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Rotting Tomatoes – Logic for precesses

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Abstract: In the literature, there are several claims about the centrality of a pro- cess ontology. It has often proved difficult,

however, to understand what a process is supposed to be, and what the central difference to thing-based ontologies amounts to. It would be useful to see what an abstract account of this difference consists in. This paper provides a sketch of what logic we should use for a process-based ontology, and argues that intuitionistic logic, and smooth infinitesimal analysis, provide a useful basis for process ontology. A brief sketch of probability for processes is made.

Introduction

Denn bleiben ist nirgends.
Rilke, *Duineser Elegien*

Some think there are two widely different ways of conceiving the world. One group peoples the world with things, enduring stuff; the other puts ongoing pro- cesses as the basic constituents. *Process ontology* or *process metaphysics* are umbrella terms for such ways of thinking of the world. Whitehead (1929/1978) put forward a process ontology, intended to be the basis of ontology instead of one based on (enduring) things. The guiding idea behind Whitehead's view is that the world simply doesn't present enough constancy to provide us with things — what we have are constantly changing events, as Heraclitus thought. Things require constancy, processes are the very embodiment of essential flux.

Some such differences in worldviews might be the result of psychological or cultural differences. It has been alleged that Asian people have a stronger tendency than Westerners to think of the world in terms of processes than in terms of things.[1] Jorge Luis Borges' story "Tlön, Uqbar, Orbis Tertius" makes the case for a process ontology in fictional form. There, a fictional language is described, where no references to constant objects are allowed. All reference is to developing events, processes:

> The world for them is not a concourse of

[1] Nisbett (2003) provides many examples.

> objects in space; it is a heterogeneous series of independent acts. ... There are no nouns in Tlön's conjectural *Ursprache*, from which the 'present' languages and the dialects are derived: there are impersonal verbs, modified by monosyllabic suffixes (or prefixes) with an adverbial value. For example: there is no word corresponding to the word 'moon', but there is a verb which in English would be 'to moon' or 'to moonate'. 'The moon rose above the river' is *hlör u fang axaxaxas mlö*, or literally: 'upward behind the onstreaming it mooned'.
> [...]
> [T]he men of this planet conceive the universe as a series of mental processes which do not develop in space but successively in time.[2]

A more concrete example might help up get a grip on the suggested difference. We can get an understanding of why a process ontology is tempting by looking at ordinary objects, but in a slightly different perspective. Consider an ordinary tomato plant over time. At first, there is just the plant, then tomatoes start sprouting, and after a while, there are lots of tomatoes on the plant. Wait longer, and the tomatoes will start rotting, and finally there will just be the pot, which contained the plant, along with some earth.[3] Given enough temporal resolution, it is less tempting to think of what is going on here as something involving definite objects. Where and when does the tomato start and end? It starts to become tempting to speak Borgesian about the tomato: "On the table, a tomatoing is manifesting". And since a longer time-

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[2] Borges (1970) pp. 32ff. In this short story, Borges stresses the connections between Uqbar language and Berkeleyan idealism, not with process thinking, but there is a connection there as well.

[3] A look at time-lapse tomatoes might be useful: https://youtube.com/shorts/61yTZD7_ I9Q?si=7BPt-PD6VeTbPEno, and for a rotting tomato: https://www.youtube.com/shorts/ Mdg59vB_-OQ.

lapse film of the table's doings would show something similar about the table, a consequent Borgesian should say something like "Over the tabling, it is tomatoing".

What does this allegedly large divide really amount to? After all, we seem to be using singular terms for both kinds of building blocks (we still talk about *a* tomatoing), so from a logical point of view, the difference may not be all that large. One logic, leaning heavily on the acceptability of singular terms, seems to rule them both.

Often, statements made on behalf of process ontologies are not particularly helpful. Consider the following:

> The being of an entity is constituted by its becoming: 'how an actual entity becomes constitutes what that actual entity is' (Whitehead 1929/1978, p. 23, emphasis in original). At first, this foundational postulate of process archaeology may seem innocent enough and in line with the traditional archaeological emphasis on the study of long-term change. But the primacy of becoming over being carries also some radical transformations into the meaning of change, time, and experience that should be made explicit at the outset. In particular: change is changing into continuous change, that is, becoming. Time is changing into duration, that is, the experience of temporality that is both memory of the past in the present and anticipation of the future (Bergson 1922/1965). Experience is changing into felt or lived experience, where the separation between 'inner' and 'outer' space vanishes and any isolated singular object 'is always a special part, phase, or aspect, of an environing experienced world — a situation' (Dewey 1938, 67). The sub-ject is changing into superject (Whitehead 1929/1978, 29) where the split between the knower and the known (subject/object) gives way to the

> continuity of hylonoetic (from Greek hyle for matter and noêsis for intelligence) consciousness. (Malafouris 2021:38)

A reader may be forgiven for not feeling too enlightened by this. Something is being asserted as importantly different from something else, but the nature and consequences of these differences are hard to grasp.[4]

Another example can be found in recent discussions about the nature of con- sciousness:

> The central dogma of cognitive science is 'consciousness is a process, not a thing'. Although the truth of this maxim is taken for granted in the field, its origin is not entirely clear. Tononi and Edelman (1998) attribute it to William James (1890), saying 'Consciousness, as William James pointed out, is not a thing, but a process or stream that is changing on a time scale of fractions of seconds'. (Pockett 2017:1)[5]

A third example from a recent study of meaning:

> Think of a word's meaning, . . . as something historical, more of a process than a thing. (Richard 2019:38)

Again, what contrast we are supposed to be looking at here is a bit hard to understand. Richard develops the claim more fully in the course of the book, and it means that we should focus on meanings as developing, making better room for such phenomena as changes in meaning. This leads to a

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[4] In fairness to Malafouris, things do get a bit clearer as we continue reading. Process archaeology is intended to have clear-cut, empirical, implications for archaeology.

[5] We can leave for some other occasion the issue of whether this really is a "dogma" of cognitive science.

certain vagueness, or perhaps indeterminacy. Questions like: When do we have sameness of meaning? When has one meaning been replaced by another? become less clear-cut, and there might not be a definite answer at times.

In effect, the allegedly special nature of processes, as an alternative to thing- based ontologies, often disappears upon closer inspection. We still think of processes in much the same way that we think of things. We say things about processes such as: they unfold in time, they are dynamic, changing, non-stable, and so on, and the world deep down consists of them (if we are engaged in process ontology). Proceeding in this way is understandable, but appears to keep holding on to a thing-based ontology. We quantify over processes in much the same way that we quantify over objects, so from a logical point of view, they still are objects. The allegedly clear distinction between things and processes tends to evaporate. Is it just that concentrating on the processes means that we emphasize change over stability? Is this like the difference between the half-full and the half-empty glasses?

The characterizations of processes above are perhaps partly metaphorical characterizations of what the process ontologist's world is like, but maybe we can get a bit further by taking a very abstract point of view, and looking at what logic and probability would be like for process ontology. Providing a background logic and probability for process ontology should be a project of some independent interest.

In this paper, I will be describing how intuitionist logic will turn out to be a very natural logic for processes, and also be saying something very brief about how probability will work out within a process framwork, built on intuitionistic logic. I am not arguing that intuitionistic logic must be seen as *the* logic for process ontology (perhaps fuzzy logic works well, perhaps we should hold on to classical logic and make modifications somewhere else), but I am arguing that we can get a better understanding of what processes are, if we think of them as characterized by intuitionistic logic. We can also look at things from the other side: if we start trying to understand intuitionistic logic, and try to figure out what the world should be like to fit intuitionistic logic, a process ontology provides a pretty good picture.

We might ask why precisely intuitionistic logic would be suitable for the task of providing a logic for process theory. Perhaps fuzzy logic would do, as well? A fuller response is developed in the rest of the paper, but a few brief remarks can be made here. Logic, as the study of valid reasoning, shouldn't by itself have ontological import. It should tell us which conclusions we are entitled to, and not what the world is made of.

In general, we adapt, revise, and expand our logical apparatus under pressures that can come from different places. The logical paradoxes have led some to embrace dialetheism; our wishes to include reasoning about modality led to the development of modal logics, making the step from "It is necessary that *p*" to *p* an instance of a logical operation. Quantum logic, as first developed by Birkhoff and von Neumann, was an attempt to provide a new logic when the surprising discoveries in quantum mechanics were discovered. Not all such attempts have proven successful — there is little interest in quantum logic these days. The use of intuitionistic logic to give a formal backing for process theory follows this tradition of logical revision.

There is a further reason to look precisely at intuitionistic logic. The intuitionist is fundamentally occupied with using the continuum as the basis for an understanding of mathematics. Beth wrote: "The central place in intuitionistic mathematics is occupied by the theory of the continuum."[6] And the same holds for the process theorist — it is not just that the world is in constant flux, but also that this flux is taking place in a continuum. Constant flux would be consistent with a pure thing ontology, as long as the *things* kept changing all the time. The process makes an additional claim about the nature of the processes, and this additional claim puts the process theorist and the intuitionistic mathematician in close contact.[7]

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[6] Beth (1959), p. 422, quoted in Kleene & Vesley (1965), p. 187.)

[7] It would also be interesting for future work to study what additions to intuitionistic logic, as is done in superintuitionistic logic, would entail. This paper will stay with intuitionistic logic,

Logic for processes

Even if the examples of process-talk mentioned in the previous section were hard to decode, there seems to be a common basic intuition lying behind the passages. This is the idea that it is preferable to view the world as in flux, changing, and that whatever permanence or constancy we find is an illusion, or perhaps a secondary phenomenon, not reflecting the true nature of the world. Change over time is what we should be focussing on. This concentration on change means that points in time, seen as fully finished and furnished, are secondary in comparison to the changes over time. The continuum — time as it progresses — is in a sense prior to individual points in time. Basic views of this kind have a long history, starting with Aristotle's discussion of Zeno's paradoxes.[8]

Consider the tomato in the films referred to in the earlier note. At what time is there a tomato on the table? Part of the time, there clearly is a tomato on the table, part of the time there is clearly no tomato on the table, while there are times when we don't really know what to say. Typical for processes is that they are not fully there at a given point in time. They come into existence, develop, and are causally active in time, and then they cease to exist. This development should be thought of as something gradual — otherwise the process seems to become too "thinglike". Processes don't usually spring suddenly into or out of existence.

This means that there should be periods of time when a given process has started to exist, yet is only on its way to being fully causally active. The tomatoing process on the table is not in possession of all the typical causal powers we associate with being a tomato all the time.[9] Perhaps we can

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since that logic is better known. For superintuitionistic logic, see Umezawa (1959) and Moschovakis (2024), § 6.1.

[8] See Hellman & Shapiro (2018), Ch. 1.

[9] Here, there is some room for choice. Being edible may differ a bit, depending on who is eating. Being fit for throwing at bad actors may be a later stage in the tomatoing process.

think of this as a period of gestation for the process, a build-up necessary for the process to become causally efficient in the way that is typical for the process we are interested in (early on, tomatoes are not edible). In that case, there will be periods of time when we don't want to say that the process is definitely not there, but we don't want to say that it is definitely there, either. Thus there will be cases when the statement that there either is a tomato or not on the table — $p \vee \neg p$ — doesn't hold.

But the above is a very sketchy picture of the process theorist's thinking, and I will try to make it more precise in what follows. Here we encountered the typical claim of intuitionistic logic, that the law of excluded middle, **LEM**, doesn't hold unrestrictedly. Other typical intuitionistic ideas will also hold: there may be periods when it is not the case that p is not in existence, yet the existence of p cannot be asserted outright. Double negation elimination, **DNE**, will fail for processes.

Given the loose ends at the beginning and end of the tomato process, the general idea here — associating intuitionistic logic with process ontology — can be seen to be related to the idea of using intuitionistic logic for handling vagueness and the Sorites paradox.[10] The tomato process can be seen as a Soritical object, and watching the beginning and the end of the process can be likened to a Sorites case — if something is a tomato at t_0, it is a tomato one nanosecond later. And yet, at some time, the tomato is gone. The intuitionist can stop the problematic step in the Sorites paradox by treating vague predicates (such as "bald", "young") the same way that non-decidable predicates in intuitionistic logic are treated. The essential move is to deny the usual step (classically valid) from $\neg \forall n Fn$ to

$\exists n \neg Fn$, since that step is not intuitionistically valid.

There are still a few differences between the application of intuitionistic logic for vague predicates and for process theory. One is that the process theorist wants to say that

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[10] See Putnam (1983) and Wright (2021).

those things we thought of as piecemeal, persisting, objects have a secondary kind of existence, compared with the truly basic processes. When discussing vagueness, we can start with something (almost) everyone agrees on: there are vague predicates, and that can cause problems if we apply classical logic to Sorites cases. The intuitionistic solution for vague predicates focusses on what is epistemically accessible; the decidable cases are those where bivalence holds. So even if there are certain formal parallels between the introduction of intuitionistic logic for vague predicates and for modelling processes, the basic impetus differs. For the process theorist, insisting on the primacy of processes is not a reaction to our cognitive or epistemic limitations.

Parallels between intuitionistic logic, treatments of vague predicates, and process ontology may be illustrative, but they don't by themselves get us very far. I think there are some substantial reasons for thinking that there *should* be some close connections between the two. Brouwer and Heyting were explicit in thinking of the mathematician's work as a process, unfolding in time. The objects of mathematics should not be seen as existing irrespective of the means for constructing them. This is naturally held to be a kind of idealism, but there is also a great deal of process thinking at work here, as some passages by Brouwer indicate, for instance:

> [T]he fundamental phenomenon of mathematical thinking, the intuition of the bare two-oneness. This intuition of two-oneness, the basal intuition of mathematics, creates not only the numbers one and two, but also all finite ordinal numbers, inasmuch as one of the elements of the two-oneness may be thought of as a new two-oneness, which process may be repeated indefinitely. (Brouwer, 1913/1999, p. 57)

The intuitionist's conception of the real numbers, and of choice sequences, is in many ways recognizably similar to process thinking in ontology.[11] For the intuitionist, a choice sequence is never *given* as something that is there to be discovered, never finished — they are developed by the creative subject. They are more process-like than thing-like. A clear example of this general attitude can be found in Dummett's characterization of the intuitionist's views on infinity:

> In intuitionistic mathematics, all infinity is potential infinity: there is no completed infinity. . . . the thesis that there is no completed infinity means, simply, that to grasp an infinite structure is to grasp the *process* which generates it, that to refer to such a structure as being infinite is to recognize that the process will not terminate. In the case of a process that does terminate, we may legitimately distinguish between the process itself and its completed output. (Dummett 1977, pp. 55f, emphasis added.)

Intuitionistic logic and mathematics are ways of developing this view of an abstract realm. Viewing the abstract world of mathematics as an unfolding process may be one thing, but can we extend this general view of an area of discourse to the tangible world, a world where most of the existing process theorists want to stress a special role for their

[11] A choice sequence is an essential tool, and object of study, for the intuitionistic mathematician. Dummett's section on choice sequences starts: "A choice sequence is an infinite sequence of natural numbers whose terms are generated in succession." (p. 418). The notion turns out to be difficult to give a fully satisfying characterization. Essentially, a few things are required for something to be a choice sequence. It is a sequence that continues indefinitely; its steps are chosen stepwise, one by one, not as a finished totality; the choice is not predetermined by a previously existing rule for selecting terms; it is incomplete, since only a finite part of the sequence is known at any given time. See Dummett (1977), § 7.4.

theories?

There is one clear difference between on the one hand physical processes, such as the tomato process, and on the other hand the kind of processes that intuitionistic mathematics develops theories about. The physical processes are given, already existing, while the intuitionistic mathematician's process of constructing a sequence of natural numbers is an activity that flows from the activity of the creative subject. Still, here, the parallels will turn out to be fruitful. Even if we don't construct the processes in the sense in which mathematical objects can be con- structed by the intuitionist mathematician, the results of such constructions will turn out to have far-reaching formal similarities with what a consistent process theorist should be saying about what processes are.

One semantics for intuitionistic logic, Scott's *sheaf semantics* (Scott 1968), can give good sense to many things we want to say about processes. We want to say that there can be processes for which it is not clear when they start, but there can be times when they *clearly* are not at hand. This general idea turns out to have a parallel in open sets of real numbers, and in this paper, I shall sketch what happens if we look at processes as open sets of numbers, considered in the way intuitionists prefer.

As a first sketch, the parallel between process theories and intuitionistic mathematics is as follows. If we think that processes lack clearly delimited starting/stopping points, processes are *open intervals* (a,b), where times $a,b \in \mathrm{R}$.

Informally, an open set of real numbers is a set U such that around every point of U there is *some* room to move — in all directions — without leaving the set. The size of "some room" will depend on where we are in the set.It will also be possible to have closed or half-open intervals, $[a,b]$, $(a,b]$ and $[a,b)$, defined in the usual way. Starting with this idea, we can now start developing a view of
processes as open sets.

> **Open set:** A set **O** of real numbers is open if for each $x \in \mathbf{O}$ there is a $\delta > 0$ such that each y

with $|x-y| < \delta$ belongs to **O**.[12]

It will be possible to show that many things that we think of as important for processes can be made sense of in a formal manner like that sketched above.

If we prefer fixed stopping or starting points for a process, this can be characterized by closed intervals, so $[a, b] = \{x : a \leq x \leq b\}$, or half-open intervals $[a,b) = \{x : a \leq x < b\}$ $(a,b] = \{x : a < x \leq b\}$ will also be in order.

Such open sets can be combined or partitioned. In order to make the formal theory more realistic, we should not be able to combine the open sets in any way whatsoever. The tomato's growing-and-rotting (excluding the middle period, where the tomato is ripe and edible) is not a good candidate for being one process. In set theory, we can combine elements any way we want to, and get a set. This should not be the case for a realistic process theory. One formal way to accomplish this is to demand that the combined or partitioned new open set is *convex*: if we connect elements in the newly combined set, all steps between the two elements would also have to be in the newly combined set. This excludes gerrymandered sets, and seems to be in concert with our everyday notion of processes, where these are held together by something, presumably a causal activity.

Some new notation is required.

$\|A\|$ is the open set of the real number line assigned to the process A. We will also for technical reasons need something we might call a "null process" $\bot$, something that never occurs — perhaps that pigs fly, or hell freezes over, or simply 0=1. $\neg A$ is then $A \rightarrow \bot$, so if $\neg A$ holds, a proof that A holds can be transformed into a proof that the null process occurs.

Equipped with these tools, we are now in a position to see processes as *open sets on the real number line*. We can then say

[12] In this paper, I will only be looking at the temporal dimension of processes, everything will be discussed in terms of the real number line. A more realistic treatment would have to introduce other aspects, such as space, making for a more complex formalism. See the discussion below, on the identity of sets.

that a process *a* exists, if it is included in R.
A process is *total* if it exists all the time.
When are two processes identical? It is tempting to say that two processes are identical if and only if they overlap completely in time. In one direction, this is obviously the case. But are two processes that overlap completely in time identical? For this paper, we will make the simplifying assumption that two processes are compared only with respect to time. This is not entirely correct: the timing of my chewing a piece of gum in Germany may overlap completely in time with the melting of an ice cube in China, but these are clearly different processes. This can be fixed by adding the further dimension of space — it is at least a step in the right direction to say that two processes that overlap completely in time *and* space in fact are identical. But perhaps not even this is enough. It seems that we can think of processes that overlap completely in time *and* space, yet still are different. Consider a ball, that has been painted with a slow-drying paint. In fact, the paint only dries when exposed to the sun. So the paint's drying and the ball being exposed to the sun are coinciding in both time and space, yet they don't seem to be the same process. It appears that a counterfactual element will have to be brought in to account for such cases. A fuller story about identifying process would have to be dealt with on some other occasion. The simplification of only considering time is for expository purposes, since it makes processes one-dimensional, and easier to describe.
If processes are to be understood as open sets, we can employ a common treatment of open sets. The *complement* of a set A, written A^c, is the set of elements not in A. The *Interior* of a set, $I(K)$, is the largest open subset of K. The stretch of time which is the non-occurrence of a process A, which we can think of as $\neg A$ (there is no tomato on the table), will then be the interior of the complement of
$\|A\|$:

$$\|\neg A\| = Int(\|A\|)^c$$

We can also combine processes, and make them depend on other processes, by using the intuitionistic interpretation of the connectives (respecting the above proviso of convexity).

A process which consists of two combined processes A and B, will simply be the union of the two:

$$\|A \vee B\| = \|A\| \cup \|B\|$$

Now it turns out that some central intuitionistic ideas make good sense on this very abstract understanding of what a process is.
First, the law of excluded middle, **LEM**, won't hold for processes. Since we are working with open sets, $A \vee \neg A$ will not hold unrestrictedly; $\|A\| \cup \|\neg A\| = \mathrm{R}$ doesn't hold for $A = \mathrm{R} - \{0\}$. In more detail: Assign to the process A the set $\mathrm{R} - 0$. In that case,

$$\|A\| = \mathrm{R} - \{0\}$$
$$\|\neg A\| = Int(\{0\}) = 0$$
$$\|\neg\neg A\| = \mathrm{R}$$

So we won't get **DNE**, $\| \neg\neg A \to A \|$, either, and extensions to predicate logic are possible.

Much of this makes good sense for processes. Consider a process A developing through time, such as the tomato, growing on the table. At some time t_1, the process is clearly not occurring (no one has put the pot on the table), and at a later time t_4, it is clearly not occurring (nothing remains of the rotten tomatoes), but there has been a period between t_1 and t_4 where we have clearly had a process occurring. This can be the period between t_2 and t_3, where $t_1 < t_2 < t_3 < t_4$. But it doesn't always have to be clear when a process starts. So we can have a period during which A occurs, a period when A doesn't occur,[13] and a period during which it is not the case that A doesn't occur. So $\| \neg\neg A \|$ won't warrant the conclusion $\| A \|$. We can get a flavour of what this would be like by considering heating. Something can be heated without any increase in temperature. So there is a period where the process of the heating of the lump of

[13] Strictly speaking, we will have two such periods: before and after the process A occurs.

material is clearly taking place (the lump is warmer), a period when we clearly are not heating it (before we switch on the stove), and a period when it is not the case that the lump is not being heated, but knowing that this last period is occurring doesn't by itself warrant our drawing the conclusion that the lump is warmer.

A smooth world

For the process theorist, the world is smooth. Change is constant, but also gradual. The usefulness of intuitionistic logic for capturing one aspect of the process theorist's view is not a mere coincidence. On a fundamental level, the process theorist and the intuitionist agree on the relative priority of the continuum over points. Their reasons may be different. The intuitionist finds the idea of the continuum consisting of a completed infinity of points inconsistent. Points are rather to be seen as something that is constructed out of the continuum. The process theorist has perhaps no qualms about infinity *per se*, but is opposed to the grittyness and unwarranted reliance on permanence of thing ontologies. The process theorist's world must be smooth. A smooth world is one where the continuum, not the points, is the basis, just as it is for the intuitionist. In intuitionistic mathematics, all functions are continuous. This view of the relation between the continuum and points has a long and distinguished history, and is prominent in the intuitionists' thinking. Brouwer wrote that "The linear continuum . . . is not exhaustible by the interposition of new units and can therefore never be thought of as a mere collection of units." (Brouwer 1913/1999, p. 57). Weyl formulates a similar sentiment: "A true continuum is simply something connected in itself and cannot be split into separate pieces; that contradicts its nature" (Weyl 1921, quoted in Bell, p. 2).

This smoothness entails that its logic cannot be classical, on pain of inconsistency. Only an intuitionistic logic will do, as the following reasoning shows. There is a technical apparatus that develops precisely these ideas, *smooth infinitesimal analysis*,

SIA.[14]
Smoothness means that we will have to use *infinitesimals*, closer to 0 than any real number, but not identical with 0. Such an infinitesimal ε is also *nilpotent*, $\varepsilon^2 = 0$ but $\varepsilon \neq 0$. R is the real number-line, with designated points 0 and 1. First, we define a set Δ: $\Delta = \{x \in R : x^2 = 0\}$. Now we can show that the logic of this theory cannot be classical.
For *reductio*, suppose that $\forall \varepsilon \in \Delta$, $\varepsilon = 0$. Now consider the function $f(\varepsilon) = \varepsilon$.
If 0 were the only member of Δ, then

$$\forall \varepsilon \in \Delta (f(\varepsilon) = f(0)) \text{ and } f(\varepsilon) = \varepsilon,$$

So

$$\forall \varepsilon \in \Delta (f(\varepsilon) = f(0 + b \cdot \varepsilon)) \text{ for both } b = 0 \text{ and } b = 1,$$

contradicting the uniqueness of b.[15] So $\neg \forall \varepsilon \in \Delta, \varepsilon = 0$. The set of infinitesimals ε thus has several members.

But using the axioms of smooth infinitesimal analysis, we can also deduce that there is no infinitesimal $\varepsilon \neq 0$.
Suppose that there is one, i. e. suppose $\exists \varepsilon \in \Delta \neg(\varepsilon = 0)$. Then $\varepsilon^2 = 0$ (since $\varepsilon \in \Delta$) and $\neg(\varepsilon = 0)$ (since that was what we assumed). Since $\neg(\varepsilon = 0)$, there is a y, such that $\varepsilon \cdot y = 1$ (the structure is a field). Then

$$0 = 0 \cdot y = \varepsilon^2 \cdot y = \varepsilon \cdot \varepsilon \cdot y = \varepsilon \cdot 1 \text{ (there is such a } y) = \varepsilon$$

Hence $\varepsilon = 0$, which is a contradiction.

So both our assumptions lead to a contradiction. Thus

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[14] See Bell (2008); Bell (2022), secs. 8–9; Rumfitt (2015), sec. 7.4. The reader who is not interested in the technical details can probably profitably skip the next few paragraphs.
[15] See Appendix in Bell (2008). What remains to be done is to show that the axioms of SIA all turn out to be intuitionistically true. This is not a completely trivial exercise, and lies beyond the scope of the present paper. See also Rumfitt, sec. 7.4, where Rumfitt shows that there are a few bumps ahead.

there is no infinitesimal $\varepsilon \neq 0$. If the logic is classical, we have a contradiction, since $\neg\forall\varepsilon \in \Delta\varepsilon = 0$ classically implies $\exists\varepsilon \in \Delta\neg(\varepsilon = 0)$. So SIA is classically inconsistent. But in intuitionistic logic, the last step is not valid. SIA is an intuitionistically consistent theory.

Probability for processes

Let us return to the tomato on the table. For simplicity, disregard the processual nature of the table that the tomato is placed on, and focus on the process that is the tomato. What is the probability that we find a tomato on the table? There must be certain indeterminacy here, given that we have to take into account the openness of the tomato process.

There will be times when there definitely is, or is not, a tomato on the table, there are times when no definite answer is to be found. "There is a tomato on the table, or there is not a tomato on the table" is thus not a logical truth, and the ordinary probability of a logical truth = 1 doesn't hold. So we would have to revise the usual Kolmogorov axioms for probability in a suitable way.

Again, probability for processes can be modelled in a satisfactory way, if we use intuitionistic logic as a base. There have already been developed logics for probability, based on intuitionistic logic (Roeper and Leblanc 1999, Chapters 1:4,9 and 10). Roeper and Leblanc stress the connection between intuitionistic logic and attempts to reflect our epistemic situation; in their account, $P(A)$ (the probability of A) reflects the evidential support received by A (p. 20). Only when $P(A)$ equals 1 do we have conclusive evidence. The process theorist will not necessarily be as impressed by the epistemic perspective here, but the formal apparatus will be possible to employ for systematic talk about probabilities for processes.

A detailed implementation of such a probability theory for processes is best left as a topic of a separate paper.

Closing remarks

Does all this mean that intuitionistic logic will provide "the" logic for processes? I'm not so sure. The notion of a process is usually not made sufficiently precise for us to say that it has a logic. It is too often used as a slogan, intended to say something different from what good old (bad old?) thing-based ontologies have been doing. What I do think has been shown here is that if we make the notion of a process more precise in the way suggested here, and if we take to heart the process ontologist's view that the world deep down is made up of processes, not things, and allow for some idealizations, then it is surprisingly straightforward to see intuitionistic logic as a logic for processes, or conversely, to see process ontology as a way to make the claims behind intuitionistic logic vivid. Other logics can very well be used to model other notions, or other aspects of processes (fuzzy logic comes to mind here).

But there are still good reasons to give the intuitionistic theory the upper hand. The triad of process theory, intuitionistic logic, and SIA fit each other in an almost organic way; they were made for each other.

A sceptically inclined person might now want to ask: "Well, yes, this is fine, but does any of it really *matter*? Does it have a direct impact on the earlier examples of process talk?" It does. Now talk of processes as fundamental, and of process theory as an alternative to a thing-based ontology, have been given a formally adequate ground, even if only a sketch has been presented here. This tells us what is involved in stressing the centrality of process talk and what it entails. Talk of processes is no longer just a metaphorical way of stressing the centrality of change over permanence; now, this metaphorical mode of expression has been put on a firmer footing. Much work remains to be done, but at least a few steps have been taken here.[16]

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References

BELL, J. L. *A Primer of Infinitesimal Analysis*, Second edition, Cambridge: Cam- bridge University Press, 2008.

—, "Continuity and Infinitesimals", *The Stanford Encyclopedia of Philosophy* (Spring 2022 Edition), Edward N. Zalta (ed.), https://plato.stanford.edu/ archives/spr2022/entries/continuity/.

BERGSON, H. *Duration and Simultaneity*. Translated by L. Jacobson. Indianapolis, IN: Bobbs-Merrill, 1922/1965.

BETH, E. W., *The Foundations of Mathematics*, Amsterdam, North Holland, 1959.

BORGES, J. L. *Labyrinths*, Harmondsworth: Penguin, 1970.

BORGES, L.E.J. "Intuitionism and formalism", *Bull. (New Series) Amer. Math. Soc.*, 37:1, 55–64, reprinted 1999 (originally in *Bull. Amer. Math. Soc.* 20

(1913)).

DEWEY, J. *Experience and Nature*. New York: W. W. Norton, 1925.

DUMMETT, M. *Elements of Intuitionism*, Oxford: Clarendon Press, 1977.

HELLMAN, G.& S. SHAPIRO, *Varieties of Continua*, Oxford: Oxford Univer- sity Press, 2018.

KLEENE, S. C. & R. E. VESLEY, *The Foundations of*

Intuitionistic Mathematics, Amsterdam: North Holland, 1965.

MALAFOURIS, L. "Making hands and tools: steps to a process archaeology of mind", *World Archaeology* 2021, 53:1, 38–55, https://doi.org/10.1080/00438243.2021.1993992
MOSCHOVAKIS, J., "Intuitionistic Logic", *The Stanford Encyclopedia of Philos- ophy* (Summer 2024 Edition), Edward N. Zalta & Uri Nodelman (eds.), https://plato.stanford.edu/archives/sum2024/entries/logic-intuitionistic/.

NISBETT, R. E. *The geography of thought : how Asians and Westerners think differently — and why*, New York: Free Press, 2003.

POCKETT, S. "Consciousness Is a Thing, Not a Process", *Appl. Sci.* 2017, 7(12), 1248; https://doi.org/10.3390/app7121248.

PUTNAM, H. "Vagueness and alternative logic", *Erkenntnis* 19 (1983), p. 297– 314.

RICHARD, M. *Meanings as species*, Oxford: Oxford University Press, 2019.

ROEPER, P. & H. LEBLANC, *Probability Theory and Probability Semantics*, Toronto: University of Toronto Press, 1999.

RUMFITT, I. *The Boundary Stones of Thought. An Essay in the Philosophy of Logic*. Oxford: Oxford University Press, 2015.

SCOTT, D. S. "Extending the topological interpretation to intuitionistic analysis". *Compositio Mathematica* 20 (1968), pp. 194–210.

UMEZAWA, T. "On logics intermediate between intuitionistic and classical pred- icate logic". *Journal of Symbolic Logic*. 24 (2)(June 1959): 141–153.

WHITEHEAD, A. N. *Process and Reality*. New York: Free Press, 1929/1978.

WRIGHT, C., "Intuitionism and the Sorites paradox", Chapter 14 in his *Riddles of Vagueness: Selected Essays*

1975?2020, Oxford: Oxford University Press, 2021.

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