SOBRAPO

MULTICRITERIA ANALYSIS OF FOOTBALL MATCH PERFORMANCES: COMPOSITION OF PROBABILISTIC PREFERENCES APPLIED TO THE ENGLISH PREMIER LEAGUE 2015/2016

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ABSTRACT. This article aims to analyze the technical performance of football teams in the FA Premier League during the 2015/2016 season. Data of twenty clubs over 38 matches for each club are considered using 23 variables. These variables have been explored in the football literature and address different features of technical performance. The different configuration of the data for teams in detached segments motivated the multi-criteria approach, which enables identification of strong and weak sectors in each segment. The uncertainty as to the outcome of football matches and the imprecision of the measures indicated the use of Composition of Probabilistic Preferences (CPP) to model the problem. "R" software was used in the modeling and computation. The CPP global scores obtained were more consistent with the final classification than those of other methods. CPP scores revealed different performances of particular groups of variables indicating aspects to be improved and explored.

Keywords: match analysis, probabilistic composition of preferences, football.

1   INTRODUCTION

Performance in football has been described as a construct with interaction between individual and collective levels (Bradley et al., 2011). Results of football matches, for example, provide objective measures of both player and team performance. Interactions within the team due to cooperation based on a strategic plan, situational variables and constraints provided by the general context of the game can be considered (Besters, van Ours & van Tuijl, 2016).

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The analysis of actions performed in a match context produces quantitative data that may become relevant information to support coaches' decisions (Carling, Williams & Reilly, 2005). Technical elements of player performance in football are measured by notational systems, designed to register and store information on players' actions, in order to identify patterns of play and critical elements of performance. For football analysis, these systems provide statistics related to players' actions (passing, shots on goal, fouls, etc.) that may provide varied information about the success of the teams (Woods, Raynor, Bruce, McDonald & Robertson, 2016).

The related literature shows contradictions between the performance of players in the execution of expected actions and the successful and unsuccessful results of the teams. This happens, for instance, with the strategy of direct play, which is described as few passes per team possession and identified by attacks involving at least one long pass, attacks with a maximum of two passes, and fast-moving attacks. Possibly, the long pass abilities and skill of players influence the effectiveness of direct play strategy (Fernandez-Navarro, Fradua, Zubillaga & Ford, 2016).

Other studies suggest that counter-attacks are more effective than planned attacks when playing against an unbalanced defense (Tenga, Holme, Ronglan & Bahr, 2010). Furthermore, Lago-Ballesteros & Lago-Peñas (2010) reported that successful teams usually touch the ball more often. In this way, ball possession has also been reported as a variable decisively affecting the performance of football teams.

Most of these studies used manual notation and computer systems to process the data. In the same way, football coaches use notational analysis to evaluate their team and opponent teams' players. A notation system enables an assessment of the relative merits of team and player performance, becoming the first step towards performance analysis (Eaves, 2015).

However, these studies access a huge amount of raw data, which is quite difficult to analyze in order to determine which teams had the best performance. In addition, uncertain outcomes and imprecise data are common in the football environment. A statistical approach, allowing for the use of estimation and simulation techniques would provide results whose uncertainty could be measured. That would certainly be strongly welcome. Nevertheless, by now, the lack of reliable models for this field makes unfeasible such a statistical approach. Instead we take here a multicriteria decision analysis (MCDA) approach.

To take into account uncertainty we apply Composition of Probabilistic Preferences (CPP), a technique that uses probabilistic reasoning to combine preferences according to separate criteria. To do that, the evaluations by the separated criteria are modeled as random variables. The application of CPP is compared to the application of two methods based on the use of fuzzy numbers instead of crisp numbers for the values and weights of the criteria in well known exact MCDA methods: Fuzzy MULTIMOORA Method and Fuzzy VIKOR Method.

In CPP, triangular distributions for the initial preference assessments are employed, while triangular fuzzy number (TFN) are used in the fuzzy methods. In spite of the conceptual differences, a comparison of the results could be done. After applying the three methods to the same matches statistics of an FA Premier League, Kendal and Spearman correlation coefficients were used to analyze the efficiency of each method with respect to the study's objectives.

These objectives are to discuss the relevance of the imprecise factors evaluated to predict the ranking of the teams at the end of the competition. CPP is used to obtain an analysis of the teams' performances in the competition, especially taking into account the decisions that may be taken to improve the performance of each team. First, it is shown how a suitable point of view in the probabilistic composition better fits the final ranking of FA Premier League season 2015/2016.

CPP is first applied on the whole set of 23 variables considered in the study taken together and, after that, on three groups of technical performance variables separately. In this last analysis, the teams are evaluated from the perspectives of goal attempts, offensive moves and defensive actions, three phases which occur in the development of a football match and are important for the knowledge and development of each team. In this way, finding out in which of the phases of the game the team has the best or the worst performance can change the way the coaches may improve their training section.

This study is organized as follows. In the next section, the methods employed including CPP, fuzzy numbers, fuzzy MULTIMOORA, fuzzy VIKOR and correlation analysis, are introduced Section 3 presents the data. The results obtained are discussed in Section 4. Finally, a section of conclusions is presented.

2   TECHNIQUES AND METHODS

In this section a brief review of the techniques employed is presented.

2.1   Composition of Probabilistic Preferences

CPP is a multi-criteria (MC) decision support method introduced by Sant'Anna & Sant'Anna (2001) that was recently expanded to different applications in decision support by Sant'Anna (2015). In general, MC methods intend to help decision-makers dealing with several possibilities in a choice set. The MC problems usually lead to a compromise between conflicting criteria, such as quality and cost, for instance. In particular, CPP has been applied to MC analysis in sport sciences (Sant'Anna & Soares de Mello, 2012).

CPP adopts a probabilistic approach to MC decision problems. The majority of MC methods apply deterministic or fuzzy evaluations of alternatives under different criteria (Mardani, Jusoh, Zavadskas, Khalifah & Nor, 2015). On the other hand, CPP explores the uncertainty that is inevitably present in preference evaluations in real problems. This uncertainty may result from processes involving expert reviews, inaccurate performance measures or processes with imperfect metric systems, among others. Adopting a probabilistic framework, the evaluation of each alternative takes the form of a probability function and the final ranking is derived from probabilities of being the best alternative (Garcia & Sant'Anna, 2015).

CPP is developed in three stages. The first refers to the 'randomization' of the evaluations. Randomization means the transformation of exact values into random variables. In Figure 1, for instance, the performance of alternative 'A' under criterion '1' is represented by an exact value

'10'. The randomization procedure changes this measurement into a probability distribution. In this case, a normal disturbance was assumed as a general behavior for evaluating alternative 'A', and the value '10' as a mode of this probability distribution. In summary, the value is taken as a single measure of location in a probability distribution that reflects the uncertainty and imprecision of the model (Sant'Anna, 2015).


	Criterion 1
Alternative A	10

(exact value)

	Criterion 1
Alternative A	(curve)

(imprecise value)
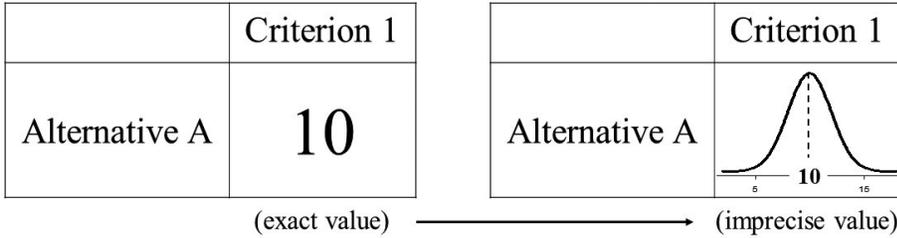
Figure 1 – CPP first stage. (Gavião, Silva, Sant'Anna & Lima (2016)).

The randomization may derive from *a priori* information obtained in similar contexts. For example, failure rates in reliability problems are generally adjusted by Exponential, Weibull or Birnbaum-Saunders distributions (Chiodo & Lauria, 2015). For a data set of preferences, the probability function can be obtained from goodness-of-fit tests for density estimation (Delignette-Muller & Dutang, 2014). Disturbances can also be described by non-parametric or empirical distributions, in the case of abundant data (Millard, 2013).

In the second stage, the joint probabilities M_{ij} and m_{ij} are computed. M_{ij} and m_{ij} denote, respectively, the probability of the i-th alternative being the best and the worst alternative according to the j-th criterion. In other words, 'M' refers to maximizing and 'm' to minimizing the preferences. Equations (1) and (2) describe the mathematical development to compute M_{ij} and m_{ij}.

$$M_{ij} = \int_{D_{X_i}} \left[\prod F_{X_{-i}}(x_{-i}) \right] f_{X_i}(x_i)dx_i \tag{1}$$

$$m_{ij} = \int_{D_{X_i}} \left[\prod \left(1 - F_{X_{-i}}(x_{-i})\right) \right] f_{X_i}(x_i)dx_i \tag{2}$$

In equations (1) and (2), F_X indicates the cumulative distribution function (cdf), f_X the probability density function (pdf) and D_{X_i} the support of the i-th alternative. The notation '$-i$' indicates all alternatives under the same j-th criterion, except the i-th alternative, which is the considered alternative to compute M_{ij} and m_{ij} (Sant'Anna, Costa, Nepomuceno & Pereira, 2016).

In the third stage, the joint probabilities 'M_{ij}' and 'm_{ij}' are composed according to different points of view of the decision maker, as described in Figure 2 and Table 1. Two axes are identified: The Progressive (P) – Conservative (C) and the Optimist (O) – Pessimist (P). These axes create four possible combinations to tailor the decision-making: PP, PO, CO and CP.
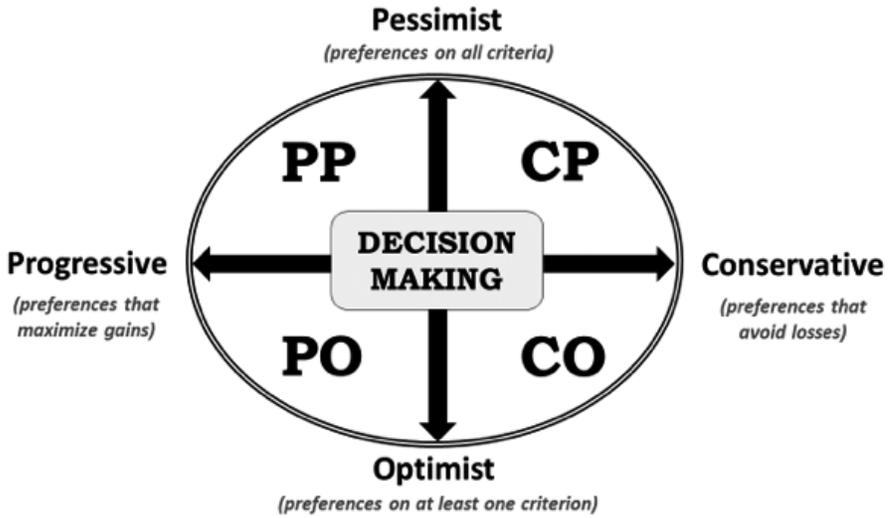
Figure 2 – Composition by axis. (Adapted from Gavião, Silva, Sant'Anna & Lima (2016)).

Table 1 – Third CPP stage equations.

Decision-Maker's Point of View	Pessimist	Optimist
Progressive	$PP_i = \prod_{j=1,\ldots,n} M_{ij}$	$PO_i = 1 - \prod_{j-1,\ldots,n}(1 - M_{ij})$
Conservative	$CP_i = \prod_{j=1,\ldots,n}(1 - m_{ij})$	$CO_i = 1 - \prod_{j-1,\ldots,n} m_{ij}$

Sant'Anna (2015).

The global scores are computed using the equations in Table 1. The global score ranking gives the final preference for each alternative. The equations reflect all decision-maker points of view, under the hypothesis of independence between all the criteria. If a correlation can be quantified, it can be introduced into the combinations. However, quantifying correlation among criteria is not an easy task, because it requires the detection of reciprocal influence between errors in the measurement of variables. For more details on the different assumptions regarding correlation between criteria, the reader is referred to Sant'Anna (2015, Section 5.2).

The progressive-conservative axis uses 'M_{ij}' or 'm_{ij}' as parameters to compose the decision-maker points of view, as depicted in Figure 2. The Progressive approach involves the concept of 'maximizing gains', plotting the decision at the frontier of excellence. For a positive-impact criterion, a progressive decision requires the use of 'M_{ij}' to combine criteria and for a negative-impact criterion, the use of 'm_{ij}' is needed.

On the other hand, the conservative approach reflects the idea of 'avoiding losses' in which the decision maker aims to differentiate alternatives near the border of worst performance. By analogy with the progressive point of view, for a positive-impact criterion to the research problem,

a conservative approach requires the use of 'm_{ij}' to combine criteria and for a negative-impact criterion, the use of 'M_{ij}' is expected.

The optimist-pessimist axis refers to the combination of 'M_{ij}' and 'm_{ij}' as strict preferences for all criteria or satisfying at least one of the multiple criteria (Sant'Anna, 2015).

2.2 Triangular Fuzzy Number

Fuzzy sets theory was developed by Zadeh (1965). The fuzzy logic has been applied to combine evaluations given by fuzzy sets in different fields of knowledge (Mardani, Jusoh & Zavadskas, 2015). A fuzzy number $\tilde{A} = (a, b, c)$, as designed in Figure 3, is called a triangular fuzzy number (TFN). The membership function $\mu_{\tilde{A}}(x)$ denotes the degree of truth that the fuzzy value is equal to x within the real interval *(lower, upper)*. The TFN $\tilde{A}$ has the core b with $\mu_{\tilde{A}}(b) = 1$ and the support $[a, c]$. Fuzziness allows a membership μ of an object to a fuzzy set vary within an interval $(0, 1)$, whereas Boolean sets allow only either a full membership or a full non-membership (Almeida, Yamakami & Takahashi, 2006; Silva, 2008).
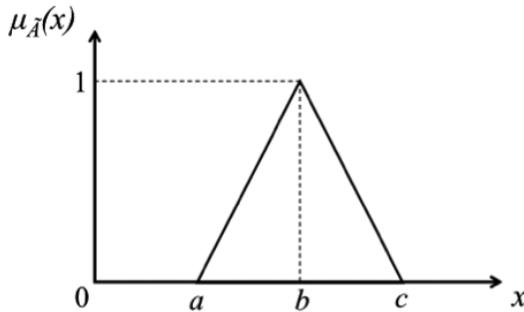
Figure 3 – Triangular Fuzzy Number. (Silva (2008)).

The TFN membership function $F_{A(x)}$ has the following form, in Equations (3):

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & x \leq b \\ (c-x)/(c-b), & x \geq b \\ 0, & x \notin [a, c] \end{cases} \tag{3}$$

Mathematical operations on a generic $\tilde{M}$ TFN are defined from Equation (4) to (12) (Opricovic, 2011):

$$\text{Summation} \sum_{i-1}^{n} \oplus \tilde{N}_i = \left(\sum_{i=1}^{n} a_i, \sum_{i=1}^{n} b_i, \sum_{i=1}^{n} c_i \right) \tag{4}$$

$$\text{Scalar summation } \tilde{N} \oplus k = (a+k, b+k, c+k) \tag{5}$$

$$\text{Subtraction } \tilde{N}_1 \text{ and } \tilde{N}_2 = (a_1 - a_2, b_1 - b_2, c_1 - c_2) \tag{6}$$

$$\text{Scalar subtraction } \tilde{N} - k = (a-k, b-k, c-k) \tag{7}$$

Scalar multiplication $k \cdot \tilde{N} = (k \cdot a, \ k \cdot b, \ k \cdot c)$, for $k \geq 0$ (8)

Multiplication $\tilde{N}_1 \otimes \tilde{N}_2 = (a_1 \cdot a_2, \ b_1 \cdot b_2, \ c_1 \cdot c_2)$, for $a_1 \geq 0$ (positive $\tilde{N}_1$)   (9)

Scalar division $\tilde{N}/k = (a/k, \ b/k, \ c/k)$, for $k > 0$ (10)

Operator MAX $MAX_i \ \tilde{N}_i = (\max_i a_i, \ \max_i b_i, \ \max_i c_i)$ (11)

Operator MIN $MIN_i \ \tilde{N}_i = (\min_i a_i, \ \min_i b_i, \ \min_i c_i)$ (12)

2.3   The Fuzzy Multimoora Method

The multi-objective optimization by ratio analysis (MOORA) method was introduced by Brauers & Zavadskas (2006) on the basis of previous research. Later, the method was extended into a more robust design, by adding a full multiplicative form to MOORA (Brauers & Zavadskas, 2010). The MULTIMOORA method is described in Figure 4, with indications of its four Equations (Baležentis & Baležentis, 2014).
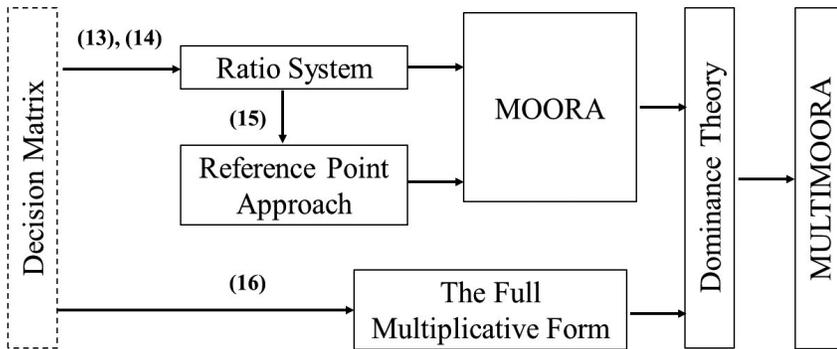
Figure 4 – Fuzzy Multimoora design. (Baležentis & Baležentis (2014)).

The Multimoora method is summarized by Equations (13)-(16).

$$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{13}$$

$$y_i^* = \sum_{j=1}^{g} x_{ij}^* - \sum_{j=g+1}^{n} x_{ij}^* \tag{14}$$

$$\min_i \left(\max_j \left| r_j - x_{ij}^* \right| \right), \ where, \ r_j = \max_i x_{ij}^* \tag{15}$$

$$U_i' = \frac{A_i}{B_i}, \ where, \ A_i = \prod_{j=1}^{g} x_{ij}, \ B_i = \prod_{j=g+1}^{n} x_{ij}, \ i = 1, 2, \ldots, m \tag{16}$$

Application of the MOORA method begins with a decision matrix, X, where x_{ij} denotes the evaluation of the i-th alternative with respect to the j-th objective ($i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$). The first step is divided in two parts: the ratio system and the reference point

approach. Equation (13) describes the ratio system, which is basically a data normalization. In Equation (14), the x_{ij} values are added (if desirable value of indicator is maximum) or subtracted (if desirable value is minimum), to yield a summarizing index y_i. The reference point approach produces a final rank derived from the deviations from the reference point, applying the *min-max* metric of Tchebycheff, as depicted in Equation (15) (Brauers & Zavadskas, 2010, 2012). The overall utility of the i-th alternative can be expressed as a dimensionless number U, in Equation (16). By this way, MULTIMOORA summarizes MOORA (i.e. ratio system and reference point) and the full multiplicative form (Baležentis & Baležentis, 2014).

Following the general practice to extend deterministic MCDA methods into the fuzzy environment, the Fuzzy MULTIMOORA method, introduced by Brauers, Baležentis & Baležentis (2011), is built replacing the initial crisp evaluations by fuzzy numbers and the numeric operations by fuzzy logic operations. A detailed description of all adaptations from the crisp approach to the Fuzzy MULTIMOORA can also be addressed at Baležentis & Baležentis (2014).

2.4 Fuzzy Vikor Method

The *VlseKriterijuska Optimizacija I Komoromisno Resenje* (VIKOR) method has been developed as an MCDM method to solve a discrete multicriteria problem with non-commensurable and conflicting criteria (Opricovic, 1998, 2011). The foundation for a compromise solution was established by Yu (1973), Duckstein & Opricovic (1980) and Zeleny (1982). The method determines a compromise ranking-list, the compromise solution, and the weight stability intervals for preference stability of the compromise solution obtained with the initial (given) weights. In providing such a compromise solution, VIKOR applies the concepts of "acceptable advantage" and "acceptable stability" to determine the maximum "group utility of the majority" and the minimum "individual regret of the opponent" (Anvari, Zulkifli, & Arghish, 2014; Opricovic & Tzeng, 2004), which are combined in a final score. The extension of VIKOR to determine a fuzzy compromise solution adopted in the present application is presented in Opricovic (2007).

The compromise-ranking algorithm of VIKOR consists of five steps. The notation, proposed by Anvari et al. (2014), is described in Equations (17) to (24). Step 1 describes the basic phase of every MCDM method, regarding the formation of a decision matrix. The alternatives are denoted by $A_1, A_2, \ldots A_i, \ldots A_m$, where m is the number of alternatives. The weight of the j-th criterion, expressing the relative importance of the criteria, is denoted w_j, where $j = 1, 2, \ldots, n$, for n representing the number of criteria. The rating (performance score) by the j-th criterion is denoted by f_{ij} for alternative A_i. In Step 2, the best f_i^* and the worst f_j^- values of all criteria are computed according as the i-th is a benefit criterion (positive impact to the final solution) – in Equation (17), or a cost criterion (negative impact to the final solution) – in Equation (18).

$$\text{Benefit criterion } f_i^* = \max(f_{ij}, \ j = 1, \ldots, n), \ f_j^- = \min(f_{ij}, \ j = 1, \ldots, n) \qquad (17)$$

$$\text{Cost criterion } f_i^* = \min(f_{ij}, \ j = 1, \ldots, n), \ f_j^- = \max(f_{ij}, \ j = 1, \ldots, n). \qquad (18)$$

In Step 3, the values of a utility measure S_j and a regret measure R_j are obtained, using the relations depicted in Equations (19) and (20), respectively. The minimum S_i emphasizes the

maximum group utility, whereas R_i emphasizes the minimum value among the maximum individual regrets

$$S_j = \sum_{j=1}^{n} w_j(f_j^* - f_{ij})\big/(f_j^* - f_j^-) \tag{19}$$

$$R_j = \text{Max}\left[w_j(f_j^* - f_{ij})\big/(f_j^* - f_j^-) \right]. \tag{20}$$

In Step 4, Q_j indices are evaluated, using the relation in Equations (21) and (22). The values of S^* may also use the best option $S^* = 0$ and $S-$ the worst value $S- = 1$. For R^* and $R-$, the best $R^* = 0$ or the worst $R- = 1$ can also been chosen. The parameter $v(0 \leq v \leq 1)$ is introduced as a weight for the strategy of maximum group utility, whereas $(1 - v)$ is the weight of the individual regret. In other words, the increase of v emphasizes group utility (i.e. $v > 0.5$), whereas $v \approx 0.5$ reflects consensus and $v < 0.5$ a veto. A way to compute v is by the formulae $(n+1)/2.n$, as proposed by Opricovic (2011). This latter approach was used in the present study leading to a value of v of 0.522)

$$Q_i = v(S_i - S^-)\big/(S^* - S^-) + (1 - v)\left[(S_i - R^-)\big/(R^* - R^-)\right] \tag{21}$$

$$S^- = \min S_i \; S^* = \min S_i \; R^- = \min R_i \; R^* = \min R_i. \tag{22}$$

In Step 5, the R, S and Q approaches are combined using the two conditions in Equation (23) and the definition (24),

$$\text{Condition 1: } Q(A_2) - Q(A_1) \geq (1/(n-1)) \tag{23}$$

where A_1 and A_2 denote the best and the second best alternatives according to the Q indices

$$\text{Condition 2: } A_1 \text{ is also the best in at least one of the } R \text{ and the } S \text{ rankings.} \tag{24}$$

Condition 1 refers to an acceptable advantage and Condition 2 reflects an acceptable stability in the decision making process, where A_1 is also the best in at least one of the R and the S rankings. Alternative A_1 is chosen if Conditions 1 and 2 are satisfied. If one of these two conditions is not satisfied, then a set of compromise solutions is proposed: (1) Alternatives A_1 and A_2, if only Condition 2 is not satisfied; (2) All the alternatives A for which $Q(A) - Q(A1) < 1/(n-1)$ if Condition 1 is not satisfied (Anvari et al., 2014).

VIKOR and the Technique of Order Preference by Similarity to the Ideal Solution (TOPSIS) use a similar approach to a MCDM problem. These methods are based on an aggregating function representing 'closeness to the ideal', which originated in the compromise programming method. The VIKOR method of compromise ranking is based on the idea of maximizing a 'group utility' that reflect the majority choice and a minimum of an individual regret for the 'opponent'. The TOPSIS method determines a solution with the shortest distance to the ideal solution and the greatest distance from the negative-ideal solution, but it does not consider the relative importance of these distances (Opricovic & Tzeng, 2004). Considering the similarity and differences of both methods, the VIKOR method was prioritized in this study.

2.5   Kendall and Spearman Nonparametric Correlation Indices

Kendall and Spearman indices are used to access the correlation between two variables calculated in a set of N alternatives. The Kendall coefficient was developed by (Kendall, 1938). This coefficient is identified with the Greek letter τ (*tau*). In Equation (25), the notation N_c represents the amount of pairs of alternatives evaluated for which the ordinal positions are concordant, while N_d indicates the number of discordant pairs. A pair of alternatives is count as concordant whenever the differences between their ordinal positions by the two variables have the same sign and is count as discordant if these differences have different signs. The denominator of equation (25) indicates the amount of possible pairs of alternatives.

$$\tau = \frac{N_c - N_d}{\frac{N(N-1)}{2}} \tag{25}$$

The Spearman coefficient is defined in Equation (26), and is represented by the Greek letter ρ (*rho*). In fact, the Spearman coefficient formula is similar to Pearson's. The difference comes from the use of ranks R_X and R_Y, instead of X and Y original values. The Spearman coefficient is obtained from the division between the covariance of R_X and R_Y and the product of their standard deviations (Spearman, 1904).

$$\rho = \frac{cov(R_X, R_Y)}{\sigma_{R_X} \sigma_{R_Y}} \tag{26}$$

The Kendall and Spearman coefficients are among the most commonly used ordinal correlation techniques in statistics (Gibbons & Chakraborti, 2011; Hauke & Kossowski, 2011). These non-parametric methods perform the calculations from the ranks obtained after ordering by the two variables. The correlation results vary in the range [1, −1], in which "−1" depicts the highest negative correlation and "1" the maximum positive correlation. The null value indicates no correlation between the ranks. In Tables 5, 7 and 9 in the next section, the ranks corresponding from the scores obtained by different multicriteria methods are employed to evaluate Kendall and spearman correlation coefficients with the PL ranking.

Similar results would be obtained by employing the nonparametric tests based on these statistics as in Leoneti (2016).

2.6   Data Collection

The data used in the study were from all 38 rounds of the FA Premier League season 2015/2016. Official Premier League performance data are collected and analyzed by Opta (optasports.com), part of Perform Content, a division of Perform Group (performgroup.com).

The live data are collected by a three-person team covering each match. Two highly trained analysts use a video-based collection system to collect information about what happens every time a player touches the ball. All the data collected are then subjected to an exhaustive post-match check to ensure accuracy. After that, the data are available on the FA Premier League web

site (premierleague.com/tables). To perform the proposed analysis, the actions of the players during each match were organized by team.

In the Premier League website, a total of 30 variables divided into an attack, defense, discipline and team play can be found. Specifically, 23 discrete technical performance variables were selected for extraction, based on their reporting in previous literature and the perception of their importance by the authors. The variables collected by Opta that are not used in previous studies and are considered in the present study are Big Chances Created, Hit Woodwork, Clean Sheets are Headed Clearance, which have positive impacts and Errors Leading to Goal and Own Goals, with negative impacts.

Table 2 presents a description of the variables employed in the analyses and a source in the literature asserting their importance. For the six variables introduced due to the perception of the authors, a link to the Premier League data is presented.

The 23 variables in Table 2 were grouped in three sets, as described in Table 3. These groups of variables were explored by several authors, such as Lago-Ballesteros & Lago-Peñas (2010); Liu, Gomez & Lago-Penas (2015); Sgro et al. (2015) and Liu et al. (2016), in order to express a team behavior by distinctive phases during a football match.

2.7   Data Analysis

CPP was applied in two types of analyses. First, the global performance was determined considering all 23 variables together as independent criteria. The Fuzzy MMOORA and the Fuzzy VIKOR, as described in the Section 2, were applied to the same data set to compare performances. A correlation with the official rank of Premier League, season 2015-2016, was a proxy measure to check the adequacy of these methods to the context, as detailed also in Section 2.

The second analysis consisted of using CPP with three dimensions of variables and combining the results thus obtained. The groups of variables (GV) are described in Table 3. Each team received a specific score for each GV. Finally, their GV scores were inputs for another round of composition by CPP. This last stage created a new global ranking. Appendices I to V list the joint probabilities M_{ij} and m_{ij} and the final scores, according to points of view described in the Table 1.

Triangular distributions were used in CPP and TFN in the fuzzy MCDA methods, as depicted in Section 2.2. The triangular distribution is a fair approximation to support decision-making in cases where the lowest, highest, and most likely values are available to describe the behavior of a random variable TFN, on the other side, are the most natural way to fuzzify exact measurements.

Both fuzzy methods were modeled in R language and the computations employed the R package "FuzzyMCDM".

Table 2 – Variables, description and references used in the analyses.

Dummy Variables/Variables			Description	Authors
V1	M	Goals	Act of kicking the ball towards the opponent's goal and making it pass the line bounded by the goal.	(Castellano, 2010; Lago-Ballesteros & Lago-Peñas, 2010)
V2	M	Shots	Trying to score a goal, made with any (legal) part of the body, on or off target.	(Lago-Ballesteros & Lago-Peñas, 2010; Liu et al., 2016)
V3	M	Shots on Target	Trying to score a goal that requires shot on target; may result in a goal or just a shot being diverted.	(Lago-Ballesteros & Lago-Peñas, 2010; Liu et al., 2016)
V4	M	Penalties Scored	Trying to score a goal from the penalty spot.	(Castellano, 2010)
V5	M	Big Chances Created	Clear chances to score a goal.	Link: premierleague.com/clubs/ 'NUMBER'/ 'TEAM'/stats Example: premierleague.com/clubs/ 1/Arsenal/stats
V6	M	Hit Woodwork	The ball has hit the frame of the goal.	Link: premierleague.com/clubs/ 'NUMBER'/ 'TEAM'/stats Example: premierleague.com/clubs/ 1/Arsenal/stats
V7	M	Clean Sheets	If a team doesn't let any goals past or doesn't let the opposition score against them.	Link: premierleague.com/clubs/ 'NUMBER'/ 'TEAM'/stats Example: premierleague.com/clubs/ 1/Arsenal/stats
V8	M	Saves	Number of goalkeeper defenses.	(Liu, Gomez & Lago-Penas, 2015)
V9	M	Tackles	Act of gaining possession from an opposition player who is in possession of the ball.	(Larkin et al., 2016; Liu et al., 2016)
V10	M	Blocked Shots	Act of blocking a shot heading on target towards goal, where there are other defending players or a goalkeeper behind the blocking player.	(Liu et al., 2016)
V11	M	Interceptions	A player intercepts a pass with some movement.	(Larkin et al., 2016; Liu, Gomez & Lago-Penas, 2015)
V12	M	Clearance	Attempt made by a player to get the ball out of the danger zone, when there is pressure (from opponents) on him to clear the ball.	(Liu et al., 2016; Liu, Gomez & Lago-Penas, 2015)
V13	M	Headed Clearance	Attempt made by a player with his head to get the ball out of the danger zone, when there is pressure (from opponents) on him to clear the ball.	Link: premierleague.com/clubs/ 'NUMBER'/ 'TEAM'/stats Example: premierleague.com/clubs/ 1/Arsenal/stats
V14	M	Aerial Battles/Duels Won	Two players competing for a ball in the air, for it to be an aerial duel both players must jump and challenge each other in the air and have both feet off the ground. The player who wins the ball is deemed to have won the duel.	(Liu et al., 2016)
V15	M	Passes	Number of passes to each player from the same team.	(Liu, Gomez & Lago-Penas, 2015; Liu et al., 2016)
V16	M	Cross	Number of crosses performed by the team against the opponent.	(Lago-Ballesteros & Lago-Peñas, 2010; Liu et al., 2016)
V17	m	Goals Conceded	The act of the opponent kicking the ball toward your goal and it passing the line bounded by the goal.	(Lago-Ballesteros & Lago-Peñas, 2010)
V18	m	Errors Leading to Goal	Errors made by the team that cause the ball kicked by the opponent to pass the goal line.	Link: premierleague.com/clubs/ 'NUMBER'/ 'TEAM'/stats Example: premierleague.com/clubs/ 1/Arsenal/stats
V19	m	Own Goals	A goal by a player in his own goal.	Link: premierleague.com/clubs/ 'NUMBER'/ 'TEAM'/stats Example: premierleague.com/clubs/ 1/Arsenal/stats

(to be continue)

Table 2 – (Continuation).

Dummy Variables/Variables			Description	Authors
V20	m	Yellow Cards	Where a player is booked by the referee due to illegal actions.	(Lago-Ballesteros & Lago-Peñas, 2010; Liu, Gomez & Lago-Penas, 2015; Liu et al., 2016)
V21	m	Red Cards	Where a player is booked by the referee due to more severe illegal actions, or after a first yellow card.	(Lago-Ballesteros & Lago-Peñas, 2010; Liu et al., 2016)
V22	m	Fouls	Where a player is fouled by an opponent.	(Lago-Ballesteros & Lago-Peñas, 2010; Liu, Gomez & Lago-Penas, 2015)
V23	m	Offside	When the ball is passed to a player ahead of all outfield players of the opponent team.	(Lago-Ballesteros & Lago-Peñas, 2010)

Adapted by the authors.

Table 3 – Three Groups of Performance Variables.

Groups		Variables
GV1	Goals scored	Goals, Shots, Shots on Target, Goals Conceded, Penalties Scored, Hit Woodwork, Errors Leading to goal, Own goals
GV2	Offensive phase	Passes, Crosses, Offsides, Big Chances Created
GV3	Defensive phase	Saves, Tackles, Blocked Shots, Interceptions, Clearances, Clean Sheets, Headed Clearances, Aerial Battles / Duels Won, Fouls, Yellow Cards, Red Cards

Adapted by the authors.

3   RESULTS AND DISCUSSION

The rankings derived from probabilistic scores are presented in Table 4 for FA Premier League 2015/2016 season. The scores were initially calculated based on the 23 variables taken together and then for the groups of variables.

These global scores represent the probabilities of having the largest value in all variables, corresponding to the pessimistic progressive point of view. The decision maker in the pessimistic progressive (PP) point of view pays attention to maximizing the probabilities of preference according to all the criteria examined. The progressive decision maker pays greater attention to small differences between the best values; in sports, this reflects the idea that the teams are oriented to seek the victory in the game. The conservative decision maker is associated with the idea of avoiding losses, which in football is important for the teams avoiding relegation.

The data base was also submitted to the Fuzzy MULTIMOORA and Fuzzy VIKOR methods. The results are also described in Table 4. The correlation between the Premier League Official ranking of the 2015-2016 season and the three MCDM methods is presented in Table 5.

The present study discusses the perspective of multiple criteria addressing the technical performance of FA Premier League teams. It was expected that the present study would improve the understanding of teams' performance depicted by final 2015/2016 FA Premiere League standings. In Table 5, these standings are compared to rankings derived from the numerical variables assessing the specific performances in the matches.

Table 4 – Ranks of different MCDA methods.

Teams	PL Official Rank	CPP-PP Rank	Fuzzy MMOORA Rank	Fuzzy Vikor Rank
Number of variables	1	23	23	23
Type of variables	Total points	Skills	Skills	Skills
Criteria weight	1	1	1	1
Arsenal	2	2	4	5
Aston Villa	20	20	16	20
Bournemouth	16	10	8	16
Chelsea	10	16	14	17
Crystal Palace	15	15	12	9
Everton	11	6	5	4
Leicester City	1	1	1	1
Liverpool	8	5	6	7
Manchester City	4	4	2	3
Manchester United	5	8	9	10
Newcastle United	18	19	13	14
Norwich City	19	18	17	18
Southampton	6	7	18	8
Stoke City	9	14	11	15
Sunderland	17	11	10	11
Swansea City	12	13	19	12
Tottenham Hotspur	3	3	3	2
Watford	13	12	7	13
West Bromwich Albion	14	17	20	19
West Ham United	7	9	15	6

Created by the authors.

Table 5 – Correlation indices.

	PL Official Rank × CPP Rank	PL Official Rank × Fuzzy MMOORA Rank	PL Official Rank × Fuzzy Vikor Rank
Spearman index	0,854	0,564	0,782
Kendall index	0,695	0,421	0,611

Created by the authors.

The top four teams, qualifying for the Champions League, are the same teams in the same order, whether ranking by points conquered or by CPP. The other methods bring small differences. Fuzzy MMOORA presents a small change in rank and Fuzzy Vikor includes Everton in the fourth position in its ranking.

In regard to relegation, the probabilistic approach is also reliable in reflecting the reality of clubs. The numbers of Newcastle United, Norwich City and Aston Villa were the worst, as reflected in the final league table (relegated to Football League Championship) and when analyzed by the CPP method. The other methods were not able to arrive at such a close adequacy in relation to PL Official Rank.

For the teams in the intermediate group, a comparison is more difficult because these teams form a more homogeneous group, so that small differences in the variables affect the ranks. There is a difference of only 1 point between the teams placed from fifth to tenth and between the 11th and 17th the maximum difference was of only three points. A global comparison may be done using Kendall and Spearman correlation coefficients.

The correlation indices between the Premier League Official and the CPP ranking are higher and reveal the adequacy of this method to the context. It is important to remark that one method cannot here be considered better than another because, despite the graphical resemblance, CPP and Fuzzy MCDA methods embody different concepts. The interpretation of CPP may be easier because uncertainty is directly modeled by CPP. One possible reason for the higher correlation found for the CPP results may be that the use of a density function takes into account in the computations the probabilities of all values while the computation based on TFN takes into account effectively only the central and extreme values. In fact, the CPP algorithm considers all values distributed by a probabilistic law. On the other hand, the fuzzy logic algorithm simplifies computation by prioritizing minimum or maximum values of a fuzzy number, thus excluding information considered in the probabilistic method. In what might seem paradoxical for methods applied to uncertain contexts, precision may have made a difference.

Thus, by the probabilistic method of CPP, a multiple criteria vision is obtained which takes into account variables expected to partially determine match results. This demonstrates that the 23 variables collected by the FA Premier League website are suitable to distinguish winning teams from losing teams.

In a second analysis, CPP was employed to analyze the teams' performances by the three GV described in Table 3. For each group, a PP point of view is represented in Table 6 and a CP point of view is represented in Table 8. A global score is obtained in both tables by combining the GV results by a PO point of view. This approach was proposed by Sant'Anna (2015, p. 40-41) and assumes that a good performance in at least one group of variables may be enough to determine a good global performance. A pessimistic composition of the three GV scores would lead to the same global scores of the PP combination of the 23 variables as a unique group.

To obtain information on which kind of variables are more important, Table 6 presents the correlation between each variable group and the 2015/2016 Premier League Final Rank.

Table 6 shows that the group of variables GV1 presents the best correlation with the final standings. This correlation is even higher than that obtained with the 23 variables as shown in Table 6. This leads to the idea that, in a football match, the most important is to make goals, no matter

Table 6 – PP Rank Composition by GV1, GV2, GV3.

Teams	PL Official Rank	PP GV1	PP GV2	PP GV3	PO Composition	PP Rank
Number of variables	1	23	23	23	23	23
Type of variables	Total points	Skills	Skills	Skills	Skills	Skills
Criteria weight	1	1	1	1	1	1
Arsenal	2	2.19E-10	3.67E-05	5.62E-15	3.67E-05	1
Aston Villa	20	3.94E-15	4.24E-06	9.01E-17	4.24E-06	6
Bournemouth	16	4.55E-13	1.00E-06	6.03E-16	1.00E-06	14
Chelsea	10	4.98E-13	1.57E-06	1.78E-17	1.57E-06	11
Crystal Palace	15	1.10E-13	3.18E-06	4.48E-17	3.18E-06	8
Everton	11	3.73E-12	6.00E-06	5.56E-16	6.00E-06	4
Leicester City	1	7.58E-09	7.32E-06	1.09E-13	7.32E-06	3
Liverpool	8	2.11E-11	2.57E-06	6.09E-16	2.57E-06	10
Manchester City	4	4.81E-10	1.68E-05	1.59E-15	1.68E-05	2
Manchester United	5	6.24E-12	3.47E-06	7.36E-17	3.47E-06	7
Newcastle United	18	8.27E-14	1.16E-06	1.67E-17	1.16E-06	13
Norwich City	19	4.17E-14	4.20E-07	1.11E-16	4.20E-07	18
Southampton	6	2.26E-11	4.32E-06	9.03E-17	4.32E-06	5
Stoke City	9	2.35E-12	3.63E-07	1.89E-17	3.63E-07	19
Sunderland	17	2.71E-13	1.62E-07	2.50E-15	1.62E-07	20
Swansea City	12	1.42E-13	7.88E-07	1.45E-16	7.88E-07	15
Tottenham Hotspur	3	4.45E-09	2.84E-06	1.54E-15	2.85E-06	9
Watford	13	2.97E-13	1.26E-06	9.27E-17	1.26E-06	12
West Bromwich Albion	14	1.12E-13	7.25E-07	4.45E-17	7.25E-07	17
West Ham United	7	7.61E-12	7.64E-07	1.67E-16	7.64E-07	16

Created by the authors.

Table 7 – Correlation Indices to PP Composition Variables.

	PL Official Rank × CPP Rank	PL Official Rank × GV1	PL Official Rank × GV2	PL Official Rank × GV3	PL Official Rank × GV Rank
Spearman index	0,854	0,949	0,558	0,453	0,558
Kendall index	0,695	0,832	0,411	0,337	0,411

Created by the authors.

how much score opportunities can be built or disarmed. What matters is the number of effective actions that contribute to directly score a goal.

In Table 6, the observer may analyze the probability of each team being better than its opponents in all the criteria of each group separately. Here, three of the four clubs that guaranteed access to dispute the Champions League in 2016/2017 appear as the top 3 of the rank.

Table 8 – CP Rank Composition by GV1, GV2, GV3.

Teams	PL Official Rank	CP GV1	CP GV2	CP GV3	PO Composition	CP Rank
Number of variables	1	23	23	23	23	23
Type of variables	Total points	Skills	Skills	Skills	Skills	Skills
Criteria weight	1	1	1	1	1	1
Arsenal	2	7.17E-01	8.86E-01	6.74E-01	9.89E-01	5
Aston Villa	20	2.34E-01	7.51E-01	4.92E-01	9.03E-01	20
Bournemouth	16	6.75E-01	8.12E-01	6.44E-01	9.78E-01	11
Chelsea	10	6.45E-01	8.46E-01	4.70E-01	9.71E-01	14
Crystal Palace	15	6.86E-01	8.38E-01	6.25E-01	9.81E-01	10
Everton	11	7.91E-01	9.09E-01	6.03E-01	9.92E-01	4
Leicester City	1	8.50E-01	8.36E-01	7.61E-01	9.94E-01	2
Liverpool	8	7.65E-01	8.61E-01	5.10E-01	9.84E-01	7
Manchester City	4	8.52E-01	9.19E-01	6.53E-01	9.96E-01	1
Manchester United	5	7.40E-01	8.88E-01	3.52E-01	9.81E-01	9
Newcastle United	18	6.05E-01	8.43E-01	5.60E-01	9.73E-01	13
Norwich City	19	5.51E-01	7.72E-01	3.39E-01	9.32E-01	18
Southampton	6	8.08E-01	8.08E-01	5.61E-01	9.84E-01	8
Stoke City	9	6.22E-01	6.94E-01	5.29E-01	9.45E-01	17
Sunderland	17	7.16E-01	5.79E-01	6.68E-01	9.60E-01	16
Swansea City	12	5.69E-01	8.35E-01	5.47E-01	9.68E-01	15
Tottenham Hotspur	3	8.39E-01	8.92E-01	6.11E-01	9.93E-01	3
Watford	13	7.26E-01	8.09E-01	5.27E-01	9.75E-01	12
West Bromwich Albion	14	4.54E-01	6.68E-01	6.20E-01	9.31E-01	19
West Ham United	7	7.04E-01	8.62E-01	6.88E-01	9.87E-01	6

Created by the authors.

It is interesting to notice that Tottenham Hotspur occupies the ninth position in this rank, which demonstrates the instability of the team throughout its campaign this season. It appears in ninth position in the offensive phase and in the fifth position in the defensive phase, which results in a lower rank in this analysis than those presented in Appendices III and IV. Its superior performance with respect to the variables in the goals scored group, presented in Appendix II – where it can be seen that Tottenham Hotspur occupies a second position with 1.82 goals per match – kept the team disputing the championship until the final rounds, but was not enough to grant it a better final rank under this point of view.

Other teams that call for analysis are Everton, which appears in the PP rank in the fourth position, Southampton, appearing in fifth position and the relegated Aston Villa, in the sixth position. All these three teams presented an interesting offensive organization, with passing accuracy of 78% to 81% and more than 400 passes per match.

Everton showed a median campaign this season, with 11 wins, 14 draws and 13 losses, demonstrated through its PP rank in ninth position in GV1, fourth position in GV2 and eighth position

in GV3. In a closer look at the variables the difference of 0.10 between goals scored per match and goals conceded per match will call attention. The balance between making and conceding goals may be a problem to be more deeply investigated by the managers of this club.

In another case, the problems that Southampton presented in its defensive line are registered by its PP rank in thirteenth position. The team is among the top 5 teams in the PP rank for both the offensive phase and construction of goals. With 18 victories, 9 draws and 11 defeats, Southampton ended the competition in sixth place, 3 points behind the fourth team.

Aston Villa is an extreme case. It had its major problems in defense with an impressive number of 2 goals conceded per match. Its construction of goals was obtained with only 0.71 goals per match, ending the competition with a goal difference of minus 49.

The irregularity of Chelsea's season may also be highlighted here, though its performance in Table 5 with respect to each group of variables is similar. A deeper analysis may reveal a large variability within the variables within the groups, primarily in the defensive phase. The numbers are very close to or worse than those of the relegated teams this season. This poor performance caused the team to change its coach and players during the season in such a way to improve its median campaign.

When the decision maker aims to differentiate alternatives near to worst performance by the idea of avoiding losses and, possibly, relegation, the CP point of view is the more adequate. In this case, none of the correlations presented in Table 8 shows a better performance than that obtained by CPP with the 23 variables. What shows that none of the groups of variables within a conservative view has a strong effect to the 2015/2016 Premier League Final Rank.

In Table 8, attention should be focused on those teams that wanted to avoid relegation. When we combine the conservative pessimistic GV scores, Newcastle United is not found among the possible candidates for relegation. Newcastle United, even with good construction in the offensive phase and being in a ninth position in this group of variables, was not successful in goal conversions, with just 1.16 goals per game and 1.71 goals conceded per match. Fundamentally, in football, goals are required. Of the last 24 available points, Newcastle United only managed to conquer 2 points, which determined the destiny of the team at the end of the season.

West Bromwich Albion was not relegated from the FA Premier League, but, when the variable groups GV1, GV2 and GV3 are analyzed, it appears in the nineteenth position in the CP Rank. This specially demonstrates the fragility of the team during the competition in building their offensive phase, totaling only 10 wins and a goal difference of −14 at the end of competition. This team also demonstrated inefficiency in its defense, with 1.26 goals conceded per match, which exceeds its mark of 0.89 goals scored per match. When analyzed from the CP point of view, West Bromwich Albion could not be better in the variables of GV1 and GV2 groups than other teams presented in Appendices II and III. Thus, even presenting an average performance in GV3, presented in Appendix IV, it appears between the teams that would have been relegated. Thus, its board should be attentive to improving these numbers next season, primarily in their offensive construction and goal conversion.

Combining analyses performed from a CP point of view through an optimistic perspective, the analysis depicted in Table 8 shows the reality of clubs who wish to stay in the elite of English Football, where they want to be better in each of the groups of variables presented. However, long term experience seems to indicate that teams fighting to avoid relegation should care about overall excellence and not only seeking to be better in one group of variables.

Table 9 – Correlation Indices to CP Composition Variables.

	PL Official Rank × CPP Rank	PL Official Rank × GV1	PL Official Rank × GV2	PL Official Rank × GV3	PL Official Rank × GV Rank
Spearman index	0,854	0,788	0,758	0,597	0,355
Kendall index	0,695	0,589	0,579	0,442	0,211

Created by the authors.

4 CONCLUSION

The purpose of this study was to explore football match related statistics in the Premier League for last season (2015/2016). Three different methods, Composition of Probabilistic Preferences (CPP), Fuzzy Multimoora Method and Fuzzy Vikor Method were employed to determine if it is possible to distinguish winning teams from losing teams on the basis of 23 performance indicators. On this point, CPP proved more efficient in predicting the final standings of the championship. The Spearman index for PL Official Rank × CPP Rank is around 0,85 and the Kendall index around 0,69.

The uncertain and imprecise football variables are well modeled by CPP with a triangular density function rather than by TFN. CPP algorithms consider all values of a probabilistic distribution while fuzzy logic, prioritizing minimum or maximum values of a fuzzy number, explores less information.

In addition, CPP analyses based on goals scored, attack phase and defensive phase from different points of view were performed. By this way, the effects of technical performance of the teams at each moment of a football match may be better understood. Thus, information is gathered to help in-depth understanding each phase of the game for different decision maker's points of view.

Evidence was found that the variables in the group GV1, related to the final offensive steps of the team, presents a better correlation with the premier League final rank. This means that the best strategy would be to prioritize being able to transpose the defense to score goals against the opponent.

Several other issues can be raised. For instance: Are the teams that make more goals always the best? Better offensive performance always results in scoring more? How much the defensive performance improves the chances of winning the matches?

As a practical application, our findings suggest that technical match analyses are an important factor in the evaluation of team performance for the following season. Being able to observe the failures in each phase by the team may drive correction of methods of training or even acquisition of players for an improvement of performance.

Future investigations should attempt to apply CPP during the championship and not only at the end of the season. Useful information might also be gathered by looking at player individual rankings. In that case, CPP should be applied not only to technical performance variables but also to physical performance variables. Furthermore, it would also be interesting to apply CPP to different leagues, to determine if different aspects may be more important in different contexts.

ACKNOWLEDGMENTS

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Appendix I – CPP 2nd stage results.

PL 15-16	V1		V2		V3		V4		V5	
	P.max	P.min	P.max	P.min	P.max	P.min	P.max	P.min	P.max	P.min
Arsenal	0.081	0.020	0.060	0.015	0.079	0.016	0.026	0.156	0.264	0.010
Aston Villa	0.012	0.237	0.019	0.171	0.023	0.164	0.029	0.072	0.022	0.177
Bournemouth	0.020	0.043	0.027	0.035	0.029	0.046	0.033	0.038	0.024	0.107
Chelsea	0.043	0.024	0.040	0.020	0.041	0.025	0.039	0.025	0.034	0.029
Crystal Palace	0.016	0.066	0.028	0.033	0.032	0.036	0.039	0.025	0.032	0.034
Everton	0.043	0.024	0.031	0.026	0.042	0.024	0.039	0.025	0.054	0.017
Leicester City	0.123	0.019	0.039	0.021	0.044	0.023	0.291	0.008	0.115	0.012
Liverpool	0.064	0.021	0.142	0.012	0.065	0.017	0.029	0.072	0.044	0.021
Manchester City	0.183	0.017	0.111	0.012	0.078	0.016	0.047	0.019	0.042	0.021
Manchester United	0.024	0.035	0.023	0.058	0.030	0.045	0.033	0.038	0.032	0.034
Newcastle United	0.019	0.046	0.020	0.118	0.032	0.038	0.033	0.038	0.026	0.064
Norwich City	0.016	0.066	0.022	0.073	0.026	0.074	0.026	0.156	0.025	0.078
Southampton	0.043	0.024	0.038	0.021	0.037	0.028	0.033	0.038	0.052	0.018
Stoke City	0.017	0.056	0.022	0.072	0.026	0.077	0.047	0.019	0.036	0.027
Sunderland	0.023	0.037	0.024	0.049	0.029	0.049	0.033	0.038	0.027	0.054
Swansea City	0.018	0.053	0.024	0.048	0.028	0.057	0.047	0.019	0.026	0.064
Tottenham Hotspur	0.141	0.018	0.233	0.011	0.261	0.011	0.047	0.019	0.059	0.016
Watford	0.017	0.061	0.025	0.044	0.029	0.050	0.060	0.015	0.026	0.071
West Bromwich Albion	0.014	0.110	0.020	0.144	0.022	0.184	0.026	0.156	0.024	0.119
West Ham United	0.081	0.020	0.052	0.016	0.048	0.021	0.039	0.025	0.036	0.027

PL 15-16	V7		V8		V9		V10		V11	
	P.max	P.min	P.max	P.min	P.max	P.min	P.max	P.min	P.max	P.min
Arsenal	0.222	0.013	0.214	0.013	0.046	0.023	0.027	0.041	0.067	0.021
Aston Villa	0.019	0.077	0.017	0.134	0.044	0.024	0.031	0.033	0.104	0.018
Bournemouth	0.021	0.058	0.019	0.086	0.026	0.088	0.027	0.041	0.032	0.032
Chelsea	0.016	0.205	0.023	0.043	0.051	0.021	0.052	0.023	0.019	0.111
Crystal Palace	0.030	0.028	0.021	0.058	0.051	0.021	0.019	0.094	0.047	0.024
Everton	0.090	0.016	0.026	0.034	0.060	0.019	0.030	0.034	0.019	0.123
Leicester City	0.027	0.032	0.069	0.017	0.030	0.048	0.196	0.016	0.217	0.016
Liverpool	0.055	0.019	0.030	0.029	0.023	0.183	0.203	0.016	0.021	0.071
Manchester City	0.046	0.020	0.099	0.015	0.025	0.123	0.030	0.034	0.030	0.035
Manchester United	0.023	0.046	0.214	0.013	0.026	0.088	0.038	0.027	0.026	0.045
Newcastle United	0.018	0.106	0.021	0.058	0.043	0.025	0.054	0.022	0.033	0.031
Norwich City	0.027	0.032	0.016	0.204	0.033	0.039	0.016	0.208	0.017	0.216
Southampton	0.034	0.025	0.035	0.024	0.028	0.065	0.027	0.039	0.052	0.023
Stoke City	0.017	0.150	0.026	0.034	0.050	0.021	0.027	0.040	0.024	0.053
Sunderland	0.025	0.038	0.019	0.086	0.265	0.011	0.041	0.026	0.023	0.056
Swansea City	0.039	0.022	0.023	0.043	0.047	0.022	0.019	0.116	0.041	0.026
Tottenham Hotspur	0.121	0.015	0.042	0.021	0.026	0.096	0.063	0.021	0.033	0.031
Watford	0.023	0.046	0.030	0.029	0.050	0.021	0.055	0.022	0.098	0.019
West Bromwich Albion	0.025	0.038	0.030	0.029	0.037	0.031	0.019	0.107	0.058	0.022
West Ham United	0.121	0.015	0.030	0.029	0.039	0.029	0.027	0.040	0.038	0.028

(to be continue)

Appendix I (continuation) – CPP 2nd stage results.

PL 15-16	V12		V13		V20		V21		V6	
	P.max	P.min	P.max	P.min	P.max	P.min	P.max	P.min	P.max	P.min
Arsenal	0.025	0.075	0.026	0.080	0.044	0.030	0.204	0.014	0.030	0.044
Arsenal	0.035	0.033	0.035	0.029	0.099	0.021	0.023	0.041	0.037	0.035
Bournemouth	0.032	0.036	0.046	0.021	0.031	0.043	0.034	0.025	0.029	0.045
Chelsea	0.022	0.139	0.024	0.149	0.045	0.029	0.081	0.017	0.027	0.051
Crystal Palace	0.034	0.034	0.035	0.029	0.046	0.029	0.018	0.080	0.214	0.019
Everton	0.037	0.030	0.038	0.027	0.024	0.075	0.041	0.022	0.034	0.037
Leicester City	0.045	0.025	0.043	0.022	0.215	0.017	0.017	0.102	0.031	0.042
Liverpool	0.026	0.061	0.025	0.111	0.067	0.023	0.085	0.017	0.034	0.038
Manchester City	0.031	0.039	0.044	0.022	0.039	0.032	0.131	0.015	0.062	0.026
Manchester United	0.020	0.199	0.023	0.171	0.022	0.119	0.102	0.016	0.035	0.037
Newcastle United	0.031	0.039	0.032	0.038	0.025	0.068	0.022	0.044	0.033	0.039
Norwich City	0.138	0.016	0.264	0.010	0.024	0.077	0.020	0.056	0.042	0.031
Southampton	0.060	0.021	0.043	0.023	0.045	0.029	0.025	0.034	0.183	0.020
Stoke City	0.023	0.094	0.027	0.064	0.036	0.036	0.028	0.030	0.022	0.077
Sunderland	0.241	0.014	0.061	0.017	0.039	0.033	0.016	0.146	0.017	0.239
Swansea City	0.048	0.023	0.052	0.018	0.019	0.206	0.044	0.021	0.027	0.050
Tottenham Hotspur	0.046	0.024	0.056	0.018	0.028	0.052	0.052	0.019	0.028	0.049
Watford	0.031	0.041	0.025	0.114	0.049	0.027	0.020	0.055	0.024	0.063
West Bromwich Albion	0.037	0.030	0.048	0.020	0.040	0.032	0.015	0.201	0.052	0.028
West Ham United	0.039	0.029	0.054	0.018	0.063	0.024	0.022	0.045	0.039	0.033

PL 15-16	V14		V15		V16		V17		V18	
	P.max	P.min	P.max	P.min	P.max	P.min	P.max	P.min	P.max	P.min
Arsenal	0.067	0.021	0.025	0.075	0.026	0.080	0.021	0.138	0.025	0.050
Arsenal	0.104	0.018	0.035	0.033	0.035	0.029	0.246	0.009	0.235	0.014
Bournemouth	0.032	0.032	0.032	0.036	0.046	0.021	0.091	0.012	0.050	0.022
Chelsea	0.019	0.111	0.022	0.139	0.024	0.149	0.036	0.022	0.042	0.024
Crystal Palace	0.047	0.024	0.034	0.034	0.035	0.029	0.033	0.024	0.042	0.024
Everton	0.019	0.123	0.037	0.030	0.038	0.027	0.039	0.019	0.050	0.022
Leicester City	0.217	0.016	0.045	0.025	0.043	0.022	0.021	0.138	0.018	0.213
Liverpool	0.021	0.071	0.026	0.061	0.025	0.111	0.032	0.026	0.063	0.019
Manchester City	0.030	0.035	0.031	0.039	0.044	0.022	0.024	0.064	0.025	0.050
Manchester United	0.026	0.045	0.020	0.199	0.023	0.171	0.020	0.161	0.019	0.147
Newcastle United	0.033	0.031	0.031	0.039	0.032	0.038	0.075	0.013	0.028	0.040
Norwich City	0.017	0.216	0.138	0.016	0.264	0.010	0.091	0.012	0.042	0.024
Southampton	0.052	0.023	0.060	0.021	0.043	0.023	0.024	0.064	0.031	0.033
Stoke City	0.024	0.053	0.023	0.094	0.027	0.064	0.039	0.019	0.021	0.098
Sunderland	0.023	0.056	0.241	0.014	0.061	0.017	0.059	0.014	0.031	0.033
Swansea City	0.041	0.026	0.048	0.023	0.052	0.018	0.034	0.023	0.031	0.033
Tottenham Hotspur	0.033	0.031	0.046	0.024	0.056	0.018	0.020	0.161	0.028	0.040
Watford	0.098	0.019	0.031	0.041	0.025	0.114	0.032	0.026	0.031	0.033
West Bromwich Albion	0.058	0.022	0.037	0.030	0.048	0.020	0.029	0.030	0.022	0.068
West Ham United	0.038	0.028	0.039	0.029	0.054	0.018	0.033	0.024	0.168	0.015

Observation: Goals Conceded (V17), Errors Leading to Goal (V18), Own Goals (V19), Yellow Cards (V20), Red Cards (V21),

Fouls (V22) and Offside (V23) are variables that have a negative impact, therefore a lower value is better for the team.

Appendix I (continuation) – CPP 2nd stage results.

PL 15-16	V19		V20		V21		V22		V23	
	P.max	P.min	P.max	P.min	P.max	P.min	P.max	P.min	P.max	P.min
Arsenal	0.051	0.014	0.014	0.243	0.047	0.020	0.025	0.048	0.051	0.023
Arsenal	0.103	0.011	0.197	0.019	0.031	0.026	0.046	0.026	0.014	0.229
Bournemouth	0.020	0.156	0.023	0.050	0.018	0.081	0.016	0.197	0.023	0.043
Chelsea	0.034	0.022	0.029	0.036	0.092	0.016	0.062	0.023	0.066	0.021
Crystal Palace	0.103	0.011	0.033	0.033	0.023	0.040	0.065	0.022	0.039	0.026
Everton	0.025	0.044	0.016	0.128	0.092	0.016	0.016	0.218	0.018	0.079
Leicester City	0.020	0.156	0.019	0.078	0.031	0.026	0.026	0.044	0.016	0.119
Liverpool	0.025	0.044	0.036	0.031	0.031	0.026	0.067	0.022	0.071	0.020
Manchester City	0.025	0.044	0.036	0.031	0.015	0.203	0.048	0.025	0.022	0.049
Manchester United	0.034	0.022	0.050	0.026	0.018	0.081	0.208	0.017	0.031	0.031
Newcastle United	0.034	0.022	0.033	0.033	0.092	0.016	0.031	0.035	0.020	0.061
Norwich City	0.034	0.022	0.036	0.031	0.031	0.026	0.026	0.046	0.084	0.020
Southampton	0.020	0.156	0.028	0.038	0.220	0.013	0.028	0.040	0.131	0.018
Stoke City	0.020	0.156	0.021	0.058	0.047	0.020	0.028	0.040	0.204	0.016
Sunderland	0.025	0.044	0.045	0.027	0.023	0.040	0.037	0.030	0.058	0.022
Swansea City	0.255	0.008	0.033	0.033	0.018	0.081	0.027	0.043	0.041	0.025
Tottenham Hotspur	0.051	0.014	0.125	0.021	0.015	0.203	0.057	0.023	0.028	0.034
Watford	0.034	0.022	0.146	0.020	0.031	0.026	0.126	0.019	0.017	0.101
West Bromwich Albion	0.051	0.014	0.050	0.026	0.031	0.026	0.037	0.030	0.025	0.039
West Ham United	0.034	0.022	0.029	0.036	0.092	0.016	0.024	0.052	0.041	0.025

Observation: Goals Conceded (V17), Errors Leading to Goal (V18), Own Goals (V19), Yellow Cards (V20), Red Cards (V21),

Fouls (V22) and Offside (V23) are variables that have a negative impact, therefore a lower value is better for the team.

Appendix II – CPP 3rd stage results GV1.

GV1	PP	PP Rank	PO	PO Rank	CP	CP Rank	CO	CO Rank
Arsenal	2.187E-10	4	5.136E-01	3	7.168E-01	9	0.999999999999741	5
Aston Villa	3.942E-15	20	1.281E-01	20	2.343E-01	20	0.999999780962030	20
Bournemouth	4.550E-13	12	2.853E-01	11	6.749E-01	13	0.999999999985799	15
Chelsea	4.978E-13	11	2.218E-01	12	6.452E-01	14	0.999999999996882	11
Crystal Palace	1.099E-13	17	1.881E-01	17	6.859E-01	12	0.999999999992223	13
Everton	3.727E-12	9	2.881E-01	10	7.913E-01	5	0.999999999999696	7
Leicester City	7.584E-09	1	6.816E-01	1	8.499E-01	2	0.999999999999983	1.5
Liverpool	2.108E-11	6	3.709E-01	6	7.652E-01	6	0.999999999999706	6
Manchester City	4.814E-10	3	4.824E-01	4	8.523E-01	1	0.999999999999981	3
Manchester United	6.244E-12	8	3.874E-01	5	7.401E-01	7	0.999999999997944	9
Newcastle United	8.274E-14	18	1.810E-01	18	6.049E-01	16	0.999999999942001	17
Norwich City	4.170E-14	19	1.626E-01	19	5.506E-01	18	0.999999999766941	18
Southampton	2.264E-11	5	3.673E-01	7	8.077E-01	4	0.999999999999803	4
Stoke City	2.345E-12	10	3.449E-01	8	6.219E-01	15	0.999999999985841	14
Sunderland	2.710E-13	14	2.043E-01	14	7.159E-01	10	0.999999999994008	12
Swansea City	1.416E-13	15	2.008E-01	15	5.686E-01	17	0.999999999983690	16
Tottenham Hotspur	4.452E-09	2	6.761E-01	2	8.395E-01	3	0.999999999999983	1.5
Watford	2.973E-13	13	2.111E-01	13	7.262E-01	8	0.999999999996923	10
West Bromwich Albion	1.121E-13	16	1.993E-01	16	4.545E-01	19	0.999999999416289	19
West Ham United	7.610E-12	7	3.423E-01	9	7.042E-01	11	0.999999999999514	8

Appendix III – CPP 3rd stage results GV2.

GV1	PP	PP Rank	PO	PO Rank	CP	CP Rank	CO	CO Rank
Arsenal	3.666E-05	1	4.446E-01	1	8.861E-01	5	0.999999692650327	3
Aston Villa	4.236E-06	6	2.902E-01	2	7.509E-01	17	0.999996422713184	15
Bournemouth	1.003E-06	14	1.234E-01	15	8.121E-01	13	0.999997195567777	14
Chelsea	1.567E-06	11	1.547E-01	12	8.464E-01	8	0.999998368503557	9
Crystal Palace	3.184E-06	8	2.722E-01	3	8.378E-01	10	0.999997994219116	11
Everton	6.003E-06	4	1.934E-01	7	9.088E-01	2	0.999999748096891	2
Leicester City	7.315E-06	3	2.579E-01	5	8.357E-01	11	0.999999136470623	6
Liverpool	2.567E-06	10	1.716E-01	9	8.609E-01	7	0.999999084912791	7
Manchester City	1.678E-05	2	2.577E-01	6	9.187E-01	1	0.999999820804166	1
Manchester United	3.467E-06	7	1.868E-01	8	8.878E-01	4	0.999999393016960	5
Newcastle United	1.160E-06	13	1.351E-01	13	8.432E-01	9	0.999997859757944	12
Norwich City	4.203E-07	18	1.031E-01	18	7.723E-01	16	0.999988473080744	17
Southampton	4.320E-06	5	2.584E-01	4	8.081E-01	15	0.999998458732760	8
Stoke City	3.627E-07	19	9.842E-02	19	6.935E-01	18	0.999987233705528	18
Sunderland	1.617E-07	20	7.961E-02	20	5.794E-01	20	0.999891385142310	20
Swansea City	7.880E-07	15	1.170E-01	16	8.348E-01	12	0.999997241864082	13
Tottenham Hotspur	2.844E-06	9	1.615E-01	11	8.922E-01	3	0.999999572365489	4
Watford	1.262E-06	12	1.622E-01	10	8.087E-01	14	0.999995829855388	16
West Bromwich Albion	7.255E-07	17	1.241E-01	14	6.676E-01	19	0.999983557029980	19
West Ham United	7.643E-07	16	1.163E-01	17	8.615E-01	6	0.999998353540662	10

Appendix IV – CPP 3rd stage results GV3.

GV1	PP	PP Rank	PO	PO Rank	CP	CP Rank	CO	CO Rank
Arsenal	5.624E-15	2	5.850E-01	3	6.735E-01	3	1.000000000000000	1
Aston Villa	9.014E-17	14	3.777E-01	11	4.917E-01	17	0.999999999999999	13
Bournemouth	6.031E-16	7	4.575E-01	9	6.439E-01	6	1.000000000000000	1
Chelsea	1.784E-17	19	3.086E-01	18	4.705E-01	18	0.999999999999996	18
Crystal Palace	4.481E-17	16	3.194E-01	16	6.254E-01	7	1.000000000000000	1
Everton	5.558E-16	8	4.938E-01	6	6.033E-01	10	1.000000000000000	1
Leicester City	1.088E-13	1	6.616E-01	1	7.614E-01	1	1.000000000000000	1
Liverpool	6.094E-16	6	5.541E-01	4	5.099E-01	16	0.999999999999999	13
Manchester City	1.586E-15	4	4.917E-01	7	6.530E-01	5	1.000000000000000	1
Manchester United	7.361E-17	15	4.308E-01	10	3.521E-01	19	0.999999999999972	20
Newcastle United	1.666E-17	20	2.966E-01	20	5.603E-01	12	0.999999999999999	13
Norwich City	1.107E-16	11	4.990E-01	5	3.392E-01	20	0.999999999999984	19
Southampton	9.027E-17	13	3.416E-01	15	5.615E-01	11	1.000000000000000	1
Stoke City	1.887E-17	18	3.017E-01	19	5.291E-01	14	0.999999999999998	17
Sunderland	2.496E-15	3	5.924E-01	2	6.679E-01	4	1.000000000000000	1
Swansea City	1.453E-16	10	3.615E-01	12	5.467E-01	13	1.000000000000000	1
Tottenham Hotspur	1.544E-15	5	4.903E-01	8	6.113E-01	9	1.000000000000000	1
Watford	9.272E-17	12	3.594E-01	13	5.271E-01	15	0.999999999999999	13
West Bromwich Albion	4.454E-17	17	3.185E-01	17	6.201E-01	8	1.000000000000000	1
West Ham United	1.673E-16	9	3.556E-01	14	6.878E-01	2	1.000000000000000	1

Appendix V – CPP 3rd stage results Composition.

Compositions of Dimensions	PP Goals	PP Offensive	PP Defensive	PP Composition	PP Rank	PO Goals	PO Offensive	PO Defensive	PO Composition	PO Rank
Arsenal	2.19E-10	3.67E-05	5.62E-15	3.67E-05	1	5.14E-01	4.45E-01	5.85E-01	8.88E-01	2
Aston Villa	3.94E-15	4.24E-06	9.01E-17	4.24E-06	6	1.28E-01	2.90E-01	3.78E-01	6.15E-01	13
Bournemouth	4.55E-13	1.00E-06	6.03E-16	1.00E-06	14	2.85E-01	1.23E-01	4.58E-01	6.60E-01	10
Chelsea	4.98E-13	1.57E-06	1.78E-17	1.57E-06	11	2.22E-01	1.55E-01	3.09E-01	5.45E-01	18
Crystal Palace	1.10E-13	3.18E-06	4.48E-17	3.18E-06	8	1.88E-01	2.72E-01	3.19E-01	5.98E-01	14
Everton	3.73E-12	6.00E-06	5.56E-16	6.00E-06	4	2.88E-01	1.93E-01	4.94E-01	7.09E-01	7
Leicester City	7.58E-09	7.32E-06	1.09E-13	7.32E-06	3	6.82E-01	2.58E-01	6.62E-01	9.20E-01	1
Liverpool	2.11E-11	2.57E-06	6.09E-16	2.57E-06	10	3.71E-01	1.72E-01	5.54E-01	7.68E-01	5
Manchester City	4.81E-10	1.68E-05	1.59E-15	1.68E-05	2	4.82E-01	2.58E-01	4.92E-01	8.05E-01	4
Manchester United	6.24E-12	3.47E-06	7.36E-17	3.47E-06	7	3.87E-01	1.87E-01	4.31E-01	7.16E-01	6
Newcastle United	8.27E-14	1.16E-06	1.67E-17	1.16E-06	13	1.81E-01	1.35E-01	2.97E-01	5.02E-01	20
Norwich City	4.17E-14	4.20E-07	1.11E-16	4.20E-07	18	1.63E-01	1.03E-01	4.99E-01	6.24E-01	12
Southampton	2.26E-11	4.32E-06	9.03E-17	4.32E-06	5	3.67E-01	2.58E-01	3.42E-01	6.91E-01	9
Stoke City	2.35E-12	3.63E-07	1.89E-17	3.63E-07	19	3.45E-01	9.84E-02	3.02E-01	5.88E-01	15
Sunderland	2.71E-13	1.62E-07	2.50E-15	1.62E-07	20	2.04E-01	7.96E-02	5.92E-01	7.02E-01	8
Swansea City	1.42E-13	7.88E-07	1.45E-16	7.88E-07	15	2.01E-01	1.17E-01	3.61E-01	5.49E-01	17
Tottenham Hotspur	4.45E-09	2.84E-06	1.54E-15	2.85E-06	9	6.76E-01	1.61E-01	4.90E-01	8.62E-01	3
Watford	2.97E-13	1.26E-06	9.27E-17	1.26E-06	12	2.11E-01	1.62E-01	3.59E-01	5.77E-01	16
West Bromwich Albion	1.12E-13	7.25E-07	4.45E-17	7.25E-07	17	1.99E-01	1.24E-01	3.18E-01	5.22E-01	19
West Ham United	7.61E-12	7.64E-07	1.67E-16	7.64E-07	16	3.42E-01	1.16E-01	3.56E-01	6.25E-01	11

(to be continue)

Appendix V (continuation) – CPP 3rd stage results Composition.

Compositions of Dimensions	PP Goals	PP Offensive	PP Defensive	PO Composition	PP Rank	PO Goals	PO Offensive	PO Defensive	PO Composition	PO Rank
Arsenal	7.17E-01	8.86E-01	6.74E-01	9.89E-01	5	1	0.999999693	1	1.00	1
Aston Villa	2.34E-01	7.51E-01	4.92E-01	9.03E-01	20	0.999999781	0.999996423	1	1.00	1
Bournemouth	6.75E-01	8.12E-01	6.44E-01	9.78E-01	11	1	0.999997198	1	1.00	1
Chelsea	6.45E-01	8.46E-01	4.70E-01	9.71E-01	14	1	0.999998369	1	1.00	1
Crystal Palace	6.86E-01	8.38E-01	6.25E-01	9.81E-01	10	1	0.999997994	1	1.00	1
Everton	7.91E-01	9.09E-01	6.03E-01	9.92E-01	4	1	0.999999748	1	1.00	1
Leicester City	8.50E-01	8.36E-01	7.61E-01	9.94E-01	2	1	0.999999136	1	1.00	1
Liverpool	7.65E-01	8.61E-01	5.10E-01	9.84E-01	7	1	0.999999085	1	1.00	1
Manchester City	8.52E-01	9.19E-01	6.53E-01	9.96E-01	1	1	0.999999821	1	1.00	1
Manchester United	7.40E-01	8.88E-01	3.52E-01	9.81E-01	9	1	0.999999393	1	1.00	1
Newcastle United	6.05E-01	8.43E-01	5.60E-01	9.73E-01	13	1	0.99999786	1	1.00	1
Norwich City	5.51E-01	7.72E-01	3.39E-01	9.32E-01	18	1	0.9999988473	1	1.00	1
Southampton	8.08E-01	8.08E-01	5.61E-01	9.84E-01	8	1	0.9999998459	1	1.00	1
Stoke City	6.22E-01	6.94E-01	5.29E-01	9.45E-01	17	1	0.9999987234	1	1.00	1
Sunderland	7.16E-01	5.79E-01	6.68E-01	9.60E-01	16	1	0.999891385	1	1.00	1
Swansea City	5.69E-01	8.35E-01	5.47E-01	9.68E-01	15	1	0.999997242	1	1.00	1
Tottenham Hotspur	8.39E-01	8.92E-01	6.11E-01	9.93E-01	3	1	0.999999572	1	1.00	1
Watford	7.26E-01	8.09E-01	5.27E-01	9.75E-01	12	1	0.99999583	1	1.00	1
West Bromwich Albion	4.54E-01	6.68E-01	6.20E-01	9.31E-01	19	0.999999999	0.999983557	1	1.00	1
West Ham United	7.04E-01	8.62E-01	6.88E-01	9.87E-01	6	1	0.999998354	1	1.00	1

Appendix VI – R Code.

```
#### PROBABILISTIC COMPOSITION OF PREFERENCES ####
########## PREMIER LEAGUE 2015-2016 ################
### Main Reference
### Sant'Anna, A. P. (2015).
"Probabilistic Composition of Preferences, Theory and Applications." New York: Springer.
### "R" packages used in the modeling
require(readxl)
require(triangle)
### Selecting the working directory where the database is located
path = "COPY AND PASTE THE WORKING DIRECTORY"
setwd(path)
#####################
### CPP 1st STAGE ###
### Randomization of evaluations with triangular distributions
### Triangular PDF parameters: dtriangle(x,"min","max","mode")
### Triangular CDF parameters: ptriangle(x,"min","max","mode")
#####################
### CPP 2nd STAGE ###
### Importing database from positive impact criteria
PL.1 = read_excel("FILE NAME.xlsx", col_names = FALSE, sheet = "SHEET NUMBER")
min.1 = apply(PL.1,2,min)
max.1 = apply(PL.1,2,max)
### Loop for computing the Prob.Max for each alternative/each positive criterion
Prob.Max.p = PL.1
 for (j in 1:ncol(PL.1))
 {
 for (i in 1:nrow (PL.1))
 {
 Prob.Max.p[i,j] = (integrate(Vectorize(function(x)
{prod(ptriangle(x,min.1[j]*0.99,max.1[j]*1.01,PL.1[,j][-i]))
*dtriangle(x,min.1[j]*0.99,max.1[j]*1.01,PL.1[,j][[i]])})),min.1[j]*0.99,max.1[j]*1.01))
$value
}}
### Loop for computing the Prob.Min for each alternative/each positive criterion
Prob.Min.p = PL.1
 for (j in 1:ncol(PL.1))
 {
 for (i in 1:nrow (PL.1))
 {
 Prob.Min.p[i,j]=(integrate(Vectorize(function(x)
{prod(1-ptriangle(x,min.1[j]*0.99,max.1[j]*1.01,PL.1[,j][-i]))
*dtriangle(x,min.1[j]*0.99,max.1[j]*1.01,PL.1[,j][[i]])})),min.1[j]*0.99,max.1[j]*1.01))
$value
}}
### Importing database from negative impact criteria
PL.2 = read_excel("FILE NAME.xlsx", col_names = FALSE, sheet = "SHEET NUMBER")
min.2 = apply(PL.2,2,min)
max.2 = apply(PL.2,2,max)
### Loop for computing the Prob.Min for each alternative/each negative criterion
Prob.Min.n = PL.2
 for (j in 1:ncol(PL.2))
 {
 for (i in 1:nrow (PL.2))
 {
 Prob.Min.n[i,j]=(integrate(Vectorize(function(x)
{prod(1-ptriangle(x,min.2[j]*0.99,max.2[j]*1.01,PL.2[,j][-i]))
*dtriangle(x,min.2[j]*0.99,max.2[j]*1.01,PL.2[,j][[i]])})),min.2[j]*0.99,max.2[j]*1.01))
$value
}}
```

```
### Loop for computing the Prob.Max for each alternative/each negative criterion
Prob.Max.n = PL.2
 for (j in 1:ncol(PL.2))
 {
 for (i in 1:nrow (PL.2))
  {
 Prob.Max.n[i,j]=(integrate(Vectorize(function(x)
{prod(ptriangle(x,min.2[j]*0.99,max.2[j]*1.01,PL.2[,j][-i]))
*dtriangle(x,min.2[j]*0.99,max.2[j]*1.01,PL.2[,j][[i]])}),min.2[j]*0.99,max.2[j]*1.01))
$value
}}
### 2nd stage results for all criteria (Progressive-Conservative axis)
Probs.PROG = cbind(Prob.Max.p,Prob.Min.n)
Probs.CONS = cbind(Prob.Min.p,Prob.Max.n)
####################
### CPP 3rd STAGE ###
### Point of view "PP" ###
PP = apply(Probs.PROG,1,prod)
PP.rank = rank(-PP)
### Point of view "PO" ###
Probs.m = 1-Probs.PROG
PO = 1-(apply(Probs.m,1,prod))
PO.rank = rank(-PO)
### Point of view "CP" ###
Probs.mm = 1-Probs.CONS
CP = apply(Probs.mm,1,prod)
CP.rank = rank(-CP)
### Point of view "CO" ###
CO = 1-(apply(Probs.CONS,1,prod))
CO.rank = rank(-CO)
```