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STOCHASTIC DISCRETE LOT-SIZING WITH LEAD TIMES FOR FUEL SUPPLY OPTIMIZATION

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ABSTRACT. We address the problem of expected cost minimization of meeting the uncertain fuel demand during a time planning horizon, where supply is provided by selecting discrete shipments with lead times. Due to uncertainty and the passage of time, corrective actions can be taken such as cancellation and postponement on supply of shipments with associated costs and delays. This problem is modeled as a stochastic multi-stage capacitated discrete lot-sizing problem with lead times. Computational experiments were performed on the resolution of various instances of the model for four information structures of uncertainty. The experimental optimal values and stochastic rating measures obtained show the validity and interest of the stochastic model, as well as the benefits that can be obtained with respect to a deterministic variant of the model that considers average demand.

Keywords: oil and gas procurement, oil supply chain, stochastic lot-sizing, multi-stage stochastic integer programming, postponement.

1 INTRODUCTION

Distribution and storage of primary products are downstream oil supply chain activities. These involve complex logistic planning under uncertainty of product features and resources. In most non-oil producing countries, or where the production is not sufficient to cover the internal demand, it is necessary to import either crude oil, or even refined products, to cover demand. One of the most important and cheapest transportation modes is by ship. This has an important impact in the supply chain, as it is necessary to negotiate not only volume and price, but also how the delivery will be carried out; shipments are of fixed sizes, ship routes are complex and travel times are usually long. This means that supply contracts must be fixed much in advance of the actual times where the fuel will be needed as stated by Moraes and Faria [20]. Additionally, the demands may vary significantly from the best forecasts; this stochastic component introduces an additional complexity and is a source of costs.

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This paper focuses on the minimization of the expectation of costs incurred in decisions made to meet the uncertain demand of fuel over a finite discrete time planning horizon. The fuel is purchased through distinguishable optional shipments with a given size and a time of delivery delay. Shipment delivery delay is relevant in the planning horizon. Due to the passage of time and the unveil of demand uncertainty, it is possible that an acquired shipment that has not been received is no longer necessary. In this case, it could be decided to cancel its acquisition or postpone its delivery; decisions, which require minimum accomplishment times and associated costs. The motivation for this model is a real problem arising in a state-owned Uruguayan oil company that deals with fuel acquisition under contractual and logistic conditions for the energy sector. The demand that the company faces is uncertain, given that thermal electricity generation, as a complement in an electrical system, is highly dependent on renewable sources [24, 32]. While this particular situation of the company focuses on the application of fuel procurement for thermal generation, the present work discusses the formulation and solution of a more general variant of the problem, which can represent all situations in which oil procurement is carried out by selecting discrete cargo options in the context of uncertain demand.

Among different approaches to assist planning decisions in the oil industry, stochastic programming has been shown to be a very successful technique, as demonstrated by Dempster et al. [10] and Pinto et al. [22]. Another successful application is presented by Al-Othman et al. [3], who developed a multi-period stochastic supply chain planning model under uncertain market demands and prices. Christiansen et al. [8] described a comprehensive review on ship routing and scheduling. A stochastic problem of shipping of oil products among ports and the management of product storage was described by Agra et al. [1]. Mixed-integer programming models are used for scheduling refined-oil shipping as shown by Ye, Liang, and Zhu [31]. Xu et al. [30] incorporated the uncertainty of crude shipping delays within crude oil planning and scheduling operations. Oliveira and Hamacher [21] considered an integrated fuel distribution network design and binary capacity expansion problem under a multi-product and multi-period setting.

The fuel-supply problem tackled in this paper can be modeled as an extension of the lot-sizing formulation of Wagner and Whitin [27]; and particularly of the variant with variable capacity or discrete lot-sizing of Fleischmann [12]. For the case in which the parameters are known with certainty (deterministic case), the size of the lot is continuous, and without capacity or with constant capacity, the problem has efficient resolution through dynamic programming as shown by Wagner and Whitin [27] and Wagelmans et al. [26]. Bitran and Yanasse [6] established that the variant with discrete sizing is a generalization of the binary knapsack problem, and belongs to the NP-hard complexity class.

In the case that the parameters are random (stochastic case), the problem can be formulated by stochastic programming [5]. Stochastic programming model decisions are over time interspersed with random events. These events are assumed discrete establishing, by the time dynamics, a tree of scenarios. The objective of the model is to determine an optimal solution that provides coverage for all the scenarios. Haugen, Løkketangen, and Woodruff [16] describe the scenario-based stochastic lot-sizing problem. Ahmed, King and Parija [2] establish an adjusted extended formulation of the non-capacitated problem and showed that the Wagner-Whitin conditions are not satisfied for the stochastic variant. Guan et al. [13, 14] propose valid inequalities and a branch-and-cut algorithm for the non-capacitated variant.

Shipping delay or delivery time of lots, due to production, transport or capacity restrictions, is modeled in some deterministic continuous no-capacitated lot-sizing problems. Lee et al. [19] present a variant in which demands have a compliance interval that has efficient resolution by dynamic programming. Brahimi and Dauzère-Pérès et al. [7, 9] present two variants according to whether the lots are or are not distinguishable with respect to delivery times. These authors propose efficient algorithms of dynamic programming for the distinguishable case and for the undistinguishable case when the order-delivery windows are not inclusive. For these variants, Wolsey [28] sets tight extended formulations. Van den Heuvel and Wagelmans [25] show the equivalence of the lot-sizing problem with production time windows. For the stochastic scenario-based case, Huang and Küçükyavuz [17] establish that the problem can be efficiently solved in the scenario tree size when delivery windows do not intersect in time. Furthermore, Jiang and Guan improve the efficiency of this procedure [18]. The peculiar aspects of the treated problem that were not found in the literature are the corrective decisions of cancellation and postponement with time delay in a stochastic modeling.

The content of the present work is described below. In Section 2 the algebraic model of the problem is presented in two subsections, first a deterministic variant is described, together with the entities of the problem, and then the stochastic variant is presented. In Section 3 experiments are established to determine validity of the model. In Section 4, conclusions and future work are discussed.

2 MODEL

The modelled problem deals with the minimization of the cost expectation incurred in decisions made to meet the uncertain demand of fuel over a finite discrete time planning horizon. The problem is reduced to a single fuel in order to simplify the proposal without loss of generality. To satisfy the demand, there are optional distinguishable shipments, denoted as cargoes, with a non-splittable quantity of the acquired product. The cargoes have relevant delivery times in the planning horizon; so that a significant amount of time elapses between the purchase decision and when the cargo is received. Due to the passage of time and the unveil of demand uncertainty, it could happen that at any given time a cargo, that was previously acquired and not received is no longer necessary. In this case it could be decided to cancel its acquisition or postpone its delivery; decisions, which, in turn, have minimum execution times in relation to the time of delivery and associated costs. After attending the demand in a given period, the amount of remaining fuel is stored up to a certain capacity, to satisfy future demand in a flexible manner.

The mathematical formulation of the problem is presented below. First, a deterministic model of the problem is developed, where definitions and general structure are established. After defining the information structure of the uncertainty from a discrete stochastic process, the stochastic variant of the model is presented.

2.1 Deterministic model

The following is a deterministic formulation of the problem, where the data are assumed to be known with certainty. A discrete time sequential decision process is considered, in which the decisions taken at a given period depend only on the information available up to that period. Main entities of the problem are described as index sets in Table 1. The planning time is represented by a set of discrete time periods, T. The set of cargoes, C, is categorized into already acquired cargoes, A, and possible cargoes to be acquired, P. Decisions are made to purchase possible cargoes, or to cancel and postpone already acquired cargoes for different periods. Due to the nature of the problem, these decisions have relevant compliance times in the planning horizon. Each cargo has a minimum delivery time, between the time the purchase decision is made and the cargo is received. Decisions to cancel and postpone an already purchased cargo must be made before a certain minimum time, prior to the receipt of the cargo. In addition, when deciding to postpone a previously purchased cargo, the time elapsed between the original reception period and the postponement period can not be less than a given minimum postponement time. All these constraints cause some latency in the decision making process.

Table 1 – Index sets.

T	periods, $t \in T := \{1,...,H\}$
A	already acquired cargoes
P	possible cargoes to be acquired
C	cargoes, $c \in C := A \cup P$

Parameters

Parameters are described in Table 2. As mentioned before, we consider a single product (or multiple products which are interchangeable). The demand for the product in each period, d_t, is known. Due to store constraints, the inventory of the product at the end of each period is restricted between a minimum volume, $\underline{s}$, and a maximum volume, $\overline{s}$, and there is an initial storage volume, s_0, at the beginning of the planning horizon.

The period in which an already acquired cargo is received, τ^c, is fixed, and it is decided in previous acquisitions (i.e. previous model resolutions). Each cargo has a given volume, q^c. Decisions on each cargo have latency times measured in periods. The delivery time of a cargo, γ^c, establishes the length of the wait time (measured in periods) between the acquisition decision and the actual arrival of the cargo. The minimum time for cancellation of a cargo, δ^c, establishes the minimum number of periods prior to the delivery period at which the cargo may be cancelled. The minimum postponement time for a cargo, ε^c, establishes the minimum number of periods after the delivery period in which the postponed cargo can be received. The achievement period of decisions on acquisition, cancellation and postponement must take place within the planning horizon.

For each cargo there are unit costs per volume associated with the decisions to acquire, ca^c, cancel, cc^c, and postpone, cp^c, it. In addition, there is a unit cost associated with storage in each period, h_t.

The already acquired volume that is scheduled to be received in each period is determined by the sum of the cargoes that are received in that period as

$$a_t := \sum_{\{c \in A | \tau^c = t\}} q^c, \quad \forall t \in T;$$

this is an auxiliary summary parameter.

Table 2 – Parameters.

d_t	demand volume in period $t \in T$
s_0	initial storage volume
$\underline{s}, \overline{s}$	minimum and maximum storage capacities by period
τ^c	period in which already acquired cargo $c \in A$ is received
q^c	volume of cargo $c \in C$
γ^c	delivery time of cargo $c \in P$, such that $0 \leq \gamma^c \leq H - 1$
δ^c	cancellation minimum time of already acquired cargo $c \in A$, such that $0 \leq \delta^c \leq \tau^c - 1$
ε^c	postponement minimum time of already acquired cargo $c \in A$, such that $0 \leq \varepsilon^c \leq H - \tau^c$
ca^c	acquisition unit cost of cargo $c \in C$
cc^c	cancellation unit cost of cargo $c \in C$
cp^c	postponement unit cost of cargo $c \in C$
h_t	storage unit cost in period $t \in T$
a_t	already acquired volume that is received in period $t \in T$

Variables

Variables are summarized in Table 3. The variables s_t represent the fuel inventory at the end of each period t. The acquisition decision of each cargo c to be acquired in period t, subject to its delivery period, is modeled by the binary variables v_t^c. Decisions to cancel each cargo c in period t, prior to their minimum cancellation time, are established using binary variables x_t^c. When postponing a cargo reception, decisions must be made about when and until when it is done. Since both decisions are independent, the decision to postpone a cargo c is modeled with a prior cancellation decision and a decision whether to delay its receipt to another period t after its minimum delay time, represented by binary variables z_t^c. In addition, there are auxiliary variables for totals per period t of acquired, u_t, cancelled, w_t, and postponed volume, y_t. These totals variables facilitate the representation of inventory balance constraints.

Table 3 – Variables.

s_t	storage volume at the end of period $t \in T$
u_t	acquired volume into period $t \in T$
v_t^c	if cargo $c \in P$ is acquired in period $t \in \{1,...,H-\gamma^c\}$, (binary)
w_t	cancelled volume out of period $t \in T$
x_t^c	if already acquired cargo $c \in A$ is cancelled in period $t \in \{1,...,\tau^c-\delta^c\}$ (binary)
y_t	postponed volume into period $t \in T$
z_t^c	if already acquired cargo $c \in A$ is postponed to period $t \in \{\tau^c+\varepsilon^c,...,H\}$ (binary)

Objective function

Our aim is to determine a minimum cost scheme of inventory, acquisition, cancellation and postponement of fuel cargoes that satisfy demand during the planning horizon. The costs of acquisitions, cancellations and postponements accrue at the time of decision making. The objective function includes budget costs of acquisition, cancellation and postponement, and inventory costs,

$$\min \sum_{t\in T} \Bigg[\sum_{\{c\in P | t\leq H-\gamma^c\}} ca^c q^c v_t^c \tag{1}$$

$$+ \sum_{\{c\in A | t\leq \tau^c-\delta^c\}} (cc^c - ca^c) q^c x_t^c \tag{2}$$

$$+ \sum_{\{c\in A | \tau^c+\varepsilon^c \leq t\}} (ca^c - cc^c + cp^c) q^c z_t^c \tag{3}$$

$$+ h_t s_t \Bigg]. \tag{4}$$

where: the expression (1) represents the costs of acquiring possible cargoes, the expression (2) represents the costs of cancellation minus the budgeted acquisition costs of already acquired cargoes that are cancelled, the expression (3) represents the acquisition costs minus the costs of cancellation plus the costs of postponement of the already acquired cargoes that are postponed, and the expression (4) represents inventory costs. The postponement cost includes the subtraction of cancellation costs, since a postponement is represented by a prior cancellation, but it does not incur cancellation costs.

Constraints

The main requirement is to satisfy demand while maintaining the inventory balance with contributions of the fuel previously stored, what was already acquired, what is acquired, and what is cancelled, postponed and available in storage,

$$s_{t-1} + a_t + u_t + y_t = d_t + w_t + s_t, \quad \forall t \in T, \tag{5}$$

where s_0 is the given initial storage.

The amount stored in each period is constrained between lower and upper bounds,

$$\underline{s} \leq s_t \leq \bar{s}, \quad \forall t \in T. \tag{6}$$

The amount of acquired fuel that is received in each period is determined by the sum of the cargo acquisitions in the possible range of the corresponding acquisition periods,

$$u_t = \sum_{\{c \in P | \gamma^c \leq t-1\}} q^c v^c_{t-\gamma^c}, \quad \forall t \in T. \tag{7}$$

If a cargo is acquired, the acquisition has been decided in a single period before or equal to its possible receiving period less its delivery time γ^c,

$$\sum_{t=1}^{H-\gamma^c} v^c_t \leq 1, \quad \forall c \in P. \tag{8}$$

The already acquired volume that is cancelled in each period is determined by the cancellations of the cargoes in the possible range of the corresponding cancellation periods,

$$w_t = \sum_{\{c \in A | \tau^c = t\}} \left(q^c \sum_{t'=1}^{\tau^c - \delta^c} x^c_{t'} \right), \quad \forall t \in T. \tag{9}$$

If a cargo is cancelled, the cancellation is decided in a single period before or equal to its receiving period τ^c minus its cancellation time δ^c,

$$\sum_{t=1}^{\tau^c - \delta^c} x^c_t \leq 1, \quad \forall c \in A. \tag{10}$$

The postponement of a cargo is modeled by the use of cancellation, that is to say, it is only possible to postpone cargoes that are cancelled. The already acquired volume that is postponed to a certain period is determined by the postponements of the cargoes in the possible range of the corresponding postponement periods,

$$y_t = \sum_{\{c \in A | \tau^c + \varepsilon^c \leq t\}} q^c z^c_t, \quad \forall t \in T. \tag{11}$$

If a cargo is postponed, it is to be received in a single period subsequent to or equal to its original receiving period τ^c plus its delay time ε^c,

$$\sum_{t=\tau^c+\varepsilon^c}^{H} z_t^c \leq 1, \quad \forall c \in A. \tag{12}$$

A cargo can be postponed, if its original arrival decision has been cancelled,

$$\sum_{t=\tau^c+\varepsilon^c}^{H} z_t^c \leq \sum_{t=1}^{\tau^c-\delta^c} x_t^c, \quad \forall c \in A. \tag{13}$$

Finally, there are domain constraints of the variables

$$\begin{aligned} & s_t, u_t, w_t, y_t \geq 0, \quad \forall t \in T, \\ & v_t^c \in \{0,1\}, \quad \forall c \in P, t \in \{1,...,H-\gamma^c\}, \\ & x_t^c \in \{0,1\}, \quad \forall c \in A, t \in \{1,...,\tau^c-\delta^c\}, \\ & z_t^c \in \{0,1\}, \quad \forall c \in A, t \in \{\tau^c+\varepsilon^c,...,H\}. \end{aligned} \tag{14}$$

This formulation is a generalization of the discrete lot-sizing problem (DLS-C) described by Wolsey [29]. The problem DLS-C is a special case of the formulation in which the acquisition variables are selected among a special ordered set of type one (cf. constraint (8)), the delivery time of each cargo is zero, and there are no decisions of cancellation or postponement. This formulation belongs to the NP-hard class, since DLS-C belongs to the same class [6].

The feasibility of this formulation is conditioned to the timely availability of fuel acquisition to satisfy its demand,

$$s_0 + \sum_{\{c \in A | \tau^c \leq t\}} q^c + \sum_{\{c \in P | \gamma^c \leq t-1\}} q^c \geq \sum_{t'=1}^{t} d_{t'}, \quad \forall t \in T.$$

2.2 Stochastic model with uncertain demand

The following model is a stochastic extension of the deterministic model in which demand is a random parameter. Previously, the uncertain information structure was established through stochastic optimization [5]. Subsequently, the entities and formulation of the stochastic model are reported.

Uncertain information structure

The uncertain demand is represented by a discrete-time stochastic process indexed in the planning periods; in such a way that each stage of the stochastic process is associated to a period. The process is defined in a finite probability space. It is assumed that the demand of the first period is deterministic, and that the demands of the remaining periods are random with known distribution function. The decisions of a period only depend on the outcomes of the random parameters of previous periods. This process is non-anticipatory of the future decisions or the realizations of the random event outcomes. This information structure can be represented by a tree structure with H levels or stages called *tree of scenarios* [23]. This is a perfect directed tree, with the root node in period $t = 1$ and with leaf nodes in period $t = H$ (identifying the scenarios).

Each node of the scenario tree describes the state of the process and is identified by a period and a scenario. An alternative abbreviated notation is to identify the nodes by a single index n in a numerable set of nodes, N. For the first period, $t = 1$, there is a unique node, called r, that represents the root of the tree. Each node $n \in N$ has an immediate predecessor $p(n)$ node; the auxiliary node 0 is defined as the predecessor of the root node, $0 := p(r)$, such that $0 \notin N$. The period corresponding to each node n is defined as $t(n)$. The probability of the state of each node n is defined as $\pi(n)$, such that $\sum_{n \in N|t(n)=t} \pi(n) = 1$, for all $t = 1,...,H$. The t-th predecessor of node n is defined as $p(n,t)$ and the nodes of the path from the root node to n as $P(n)$. The successors of node n are defined as $S(n) := \{n' \in N, t = 1,...,H - t(n) | n = p(n',t)\}$. The nodes of the path from a given node n_1 to a successor node $n_2 \in S(n_1)$ is defined as $P(n_1,n_2)$. Leaf nodes are defined as $L := \{n \in N | t(n) = H\}$.

Stochastic model formulation

From the scenario tree notation, the formulation of the deterministic model (cf. Section 2.1) is extended into a multi-stage stochastic optimization scheme considering the uncertainty in demand.

New index sets are established according to Table 4. First of all, the set of nodes and their subset of leaf nodes are incorporated into the formulation. Subsets of the set of nodes are established to abbreviate the denomination of nodes where it is possible to acquire each cargo, N^c_γ, and where it is possible to cancel and postpone each cargo, N^c_δ. In addition, subsets of periods to where it is possible to postpone each cargo are established, T^c_ε. With the purpose of disambiguation with respect to the original sets, these subsets are named with the parameters that define them as suffixes.

Table 4 – Stochastic model index sets.

N	nodes of the scenario tree
L	leaf nodes of the scenario tree, $L := \{n \in N \| t(n) = H\}$
N^c_γ	nodes where it is posible to acquire cargo $c \in P$
	$N^c_\gamma := \{n \in N \| t(n) \leq H - \gamma^c\}$
N^c_δ	nodes where it is posible to cancel and postpone cargo $c \in A$
	$N^c_\delta := \{n \in N \| t(n) \leq \tau^c - \delta^c\}$
T^c_ε	periods to where it is possible to postpone cargo $c \in A$
	$T^c_\varepsilon := \{t \in T \| t \geq \tau^c + \varepsilon^c\}$

The random parameter and mapping operators on the nodes of the stochastic model are established in Table 5. The demand parameter, d_n, which in the deterministic model depends on the periods, in the stochastic model depends on the nodes of the tree.

Table 5 – Parameters and operators of the stochastic model.

d_n	demanded volume in node $n \in N$
$t(n)$	period of node $n \in N$
$p(n)$	immediate predecessor node of node $n \in N$ in the tree
$p(n,t)$	t-th predecesor node of node $n \in N$ in the tree
$\pi(n)$	probability of node $n \in N$
$P(n)$	nodes in the path from root node to node $n \in N$
$S(n)$	successor nodes of node $n \in N$ in the tree
$P(n_1,n_2)$	nodes in the path from node n_1 to a successor node $n_2 \in S(n_1)$

In the stochastic model, most decisions depend on the nodes of the tree according to Table 6. In contrast to the deterministic model, the decision to postpone a cargo, into a given period, could be taken in different nodes; this is modeled by variable z^c_{nt}.

Table 6 – Variables of the stochastic model.

s_n	stored volume at the end of period in node $n \in N$
u_n	acquired volume into node $n \in N$
v^c_n	if cargo $c \in P$ is acquired in node $n \in N^c_\gamma$, (binary)
w_n	cancelled volume out of node $n \in N$
x^c_n	if already acquired cargo $c \in A$ is cancelled in node $n \in N^c_\delta$ (binary)
y_n	postponed volume into node $n \in N$
z^c_{nt}	if already acquired cargo $c \in A$ is postponed in node $n \in N^c_\delta$
	to period $t \in T^c_\varepsilon$ (binary)

The indexes of periods in deterministic parameters or variables are referred to the temporary realization of a node n by $t(n)$. This is the case for the already acquired volume parameter, which is indexed as $a_{t(n)}$.

From the previous definitions, the formulation of the multi-stage stochastic optimization model is

$$\min \sum_{n\in N} \pi(n) \Big[\sum_{\{c\in P|n\in N^c_\gamma\}} ca^c q^c v^c_n \tag{15}$$

$$+ \sum_{\{c\in A|n\in N^c_\delta\}} (cc^c - ca^c) q^c x^c_n \tag{16}$$

$$+ \sum_{\{c\in A, t\in T^c_\varepsilon|n\in N^c_\delta\}} (cp^c + ca^c - cc^c) q^c z^c_{nt} \tag{17}$$

$$+ h_{t(n)} s_n \Big], \tag{18}$$

s.a.

$$s_{p(n)} + a_{t(n)} + u_n + y_n = d_n + w_n + s_n, \quad \forall n \in N, \tag{19}$$

$$\underline{s} \le s_n \le \bar{s}, \quad \forall n \in N, \tag{20}$$

$$u_n = \sum_{\{c\in P|\gamma^c+1\le t(n)\}} q^c v^c_{p(n,\gamma^c)}, \quad \forall n \in N, \tag{21}$$

$$\sum_{n'\in P(n)} v^c_{n'} \le 1, \quad \forall c \in P, \forall n \in N|t(n) = H - \gamma^c, \tag{22}$$

$$w_n = \sum_{\{c\in A|t(n)=\tau^c\}} \left(q^c \sum_{\{n'\in P(n)|t(n')\le \tau^c-\delta^c\}} x^c_{n'} \right), \quad \forall n \in N, \tag{23}$$

$$\sum_{n'\in P(n)} x^c_{n'} \le 1, \quad \forall c \in A, \forall n \in N|t(n) = \tau^c - \delta^c, \tag{24}$$

$$y_n = \sum_{\{c\in A|t(n)\ge \tau^c+\varepsilon^c\}} \left(q^c \sum_{\{n'\in P(n)\cap N^c_\delta\}} z^c_{n't} \right), \quad \forall n \in N, \tag{25}$$

$$\sum_{\{n'\in P(n), t\in T^c_\varepsilon\}} z^c_{n't} \le 1, \quad \forall c \in A, \forall n \in N^c_\delta, \tag{26}$$

$$x^c_n \ge z^c_{nt}, \quad \forall c \in A, \forall n \in N^c_\delta, \forall t \in T^c_\varepsilon, \tag{27}$$

$$s_n, u_n, w_n, y_n \ge 0, \quad \forall n \in N,$$

$$v^c_n \in \{0,1\}, \quad \forall c \in P, \forall n \in N^c_\gamma,$$

$$x^c_n, z^c_{nt} \in \{0,1\}, \quad \forall c \in A, \forall n \in N^c_\delta, \forall t \in T^c_\varepsilon.$$

This formulation takes into account the same structural properties of the deterministic model extended with the information structure of the scenario tree. It minimizes the expectation of acquisition costs (15), cancellation costs minus acquisition costs in case of cancellation (16), postponement costs plus acquisition costs minus postponement costs (a postponement is modeled in conjunction with a cancellation) and storage costs (18).

The constraint (19) sets the volume balance for each node. The lower and upper storage bounds at each node are determined by (20). The amount of acquired fuel that is received in each node is determined by acquisitions of cargoes in the possible range of the corresponding acquisition periods according to (21). The constraint (22) states that each cargo is acquired at a single node in each path from the root node to a node whose period coincides with the receiving period minus the delivery time of the cargo. The fuel previously acquired that is cancelled at each node is determined by the cancellations of the nodes in the path from the root node to the node, whose cancellation periods are less than the delivery period less the cancellation time, according to (23). The constraint (24) states that each cargo to be cancelled is at a single node in each path from the root node to a node whose period coincides with the receiving period minus the cancellation time of the cargo. The postponement of the cargoes is modeled in conjunction with the cancellation, i.e. only cancelled cargoes can be postponed, (27). The already acquired volume that is postponed in a node is determined by the postponements of the cargoes in the nodes in the path from the root to the node for all periods superior to the period of reception plus the delay time of the node, according to (25). The constraint (26) states that each cargo to be postponed is at a single node in each path from the root node to a node in some period greater than the receiving period plus the time of postponement of the node. The formulation does not need nonanticipativity constraints because the direct indexing of the decisions on the scenario tree implicitly establishes this condition.

The stochastic formulation does not guarantee that all solutions that satisfy constraints at initial stages are also feasible in the remaining stages; that is, it does not have *complete recourse* [5]. This formulation is a generalization of the deterministic one. The deterministic formulation is a special case of the stochastic for the case of a unique scenario in which the tree of scenarios is reduced to a path. Therefore, the feasibility of the stochastic formulation is conditioned to the timely availability of fuel to satisfy its demand along the path of each node of the tree,

$$s_0 + \sum_{\{c \in A | \tau^c \leq t(n)\}} q^c + \sum_{\{c \in P | \gamma^c + 1 \leq t(n)\}} q^c \geq \sum_{n' \in P(n)} d_{n'}, \quad \forall n \in N.$$

3 EMPYRICAL ANALYSIS

In order to generate a number of diverse test instances, four scenario tree structures were considered. Each structure, depicted in Table 7, is determined by the number of direct descendants of each node (tree arity) and the number of periods of the planning horizon. The time size of the periods is measured in weeks. For each tree structure with arity g and H periods there are g^{H-1} escenarios and $(g^H - 1)/(g - 1)$ nodes.

Table 7 – Size of scenario tree structures.

Arity	Periods	Scenarios	Nodes
2	5	16	31
2	6	32	63
3	5	81	121
3	6	243	364

In order to show how the stochastic process is considered, the scenario tree structure with arity $g = 2$ and $H = 5$ periods is depicted in Figure 1.

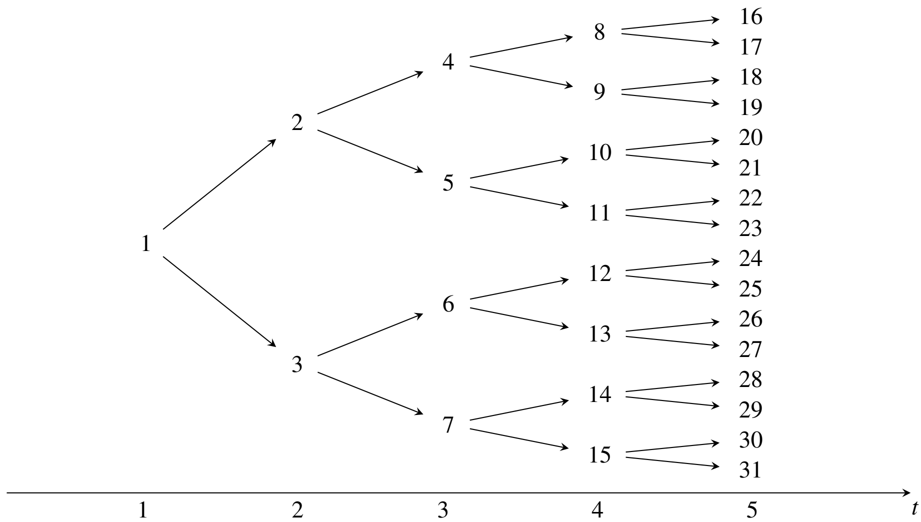

Figure 1 – Tree structure with arity $g = 2$ and $H = 5$ periods where the set of 31 nodes is numbered sequentially by stage increase and the 16 scenarios are defined as the paths from root node 1 to each leaf node 16 to 31.

The size of each tree structure model instance (number of equations and variables) for a given distribution of already acquired cargoes (A) and possible cargoes to be acquired (P) is shown in Table 8.

Table 8 – Instance size of scenario tree structures by cargo distribution.

Arity	Periods	Cargoes $[\lvert C\rvert\ (\lvert A\rvert + \lvert P\rvert)]$	Equations	Variables	(binary)
2	5	10 (2+8)	225	249	(124)
2	6	12 (3+9)	480	549	(296)
3	5	10 (3+7)	827	809	(324)
3	6	12 (4+8)	2542	2485	(1028)

Three data instances were generated for each tree structure and cargo distribution, totaling 12 instances. Each instance has an initial storage, $s_0 = 20$, and a lower and an upper bound storage, $\underline{s} = 0$ and $\bar{s} = 80$, respectively. For each cargo $c \in C$ there is an uniformly distributed volume, $q^c \sim U[10,50]$, and there are costs evenly distributed according to the operations of acquisition, $ca^c \sim U[150,250]$, cancellation, $cc^c \sim U[30,50]$, and postponement, $cp^c \sim U[5,12]$. Each already acquired cargo $c \in A$ has delivery period $\tau^c = 1$ or 2 with equal probability. Each cargo $c \in C$ has delivery time $\gamma^c = 1$, cancellation time $\delta^c = 1$ and delay time $\varepsilon^c = 1$. The unit storage cost in each period t is $h_t = 1$. For each scenario $n \in L$ (leaf node), a probability of state $\pi(n)$ is established from a distribution $Beta(\alpha = 2, \beta = 2)$; the probability of the remaining nodes is obtained from the sum of the probabilities of their corresponding immediate successor nodes. Finally, the demand for each node is evenly distributed, $d_n \sim U[10, 50]$.

The computational implementation was performed using AMPL [4] for the algebraic coding of the stochastic model, and GUROBI 6.5 [15] for the resolution of the instances through its branch and cut solver. The calculations were carried out on an Intel Core i7 5960X 3.5GHz computer with 20MB cache and 64GB RAM, operating with CentOS-7 Linux system.

Optimal values of the instances are shown in Table 9. The optimal value and time of resolution of the stochastic model, denoted as recourse problem, is depicted at attributes RP and Time, the optimal value of the "wait and see" approach is represented with attribute WS, and the uncertainty measure "expected value of perfect information" is shown at attribute EVPI [5]. WS represent the lower bound of the optimum in the idealized case of knowing the future with certainty; and EVPI is defined as the difference between RP and WS. It can be seen that EVPI shows a good performance, since large values validate the use of the stochastic programming approach. Unsurprisingly, instances of larger problems take longer execution time. Particularly the instances 10 and 11 reach the time limit of 900 s. Their best feasible solutions do not change after 48 and 294 s, respectively, and both optimality MIP gap's marginal rate reduction, at the time limit, are 5×10^{-6} per second.

Table 9 – Optimal values of recourse problem (RP) and "wait and see" approach (WS), and expected value of perfect information measure (EVPI) of the instances.

Instance	Arity	Periods	Cargoes	RP	Time(s)	WS	EVPI
1				10,254	0.07	8,993	1,261
2	2	5	10	11,699	0.32	10,428	1,271
3				12,755	0.66	10,601	2,154
4				8,091	6.65	6,961	1,130
5	2	6	12	11,760	14.76	10,415	1,345
6				17,014	31.59	15,561	1,453
7				8,186	1.14	6,177	2,009
8	3	5	10	13,284	20.76	10,945	2,339
9				16,719	17.05	14,294	1,632
10				12,371	(2.34%)†	10,008	2,363
11	3	6	12	6,346	202.68	4,468	1,878
12				11,580	(1.60%)†	9,003	2,577

(†) MIP gap for instances that reach the time limit of 900 s.

The "expectation of the expected value problem" for each period t, EEV_t, is obtained by solving the problem while fixing the solution of its variables up to time t with the solution of a variant of the problem in which the uncertain parameter is substituted by the mean. Then the "values of the stochastic solution" measure corresponding to period t, VSS_t, is defined as the difference between each EEV_t and RP. The values of VSS_t for the periods of the first two instance are reported in Table 10. These values measures the importance of using the distribution of the uncertain outcomes, and they are calculated according to the proposal of Escudero et al. [11]. The VSS_t values depicted confirm the model time-step advantage with respect to the solution of the expected value problem. For the case of Instance 1 an $t = 3$, the value of VSS_3 indicates that the stochastic model obtains a solution with an expected reduced cost of 3,489 with respect to a deterministic variant of the model that consider expected value demand. For larger values of t, the tightening of the solution of the expected value problems turn the EEV_t values infeasible, since the formulation does not have complete recourse and the settling of the variable makes the EEV_t problems increasingly restrictive as t increases.

Table 10 – Optimal value of expected result of using the expected value problem (EEV) and value of the stochastic solution measure (VSS) for the periods by selected instances.

Instance	Arity	Periods	Cargoes	t	EEV_t	VSS_t
				1	10,254	
				2	10,524	270
1	2	5	10	3	13,743	3,489
				4	infeas.	∞
				5	infeas.	∞
				1	11,699	
				2	11,699	0
2	2	5	10	3	12,125	426
				4	12,146	21
				5	infeas.	∞

Note that EEV_1 = RP, therefore VSS_1 = 0.

Table 11 shows a summary of the optimal solution of the instances. For each instance, it depicts the number of cargoes that are acquired, $\#\text{AcqCrg} = |\{c \in P : n \in N^c_\gamma, v^c_n = 1\}|$, among the available ones that total between 7 and 8 depending upon the instance (cf. Table 8). The number of decisions on acquisition made is depicted in $\#\text{Acq} = |\{c \in A, n \in N^c_\gamma : v^c_n = 1\}|$. The results show a high granularity of the acquisition process, because the optimal acquisition decisions are made at many different nodes; and the number of nodes where acquisition decisions are made grows as the scenario tree size increases. The number of decisions on cancellation and postponement, among the already acquired cargoes that total between 2 and 4 depending upon the instance (cf. Table 8), made are depicted in $\#\text{Can} = |\{c \in A, n \in N^c_\delta : x^c_n = 1\}|$ and $\#\text{Pos} = |\{c \in A, n \in N^c_\delta, t \in T^c_\varepsilon : z^c_{nt} = 1\}|$, respectively, for each instance. As can be seen that these decisions are sporadic, which is valuable from the commercial and logistical point of view. Furthermore, it may be difficult to detect them in advance without the support of a model. The model's advantage of detecting all these decisions in advance allows an early probabilistic consideration of their execution.

Table 11 – Optimal solution summary with cardinality of selected cargos and cardinality of acquisition, cancellation and postponement decisions by instances.

Inst.	Arity	Periods	Cargoes	#AcqCrg	#Acq	#Can	#Pos
1				4	8	0	0
2	2	5	10	4	12	0	0
3				5	11	0	0
4				7	32	1	0
5	2	6	12	6	36	1	0
6				7	30	0	0
7				4	36	0	0
8	3	5	10	6	36	1	0
9				6	31	0	0
10†				7	74	3	0
11	3	6	12	6	87	1	1
12†				6	81	2	1

(†) Instances that reach the time limit of 900 s.

4 CONCLUSIONS

In this paper, we proposed a stochastic multi-stage capacitated discrete lot-sizing model formulation for a discrete cargo fuel supply with lead times problem. The decisions of the problem were represented in detail with their delay time, aspect that for cancellation and postponement decisions is not covered in previous literature. The structure of the uncertain information was modeled by a discrete time stochastic process with finite probability, summarized in a scenario tree. Stochastic programming methodology with entities indexed by nodes of the scenario tree was used to formulate the model. The model extends deterministic models of the literature, which implied the revision of the definitions of the variables and the restrictions to take into account the structure of the scenario tree.

Computational experiments where carried out for several instances with scenarios of a variety of sizes and characteristics. Most of these experiments were solved to optimality for the medium-size generated instances. The experimental results have shown the validity of the model. Results shows a considerable advantage over the expected value of corresponding deterministic models. Although medium-sized instances could be solved, computational times grew significantly, and for scenarios with more stages or more decisions per stage it is important to improve computational efficiency. Therefore, for future work, it is recommended to study reformulations of the stochastic model with tight high level relaxations, and develop resolution techniques according to them.

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REFERENCES

[1] AGRA A, CHRISTIANSEN M, DELGADO A & HVATTUM LM. 2015. A maritime inventory routing problem with stochastic sailing and port times. *Computers and Operations Research*, **61**: 18–30, http://www.sciencedirect.com/science/article/pii/S0305054815000210.

[2] AHMED S, KING AJ & PARIJA G. 2003. A Multi-Stage Stochastic Integer Programming Approach for Capacity Expansion under Uncertainty. *Journal of Global Optimization*, **26**(1): 3–24, http://dx.doi.org/10.1023/A:1023062915106.

[3] AL-OTHMAN WB, LABABIDI HM, ALATIQI IM & AL-SHAYJI K. 2008. Supply chain optimization of petroleum organization under uncertainty in market demands and prices. *European Journal of Operational Research*, **189**(3): 822–840, http://www.sciencedirect.com/science/article/pii/S0377221707006595.

[4] AMPL. AMPL Optimization Inc. http://www.ampl.com, online; accessed July, 2018.

[5] BIRGE JR & LOUVEAUX F. 1997. *Introduction to Stochastic Programming*. Springer, New York, springer Series in Operations Research and Financial Engineering.

[6] BITRAN GR & YANASSE HH. 1982. Computational Complexity of the Capacitated Lot Size Problem. *Management Science*, **28**(10): 1174–1186, http://www.jstor.org/stable/2630946.

[7] BRAHIMI N, DAUZÈRE-PÉRÈS S & NAJID N. 2005. Capacitated multi-item lot-sizing problems with time windows. Tech. rep., Ecole des Mines de Nantes.

[8] CHRISTIANSEN M, FAGERHOLT K, NYGREEN B & RONEN D. 2013. Ship routing and scheduling in the new millennium. *European Journal of Operational Research*, **228**(3): 467–483, http://www.sciencedirect.com/science/article/pii/S0377221712009125.

[9] DAUZÈRE-PÉRÈS S, BRAHIMI N, NAJID N & NORDLI A. 2005. Uncapacitated lot-sizing problems with time windows. Tech. rep., Ecole des Mines de Saint-Etienne.

[10] DEMPSTER MAH, HICKS PEDRÓN N, MEDOVA EA, SCOTT JE & SEMBOS A. 2000. Planning logistics operations in the oil industry. *Journal of the Operational Research Society*, **51**(11): 1271–1288, https://doi.org/10.1057/palgrave.jors.2601043.

[11] ESCUDERO LF, GARÍN A, MERINO M & PÉREZ G. 2007. The value of the stochastic solution in multistage problems. *TOP*, **15**(1): 48–64, https://doi.org/10.1007/s11750-007-0005-4.

[12] FLEISCHMANN B. 1990. The discrete lot-sizing and scheduling problem. *European Journal of Operational Research*, **44**(3): 337–348, http://www.sciencedirect.com/science/article/pii/0377221790902457.

[13] GUAN Y, AHMED S, MILLER AJ & NEMHAUSER GL. 2006. On formulations of the stochastic uncapacitated lot-sizing problem. *Operations Research Letters*, **34**(3): 241–250, http://www.sciencedirect.com/science/article/pii/S016763770500057X.

[14] GUAN Y, AHMED S, NEMHAUSER GL & MILLER AJ. 2006. A branch-and-cut algorithm for the stochastic uncapacitated lot-sizing problem. *Mathematical Programming*, **105**(1): 55–84, http://dx.doi.org/10.1007/s10107-005-0572-9.

[15] GUROBI. Gurobi Optimization Inc. http://www.gurobi.com/, online; accessed July, 2018.

[16] HAUGEN KK, LØKKETANGEN A & WOODRUFF DL. 2001. Progressive hedging as a meta-heuristic applied to stochastic lot-sizing. *European Journal of Operational Research*, **132**(1): 116–122, http://www.sciencedirect.com/science/article/pii/S0377221700001168.

[17] HUANG K & KÜÇÜKYAVUZ S. 2008. On stochastic lot-sizing problems with random lead times. *Operations Research Letters*, **36**(3): 303–308, http://www.sciencedirect.com/science/article/pii/S0167637707001460.

[18] JIANG R & GUAN Y. 2011. An $\mathscr{O}(N^2)$-time algorithm for the stochastic uncapacitated lot-sizing problem with random lead times. *Operations Research Letters*, **39**(1): 74–77, http://www.sciencedirect.com/science/article/pii/S0167637710001422.

[19] LEE CY, ÇETINKAYA S & WAGELMANS APM. 2001. A Dynamic Lot-Sizing Model with Demand Time Windows. *Management Science*, **47**(10): 1384–1395, http://www.jstor.org/stable/822493.

[20] MORAES LA & FARIA LF. 2016. A stochastic programming approach to liquified natural gas planning. *Pesquisa Operacional*, **36**: 151–165, http://www.scielo.br/scielo.php?script=sci_arttext&pid=S0101-74382016000100151&nrm=iso.

[21] OLIVEIRA F & HAMACHER S. 2012. Stochastic Benders decomposition for the supply chain investment planning problem under demand uncertainty. *Pesquisa Operacional*, **32**: 663–678, http://www.scielo.br/scielo.php?script=sci_arttext&pid=S0101-74382012000300010&nrm=iso.

[22] PINTO J, JOLY M & MORO L. 2000. Planning and scheduling models for refinery operations. *Computers and Chemical Engineering*, **24**(9): 2259–2276, http://www.sciencedirect.com/science/article/pii/S0098135400005718.

[23] RÖMISCH W & SCHULTZ R. 2001. Multistage Stochastic Integer Programs: An Introduction. In: GRÖTSCHEL M, KRUMKE SO & RAMBAU J (Eds.), *Online Optimization of Large Scale Systems*, Springer Berlin Heidelberg, pp. 581–600, http://dx.doi.org/10.1007/978-3-662-04331-8_29.

[24] TESTURI CE, ZIMBERG B & FERRARI G. 2012. Modelado estocástico múltiple etapa de adquisición de combustible para la generación de electricidad bajo demanda incierta. Tech. Rep. INCO RT 12-07, Instituto de Computación, Facultad de Ingeniería, Universidad de la República, https://www.fing.edu.uy/inco/pedeciba/bibliote/reptec/TR1207.pdf.

[25] VAN DEN HEUVEL W & WAGELMANS AP. 2008. Four equivalent lot-sizing models. *Operations Research Letters*, **36**(4): 465–470, http://www.sciencedirect.com/science/article/pii/S0167637708000035.

[26] WAGELMANS A, HOESEL SV & KOLEN A. 1992. Economic Lot Sizing: An $\mathscr{O}(n \log n)$ Algorithm That Runs in Linear Time in the Wagner-Whitin Case. *Operations Research*, **40**: 145–156, http://www.jstor.org/stable/3840844.

[27] WAGNER HM & WHITIN TM. 1958. Dynamic Version of the Economic Lot Size Model. *Management Science*, **5**(1): 89–96, http://mansci.journal.informs.org/content/5/1/89.abstract.

[28] WOLSEY AL. 2006. Lot-sizing with production and delivery time windows. *Mathematical Programming*, **107**(3): 471–489, http://dx.doi.org/10.1007/s10107-005-0675-3.

[29] WOLSEY LA. 2002. Solving Multi-Item Lot-Sizing Problems with an MIP Solver Using Classification and Reformulation. *Management Science*, **48**(12): 1587–1602, http://www.jstor.org/stable/822525.

[30] XU J, QU H, WANG S & XU Q. 2017. A New Proactive Scheduling Methodology for Front-End Crude Oil and Refinery Operations under Uncertainty of Shipping Delay. *Industrial and Engineering Chemistry Research*, **56**(28): 8041–8053, https://doi.org/10.1021/acs.iecr.7b01496.

[31] YE Y, LIANG S & ZHU Y. 2017. A mixed-integer linear programming-based scheduling model for refined-oil shipping. *Computers and Chemical Engineering*, **99**: 106–116, http://www.sciencedirect.com/science/article/pii/S0098135417300315.

[32] ZIMBERG B, TESTURI CE & FERRARI G. 2019. Stochastic modeling of fuel procurement for electricity generation with contractual terms and logistics constraints. *Computers and Chemical Engineering*, **123**: 49–63, https://doi.org/10.1016/j.compchemeng.2018.12.021.