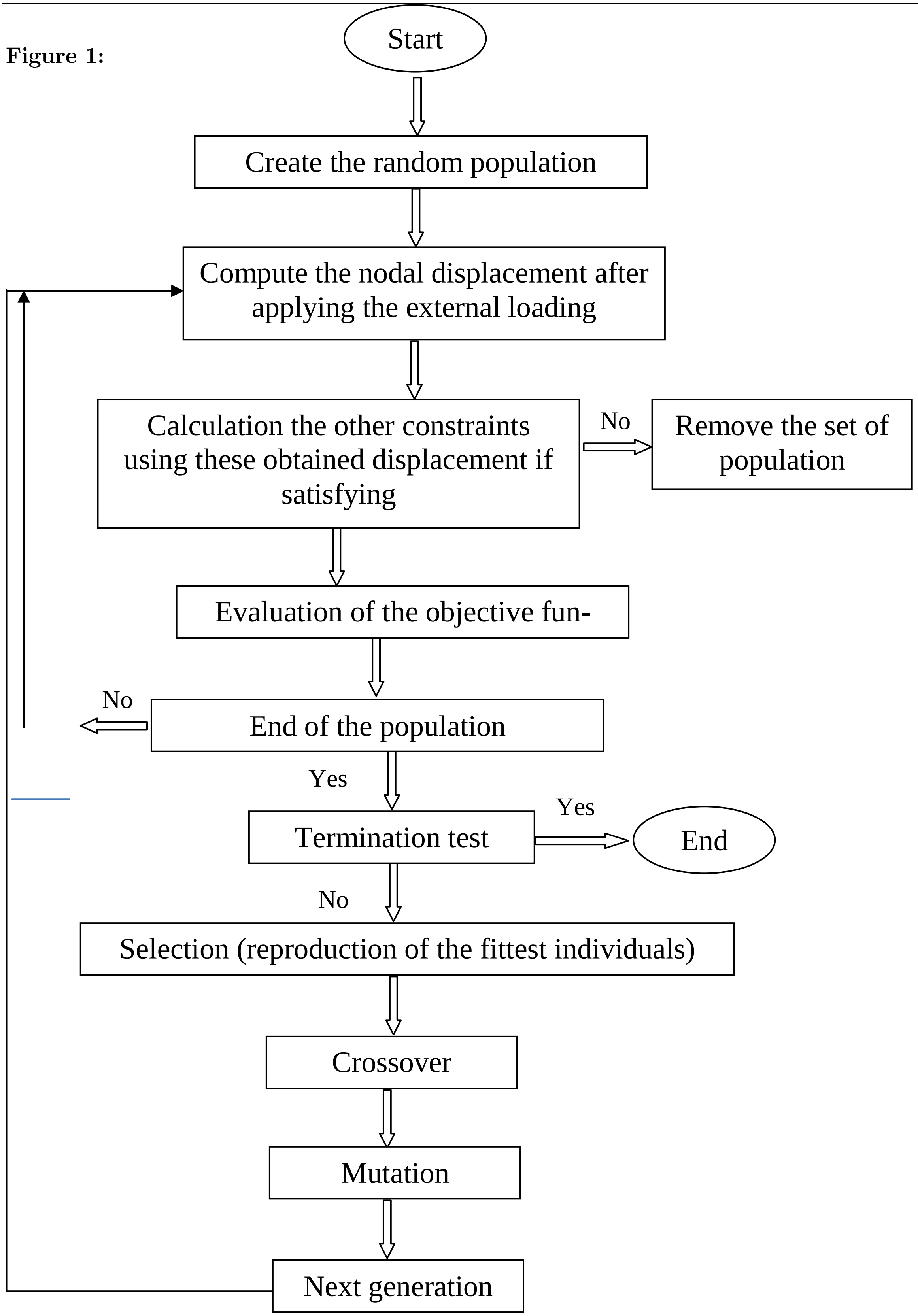
Figure 1:


Start
Create the random population
Compute the nodal displacement after applying the external loading
Calculation the other constraints using these obtained displacement if satisfying
No
Remove the set of population
Evaluation of the objective fun-
No
End of the population
Yes
Termination test
Yes
End
No
Selection (reproduction of the fittest individuals)
Crossover
Mutation
Next generation