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ARTICLES

PORTFOLIO IMPACT INVESTMENT MANAGEMENT USING MULTI-OBJECTIVE OPTIMIZATION

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ABSTRACT. The main goal of this paper is to propose a methodology for selecting investment portfolios designed to generate financial returns and a socio-environmental impact. This work suggests a seven-step protocol that prioritizes the use of scientific evidence, quality data sources and the interdependence of investment alternatives. In particular, it proposes a multi-objective evolutionary optimization model to estimate the efficient frontier and select the portfolio according to the decision maker's preferences. By an innovative extension of the concept of portfolio diversification, already well understood in traditional portfolio management, this study concludes that it is possible for impact investors to build more efficient portfolios. Due to the scarcity of similar studies in the literature and the rare use of rigorous methods for implementing impact investments, this study, therefore, presents an academic contribution to the field and illustrates the use of the proposed methodology to improve management decisions.

Keywords: portfolio selection, social finance, multi-objective optimization.

1 INTRODUCTION

This article discusses and analyze the conceptual issues related to the management of portfolios formed by impact investments. The analysis's focus is more on the joint result of the portfolio's assets rather than the management of a singular investment itself. To this end, it will propose and analyze a methodology for building portfolios.

Impact investments seek to meet the desires of individuals, corporations and governments that wish to contribute to society's welfare through their allocations of financial resources. However, managing impact investment portfolios is challenging and involves an attempt to answer the following questions: i) How to estimate the expected socio-environmental impact of a given investment?; ii) How to measure the risk that this impact will not occur as expected?; iii) How

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to choose among the different types and possibilities of measuring socio-environmental impacts?; iv) How to define the extent to which financial returns are sacrificed to achieve socio-environmental impacts?; and v) How to form a portfolio that maximizes the expected utility of an investor?

Several theoretical frameworks will be explored in this article to elucidate the possible answers to each of the questions raised above. In this sense, in relation to the first two questions, the techniques for measuring the social program's impact, discussed in Gertler, Martinez, Premand, Rawlings and Vermeersch (2017) and Glennerster and Takavarasha (2013), will be presented. Due to the impossibility of generalizing the estimates (Banerjee et al., 2017; Athey & Imbens, 2017), it will be introduced a discussion of how Bayesian statistics can be useful in transporting the results of impact studies to the context of the investments under analysis.

Regarding the last-mentioned questions, the main problem to be faced is to elucidate the preferences of the investor, here understood in a broad sense to be an individual or group which makes the final decisions regarding the portfolio, which may be a traditional investor, a government official, or an organized group of society in general. In this article, the multi-objective evolutionary optimization is the theoretical framework chosen to address the issues of this section. In particular, a parallel will be drawn between the traditional optimization of mean-variance portfolios proposed by Markowitz (1952) and the optimization of portfolios with multiple objectives.

Therefore, the contribution of this work is that it tries to fill a gap, both from a theoretical and empirical point of view, in terms of the use of scientific evidence in the design and management of impact investment portfolios. In addition, the present work innovates by structuring a single protocol that incorporates previous impact estimates to inform decision making, taking into account the expectations of return and uncertainties related to the set of allocation alternatives. It is also worth mentioning that this type of theoretical and practical approach contributes to the diversification of publications on impact investments, since the bibliometric review carried out by Agrawal and Hockerts (2019) points out that most of the literature in this field of research is still concentrated in terms of definitions and terminology.

Due to the multiple goals of these types of investments, framing the problem as a multi-objective optimization problem can be a useful contribution for industry practitioners and academic researchers interested in this subject. Eden and Ackermann (2018) stated that operational research has had less impact on the strategic decision-making of senior management teams, since much of the research carried out in this field is not intended to impact management practice. Thus, the research contributes to operational research itself, since this work deals with a genuine problem of interest to managers.

This study is organized as follows: 1) a description of impact investments; 2) exposure to impact measurement techniques; 3) a presentation of the generalization problem and the use of Bayesian statistics as a workaround; 4) a presentation of the multi-objective optimization techniques and the theory of choices; 5) the application of multi-objective optimization techniques for portfolio selection; 6) the definition of the methodology for the selection of impact portfolios and a description of a test example of the asset selection protocol; 7) the presentation of numerical results for the suggested methodology in forming impact investment portfolios; and 8) final considerations of the article with a summary of the conclusions and suggestions for future work.

2 LITERATURE REVIEW

2.1 Impact investments

According to the definition of Wood, Thornley and Grace (2013), impact investments are investments made with the intention of creating measurable social and environmental benefits in addition to financial returns. According to the Global Impact Investing Network (GIIN, 2019), impact institutions must adhere to four practices, namely:

1. Explaining their intention to generate a positive social and environmental impact on their investments in addition to financial returns: the institution must precisely define its financial and social objectives, and map the entire chain of events necessary between the policy employed and the desired final results;

2. Using evidence and data in choosing and designing their investments: the institution's actions must be based on empirical evidence and well-established scientific knowledge; and yet, the possible harm resulting from their interventions must also be well understood and made explicit;

3. Managing the results of their investments: institutions must monitor the intended social results in order to inform their decision-making process and mitigate unexpected negative effects of their investments; and yet, the effective performance of the program must be easily available to the community; and

4. Contributing to the growth of the sector: institutions must adhere to procedures, conventions and quality standards in order to monitor and evaluate their investments. In addition, they must share data and results, positive or negative, with other interested institutions.

It is worth noting that although the survey conducted by Agrawal and Hockerts (2019) suggests that there is a consensus among impact investment publications to define these instruments according to their social and commercial performance, the authors still point out that there are many ambiguous definitions and terminologies. Grim and Berkowitz (2018) confirm this understanding and argue that there is a myriad of terminologies related to the impact investment sector, sometimes used interchangeably, without these terms referring to the same things. Thus, the common terms related to the sector are: Environmental, Social and Governance Integration (ESG), Socially Responsible Investment and Impact Investments.

ESG integration is an additional practice in financial analysis which seeks to identify characteristics that promote some material impact on investment performance. The occurrence of negative characteristics does not necessarily rule out investments (Grim & Berkowitz, 2018).

On the other hand, adherents of socially responsible investment have the habit of screening existing assets to exclude or decrease participation in investments with certain characteristics considered immoral or inappropriate, such as businesses involving arms, tobacco, alcoholic beverages, pornography, pesticides, child labor, damage to the environment or corrupt practices. In addition, there is filtering in the opposite direction, favoring investments with positive characteristics for society, such as clean energy generating companies, recycling plants, etc.

Impact investments are a subset of the previous terms, as its investors aim to achieve, in a deliberate way, positive socio-environmental returns with their assets, in addition to financial returns. Thus, investments such as Green Bonds and Social Impact Bonds are among the many vehicles used by the sector when relating financial gains to meeting pre-established goals in the social or environmental areas.

Mudaliar and Dithrich's study (2019) estimates that 1,340 institutions manage US $ 502 billion in impact investments around the world. Of the total financial managed, 51% are managed by fund managers and 27% are managed by development banks, which includes regional and national development banks, multilateral institutions and international financial institutions. In addition to these actors, banks, pension funds, insurers, foundations, family offices, private foundations, individual investors, non-governmental organizations (NGOs) and religious institutions also participate.

In a broader concept of investments with social concerns - including ESG practices and socially responsible investments - Aitken et al. (2018) report that in 2018 just in the United States, of the US $ 46.6 trillion of investments managed by professionals, about US $ 12 trillion were linked to this category of investments. In addition, the growth of this market during the period of 1995-2018 was 1,778%, or an average of 13.6% growth per year.

Also, according to Mudaliar and Dithrich (2019), most impact institutions are located in developed markets, including the US and Canada (58%) and Western, Northern and Southern Europe (21%). It's worth noting that the number of institutions in Latin America and the Caribbean corresponds to only 4% of the impact institutions in the world.

Regarding the financial performance of the sector and considering only the stock market, potential gains or losses related to an investment strategy in socially responsible companies is a controversial topic. As Fabozzi, Ma and Oliphant (2008) argue, it is possible to believe that companies with "sinful products" are overvalued or undervalued in the market. Factors that contribute to the identified loss of profitability are due to the costs of conforming to a social standard (such as recalling defective products, controlling pollutants, environmental repair programs, etc.), in addition to pure rejection of the part of investors, which increases a company's funding costs. On the other hand, barriers to entry resulting from the rigidity of the dictates related to these businesses create a monopoly for established companies that increases their profit margins.

According to the empirical tests carried out by Fabozzi, Ma and Oliphant (2008), there is a loss resulting from socially responsible investment. A study by Adler and Kritzman (2008) also corroborates this hypothesis by estimating an average loss of 2.4% of return per year due to the restriction of investment in companies with negative evaluations in terms of social or environmental factors. To the authors, investors could in an equivalent way, instead of avoiding investments in non-socially responsible companies, redirect the gains resulting from these investments to alleviate the damages caused by these companies. However, Humphrey and Tan (2014) found no evidence that ethical investments are positive or negative for the generation or destruction of the value of a stock portfolio.

Although the tools explored in this study apply to different types of institutions that invest their resources in order to generate economic and social impact through their actions, it is emphasized that the state is the most relevant actor to occupy the role in question. In this sense, Wood et al. (2013) points out that the government generally plays a fundamental role as a subscriber, co-investor, regulator, and purchaser of goods and services or supplier of subsidies and technical assistance for the promotion of the impact investment sector. As an example, the UK Government formally supports the contribution of impact investments to solving its social problems and is the founder of institutions created for the promotion and development of Social Impact Bonds (SIBs), such as the Center for Social Impact Bonds and the Government Outcomes Lab (HM Treasury UK, 2018). Despite the importance of the state's role, Roundy (2019) also points out a number of aspects present in impact investment ecosystems that help the sector flourish in certain regions, such as the characteristics of its investors, the presence of support organizations and the region's cultural values.

At the same time, as an alternative to the debate on differing worldviews that arise from various aspects of economic development theories, there is a growing trend towards the use of evidence to guide state action, which, for some authors, represents a true revolution. in the economy and in the design of public policies (Angrist & Pischke, 2010; Gueron, 2017).

2.2 Impact evaluation methodologies

The search for a correct estimate of the impact of an action is a fundamental point in impact investment frameworks. According to Gertler et al. (2017), the estimate of the average effect of changes in the welfare of individuals resulting from a project or public policy is the objective of impact measurement techniques. To this end, the establishment of a causal relationship between a policy and a change in welfare is done by isolating the change generated by the policy from all other effects that occur concurrently in the lives of individuals.

According to Deaton and Cartwright (2018), an interesting way to understand the impact of a treatment estimation - that is, the effect of a public policy or intervention - is to use a linear causal model, as expressed in the following equation (1):

$$Y_i = \beta_i T_i + \sum_{j=1}^{J} \gamma_j x_{ij} \tag{1}$$

where:

- i is the observation unit. It is dependent on the context of the problem and can be represented by an individual, a company, a city, etc.

- T_i = is a binary variable, which indicates whether the unit received (i.e., $T_i = 1$) or did not receive (i.e., $T_i = 0$) a particular treatment. The treatment corresponds to a well-defined action, such as "committing to a 5-year term financing at a rate of 1% interest per month";

- Y_i is a variable that measures the outcome of a policy, for example, "a company's profitability after two years of financing";

- x_{ij} are observable and unobservable variables, which also affect the dependent variable Y_i. Examples of observable variables: a "company's sector of activity", its "investment rate" or its "level of indebtedness". Examples of unobservable or latent variables: the "organizational culture of a company" or the "managerial ability of its decision makers";

- γ_j is the impact of the variation of one unit of the variable x_{ij} on the dependent variable; and

- β_i is the treatment effect on individual i.

Using the terminology of Rubin's Causal Model (Rubin, 2005), for each unit i, it is possible to conceive two potential states: 1) $Y_i\,(1)$, related to the individual's potential result with the treatment effect; and, 2) $Y_i(0)$, related to the potential result of the individual without the treatment effect. Obviously, the treatment effect for each individual β_i could be obtained by the difference $Y_i\,(1) - Y_i\,(0)$, however, at every moment in time, it is only possible to know the value of $Y_i\,(1)$ or $Y_i(0)$. The impossibility of knowing what is the difference between an individual's potential outcome with the treatment effect, $Y_i\,(1)$,, and that individual's potential outcome without the treatment effect, $Y_i(0)$, is known as the "fundamental problem of causal inference" (Holland, 1986, p. 947).

If a valid comparison group (also called a counterfactual group) is found, the differences between the treated and untreated units is equal to the average effect of the treatment, since there is a cancellation of the effects of the observed and unobserved variables in a situation of perfect balance between groups. Therefore, the Average Treatment Effect, $\overline{\beta}$, is given by Equation (2), which is as follows:

$$\overline{Y}\,(1) - \overline{Y}\,(0) = \overline{\beta} + \sum \gamma_j (\overline{x}_{ij}\,(1) - \overline{x}_{ij}(0)) = \overline{\beta} \tag{2}$$

Several methodologies are possible for obtaining the estimate given by equation (2). These methodologies are divided into experimental studies, such as the randomized controlled trial (RCT) method, systematized in Fisher (1925), and observational (or quasi-experimental) studies, such as the before-and-after method, pairing method, matching method, difference-in-differences, multiple regression and discontinuous regression.

According to Athey and Imbens (2017), experimental studies are usually considered superior to observational studies, since the researcher's greater control over treated and untreated individuals practically eliminates the problem of selection bias. Details of the functioning of the methods in question can be found in Glennerster and Takavarasha (2013) and Gertler et al. (2017).

According to Gueron (2017), the countless studies that have been conducted based on randomized controlled experiments to evaluate social programs since the 1970s have been essential in transforming the United States. In addition, as reported by Cameron, Mishra and Brown (2016), there has been a substantial growth in impact assessment publications in the area of economic development in recent decades.

2.3 The generalization problem and Bayesian inference

Several scholars of impact assessment methodologies argue that it is not possible to generalize the results to a new context, a problem which is also known as the transportability of results (Banerjee et al., 2017; Athey & Imbens, 2017). In addition, several factors constitute a source of uncertainty in assessing the validity of estimated policy impacts.

The article by Banerjee et al. (2017) points out several challenges for the generalization of results obtained from small-scale experiments, generally implemented as proofs of concept, for forecasting results in large-scale programs. In particular, the authors highlight the dependence on the context of the estimates, since pilot programs generally take place in a given location, are managed by specific organizations, and surrounded by a series of local characteristics and temporal situations that may not be repeated in the replication of the program in other locations.

As pointed out by Banerjee et al. (2017), one way to consolidate the conclusion of the collected results is to carry out meta-analyzes. An example of this methodology applied to Impact Investments is the study on the impact of microcredit carried out by Meager (2019), who applied the Bayesian Hierarchical Model popularized by Rubin (1981) to consolidate the results of six other studies, assuming that the effect of the policy, estimated in each location, came from a normal distribution.

From the decision maker's point of view, the use of the results of impact assessments to subsidize their actions has also been the subject of wide debate. In this sense, some proposals have emerged to deal with the uncertainty of the expected effect of a policy when it is replicated in a new context. Some of these approaches include the Bayesian Hierarchical Model (Rubin, 1981), the use of Causal Diagrams in the Data Fusion Theory (Bareinboim & Pearl, 2016), the mapping of the Theory of Change for the analysis of the similarity of the chain of events between the proposed treatment and the final observed effect (Gertler et al., 2017; White, 2009), techniques that mix different types of information sources, including conclusions from experimental, observational and theoretical studies (Deaton & Cartwright, 2018), or even a suggestion favorable to the subjective freedom of the decision maker (Banerjee, Chassang, et al., 2017).

Behind these suggested approaches lies an epistemological difference regarding the validity of the inferential methods used, given in terms of a frequentist or Bayesian view of the researcher. In the Bayesian Hierarchical Model and the Data Fusion Theory, the extrapolation of knowledge is made explicitly dependent on Bayesian statistics. On the other hand, Banerjee et al. (2017)

do not endorse any valid inferential method as possible to transport the impact results to another context based on previous experiments, although it uses Bayesian reasoning to infer the optimal design of a decision maker's experiment.

As Poirier (1995) points out, frequenters argue that situations that do not allow repetition under essentially identical conditions are not within the scope of statistical inference. Thus, according to the frequentist view, the inference cannot be applied: i) to unique and exclusive phenomena, ii) to theories (for example, to the Monetarist Theory or to the Keynesian Theory, or iii) to past situations that have occurred. On the other hand, such inferences are possible according to the Bayesian view of statistics.

According to Koop (2003), the frequentist view is the dominant view in science. This is due to two main reasons: the controversy over the use of prior knowledge in the development of inferential models and the computational difficulties of Bayesian statistics. Regarding the first point, the author argues that there is a strong objection from many researchers to the use of subjective probabilities and that they threaten the objectivity sought by science. In terms of the second point, the author argues that Bayesian statistics are computationally difficult for many classes of problems, which greatly hinders their use by researchers.

In defense of Bayesian statistics, Koop (2003) argues that the model construction process involves a series of pieces of information which are not contained in the data (for example, which model to use, which criteria to use to compare the models, which results should be reported, etc.). In this sense, the Bayesian approach is very precise and transparent about how this information is used in the construction of models. In addition, it is possible to use prior non-informative knowledge in Bayesian models that imply results and conclusions identical to those obtained by the inferential models of frequentist statistics. Regarding the computational difficulty, the author argues that the evolution of computers and the development of new algorithms have made the use of Bayesian statistics much easier.

Other advantages pointed out by the author to the use of Bayesian statistics are the convergence properties of the data (which makes the prior information irrelevant with the addition of lots of new information), the possibility of using and reconciling multiple models, the possibility of updating knowledge based on objective mathematical laws, and the ability to carry out sensitivity analyses of the worldviews explained by researchers.

According to DeGroot (1988) there is a great need to quantify uncertainty in the development of effective decision-making methods. In the author's view, Bayesian statistics is the only one capable of providing a coherent system for decision making in an uncertain environment. In this sense, the author argues in favor of the Bayesian approach to quantify uncertainty and describes methodologies that use this tool to compare and consolidate expert opinions, defined as mathematical models or scholarly opinions on a given topic.

More formally, we have the fact that the Bayesian statistic is based on the Bayes Rule, defined by equation (3), where A and B are random variables:

$$p(B|A) = \frac{p(A|B)\,p(B)}{p(A)} \tag{3}$$

Thus, if we substitute in (3) A for $\boldsymbol{X}$, which defines the set of observed data, and B for $\boldsymbol{\theta}$, which defines the model parameters that explain the occurrence of $\boldsymbol{X}$, we will arrive at the equation:

$$p(\boldsymbol{\theta}|\boldsymbol{X}) = \frac{p(\boldsymbol{X}|\boldsymbol{\theta})\,p(\boldsymbol{\theta})}{p(\boldsymbol{X})} \tag{4}$$

This mathematical relationship allows us to answer how to infer what can be known about $\boldsymbol{\theta}$, which is unknown, which is conditioned on what is already known based on the observed data, explained on the right side of the equation (4).

According to Higgins and Sally (2008), in the field of medicine it is very common to use Bayesian statistics to perform meta-analyses to consolidate the results of several studies that analyze the effects produced by a given treatment. In the field of social sciences, Rubin's (1981) article presents an application of Bayes's equations in a meta-analysis technique for consolidating estimates from parallel studies of the same phenomenon. In this article, the author assumes that each of the estimates μ_i inferred from a number i of RCTs, originates from a normal distribution with mean μ_* and variance V_*. Thus, the author deduces from the Bayesian statistical equations that each μ_i is independently distributed with a common mean μ_* and variance $V_i + V_*$, which means that μ_* is centered close to each $\widehat{\mu}_i$ (the estimated treatment effect based on the data in the *ith* experiment) and V_* reflects an extra variability of each $\widehat{\mu}_i$ in addition to the sample variability V_i. By setting V_* to 0, the author presents the following equations for consolidating the μ effect of n experiments:

$$\mu = \frac{1}{\sum_{i=1}^{n} \frac{1}{V_i}} \sum_{i=1}^{n} \frac{\mu_i}{V_i} \tag{5}$$

and with variance V given by:

$$V = \left(\sum_{i=1}^{n} \frac{1}{V_i} \right)^{-1} \tag{6}$$

In short, Bayesian statistics allows the decision maker to deal explicitly with that individual's hypotheses and worldview, by expressing uncertainties in the format of a probability distribution. Thus, in possession of such distributions, the decision maker will be able to use them to feed even more complex computational decision support systems.

2.4 Multi-objective optimization

According to Cohon (1978), multi-objective optimization is an excellent tool for planning public policies, since the public decision-making process is concerned with a large number of economic and social interests, such as decisions with impacts on economic efficiency, social equity and the quality of the environment. A general problem of multi-objective optimization with n decision variables, m restrictions and p objectives is given by the following equation:

$$\begin{array}{lrl} \textit{maximize} & \boldsymbol{Z}(x_1,x_2,\ldots,x_n) = [Z_1,Z_2,\ldots,Z_p] & \\ \textit{subject to} & g_i(x_1,x_2,\ldots,x_n) \leq 0, & i=1,2,\ldots,m \\ & x_j \geq 0, & j=1,2,\ldots,n \end{array} \tag{7}$$

where $\boldsymbol{Z}(x_1,x_2,\ldots,x_n)$ is the multi-objective function and $Z_1,Z_2,\ldots,Z_p$ are the p individual objective functions. Cohon (1978) stresses that the individual objective functions are merely listed in (8), they are not added, multiplied or combined in any way.

Obviously, as pointed out by Cohon (1978), a first difficulty that arises is the definition of objectives and their measurement. For example, measuring the aesthetic value of the environment is extremely difficult. Even if a metric can be invented, there is no guarantee that it will be meaningful to a set of decision makers. In relation to this, the author exemplifies that one person can see a river as an aesthetically valuable scene, while another can value the trees that are on its banks.

Cohon (1978) lists two initial approaches to the solution of (7): a) the modeling of decision-makers' preferences using a utility function, making the $\boldsymbol{Z}$ vector a scalar in practice; or b) the construction of an efficiency frontier, composed of non-inferior solutions among the objectives, whose objective is to provide the necessary inputs for managers to decide at a future time their preferences in terms of the existing trade-offs in the search for each of the objectives.

According to the author, there are advantages and disadvantages related to each of the approaches. Regarding the construction of the utility function, the author mentions that the elucidation of the decision makers' preferences for the construction of a mathematical function is a very complex activity for the analyst and one that demands time from decision makers. In addition, the author mentions that many decision makers feel uncomfortable in voicing some types of preferences, such as, for example, when they are urged to quantify the financial value of a human life to gauge the trade-off between the financial cost of a highway and the safety of its users. On the other hand, the construction of a utility function for ordering preferences greatly simplifies the optimization problem, as it transforms an objective vector function into a scalar one.

Cohon's (1978) preference is that the modeling of public management problems should be done via multi-objective optimization, since such an approach frees the analyst from all the difficulties involved in modeling the preferences that would be assigned to decision makers in the political choice between the efficiency frontier alternatives presented by the analyst.

Considering two possible objective vectors $\boldsymbol{Z}^A$ and $\boldsymbol{Z}^B$, as defined in Collete and Siarry (2003), it is said that vector $\boldsymbol{Z}^A$ dominates vector $\boldsymbol{Z}^B$ if: 1) $\boldsymbol{Z}^A$ is at least as good as $\boldsymbol{Z}^B$ for all objectives, and 2) $\boldsymbol{Z}^A$ is strictly better than $\boldsymbol{Z}^B$ for at least one objective. Thus, the set of non-dominated vectors forms the efficient frontier (also called Pareto's frontier) of the multi-objective problem.

Ordering the vectors itself plays an important computational burden in the algorithms. The non-domination of each vector can be checked with a double nested loop, but it has a computational complexity of order $O\left(n^2\right)$ (Daskalakis et al., 2011; Emmerich & Deutz, 2018). Nevertheless, the algorithm presented in Kung et al. (1975) can find the non-dominated set with complexity $O(n(\log n)^{d-2})$, when the vector $\boldsymbol{Z}$ has a dimension $d \geq 3$.

For the construction of the efficient frontier, several methods are proposed by Cohon (1978), of which the method of weights and the method of restrictions stand out in terms of simplicity.

The weights method assigns arbitrary weights to the objectives, for example, for a function with two objectives Z_1 and Z_2, the boundary is calculated by assigning weights (w_1, w_2) to the function $w_1 Z_1 + w_2 Z_2$, for example, making (w_1, w_2) equal to $(1,0), (0,1), (1,1), (1,2), (1,3)$ and $(1,4)$. As the author sets forth, the absolute values of the weights are irrelevant for the definition of the frontier, because what matters are their relative values. Marler and Arora (2010) highlight that the use of rescheduling techniques aiming at the comparability of objectives is sometimes necessary to allow the use of the weight method for multi-objective optimization.

The constraint method works by maintaining only one objective in the objective function and repositioning the other objectives as arbitrary restrictions of the operational research problem. The efficient frontier is then determined by varying the values of the constraints for each of the objectives. The problem with this approach, as Cohon (1978) argues, is that the method involves solving numerous problems whose solutions are not viable.

The weight method and the constraint method work with traditional scalar optimization algorithms. There are, however, some adaptations to these algorithms to deal directly with multi-objective problems. As the survey of Fukuda and Graña Drummond (2014) points out the steepest descent (Graña Drummond & Svaiter, 2005), the projected gradient (Fukuda & Graña Drummond, 2011) and the Newton (Fliege, Graña Drummond, & Svaiter, 2009) descent vector-valued optimization methods do not have the disadvantage of converging to unbounded solutions for an arbitrary number of initial parameters.

Another approach to solving multi-objective problems is by using heuristic search algorithms, many inspired by biological processes, such as the algorithms: particle swarm optimization (Reyes-Sierra & Coello Coello, 2006), ant colony optimization (Lopez-Ibanez & Stutzle, 2012), and evolutionary algorithms (Deb, 2001). As an example of the last-mentioned category, genetic algorithms are extensively used by researchers and practioners to solve multi-objetive problems since they generate a population of solutions, allowing an approximation of the entire Pareto Frontier in a single run of the algorithm instead of seeking a single solution at a time (Goh & Tan, 2009).

Genetic algorithms are based on the idea of a population of solutions (decision vectors) that evolve in terms of their fitness, according to defined rules and operators, such as reproduction, crossover, elitism and mutation. As in the scalar version of the algorithm, multi-objective genetic algorithms tend to perform relatively well on non-convex functions and not getting trapped in local optima (Deb, 2001). However, in the multi-objective genetic algorithm, the fitness not only comprises the performance in terms of the objective function, but it also tends to value population diversity in order to cover all the objective space, as proposed in the NSGA-II algorithm (Deb, Pratap, Agarwal, & Meyarivan, 2002).

As Emmerich and Deutz (2018) pointed out, in some cases, reducing the decision space to the Pareto Frontier is not enough for the subsequent decision-making, since too many possible solutions can coexist. In this situation, the authors recommended some preference elicitation methods, such as those discussed in Multiple Criteria Decision Analysis (MCDA). An empirical comparison of the preference ordering results of the main MCDA algorithms and their caveats can be found in Leoneti (2016) and Aires e Ferreira (2018).

The pseudo-weights algorithm presented in Deb (2001) illustrates a possible type of post-optimality analysis. By this algorithm, decisions can be ranked by multiplying the vector of weights for the objectives, which is chosen by the decision maker, by the relative importance of each objective in a decision vector $\boldsymbol{Z} = [z_1, \ldots, z_p]$, given by the formula:

$$w_i = \frac{z_i^{max} - z_i}{z_i^{max} - z_i^{min}} \Big/ \sum_{j=1}^{p} \frac{z_j^{max} - z_j}{z_j^{max} - z_j^{min}} \tag{8}$$

2.5 Forming portfolios with multiple objectives

Traditional portfolio management originates from the work of Markowitz (1952), in which the author proposed a methodology for building portfolios based on the following optimization:

$$\begin{array}{ll} \textit{minimizing} & \boldsymbol{p}'\boldsymbol{\Sigma}\boldsymbol{p} \\ \textit{subject to} & \boldsymbol{p}'\boldsymbol{r} \geq H \\ & \boldsymbol{p}'\mathbf{1} = 1 \end{array} \tag{9}$$

where $\boldsymbol{p}$ is a vector with the proportions invested in each asset, $\boldsymbol{r}$ is the vector of expected returns on each asset, H is the desired portfolio return target and $\boldsymbol{\Sigma}$ is the covariance matrix of the returns of the assets.

Thus, in the Mean-Variance model proposed by Markowitz, there are already two objectives implicit in investor's preference, namely: gaining financial returns and minimizing risk. Therefore, it is a multi-objective problem and can be addressed with traditional techniques concerning this methodology, that is, transforming the multi-objective problem into a single objective with an *a priori* definition of preferences (for example, setting an ideal return k, assuming a utility function with a degree of aversion to fixed risk, etc.); or calculating the entire Pareto frontier so that the decision maker can choose his preferred investment point at a future moment. According to Engau (2009), it is also possible to use interactive methods to solve the problems of portfolios with multiple objectives. Through this approach, the decision maker is constantly presented with subproblems, which would facilitate the articulation of his preferences in the convergence of the algorithm in the search for the optimal solution.

To Fliege and Werner (2014) and Kolm, Tütüncü and Fabozzi (2014), estimation risk represents a popular and much researched topic, especially in the context of portfolio optimization, as the return estimates and covariance used in the Mean-Variance optimization model are very sensitive to changes in the input data. To deal with this problem, the authors list several Bayesian methods to obtain a more efficient estimator. Another approach suggested by Fliege and Werner (2014) is the use of robust optimization to deal with the uncertainty of estimates. To Kolm et al. (2014), the comprehensive view provided for portfolio management made possible by the Bayesian structure is very valuable in practice, as it allows forecasting systems to use external information sources and subjective interventions (i.e., modification of the model due to judgment), in addition to traditional information sources. The portfolio management model of Black and Litterman (1991), for example, makes use of Bayesian statistics to incorporate expert opinions about the expected return of one or more assets in the portfolio and also makes it possible to assign different degrees of confidence to each one of them.

Saltuk and Idrissi (2012) propose that the inclusion of socio-environmental objectives can be achieved by adding more dimensions to the portfolio selection model in order to build efficient borders of multiple dimensions. In the specific case of their study, the authors work with an efficient three-dimensional frontier, because in addition to the traditional variables involving financial return and investment risk, the authors suggest incorporating into the model a generic variable called "impact" to reflect the expected socio-environmental return of each investment alternative.

According to Agrawal and Hockerts (2019), the discussion of the risks and rewards of impact investments can be made using the portfolio theory, however the lack of longitudinal data (when data is collected sequentially from the same units over a period of time) and a sufficient population of impact investments make it difficult to explore the performance of this sector.

Although the parallel between traditional portfolio theory and impact portfolio management is possible, it is convenient to point out some differences between these types of investments. For this purpose, Table 1 presents the various characteristics of these types of investments.

Among the differences pointed out in Table 1, the difficulty of quantifying the socio-environmental objectives in the impact investment portfolios stands out, both in terms of expectations, and in the measurement of their effects when a certain period after an intervention has elapsed.

In relation to the measurement units, financial returns are calculated directly from the portfolio's final value in relation to its cost. Regarding the socio-environmental return, it is common to measure gains in terms of standard deviations in relation to the counterfactual group, for a cost base for implementing the policy. Such a form is good practice, as it allows for an easier comparison of the results estimated in different studies. Thus, the formula for determining the gain of an intervention for a given monetary cost of implementation is given by:

$$\frac{\widehat{e}_2^T - \widehat{e}_2^C}{\widehat{e}_1^C} \tag{10}$$

Table 1 – Differences between Traditional Portfolio Theory and Impact Portfolio Management.

Traditional Portfolio Theory	**Impact Portfolio Management**
Ex-post return is easy to measure and takes place instantly.	*Ex-post* return is difficult, expensive and time-consuming to measure.
A negative return is always possible.	Normally, an absence of impact is the worst possible case.
Easy to define measurement units (return and financial risk)	Difficult to define the measurement unit.
Impact of the action takes place at a precise moment.	Impact of the action occurs at an imprecise moment.
Expected return (*ex-ante* return) is based on historical data and / or discounted cash flows.	Expected return (*ex-ante* return) is due to empirical studies (RCT, quasi-experiments, etc.) and premises for extrapolating the context; or simply Economic Theory and / or personal beliefs.
Risk is generally measured based on the volatility of past financial returns.	Risk is associated with the uncertainty of the estimate (standard error of the estimate).
Does not worry about unmeasured variables or side effects.	It is concerned with unmeasured variables and side effects.
Impact occurs in a single variable.	Impact can occur in multiple variables.
Investment horizon is irrelevant for portfolio decisions (in Markowitz Portfolio Theory)	Time can be relevant in defining the portfolio (knowledge accumulation and risk reduction)
Correlation between assets is generally greater than zero.	Correlation between investments is generally equal to zero.
Investment cost is accurate.	Investment cost is imprecise.
Investment size is generally irrelevant.	Investment size can affect the pattern of returns and risks.
Measuring the investor's utility function (preferences) is of medium difficulty.	It is very difficult to define the investor's utility function.
Measurement in terms of percentage change in the amount invested	Measurement generally takes place in terms of gains from standard deviations from the base year of the counterfactual group.

Source: Elaborated by the authors, with information from Markowitz (1952), Kolm, Tütüncü and Fabozzi (2014), Agrawal and Hockerts (2019), Glennerster and Takavarasha (2013) and Gertler et al. (2017).

where $\widehat{e}_2^T$ is the average value of the variable measured in the treatment group at t+1; and $\widehat{e}_2^C$ is the average value of the variable measured in the control group at t+1; and $\widehat{e}_1^C$ is the average value of the variable measured in the control group before the intervention (t=1).

Finally, it is worth noting that the gain from diversification should be even greater with socio-environmental objectives, since, normally, the correlation between the effects of different programs should be close to zero.

3 METHODOLOGY

In light of the discussion concerning the theoretical framework, the purpose of this section will be to define a consistent protocol for the selection of portfolios with the objective of achieving financial and socio-environmental goals. It is proposed that the decision-making process follows a series of predefined steps in order to guarantee the robustness of the chosen final investment portfolio, providing transparency about the assumptions made and ensuring greater supervisory power and continuous improvement of the modeling phases.

It is suggested, therefore, that the selection of impact portfolios be made through a series of steps, as shown in Figure 1.

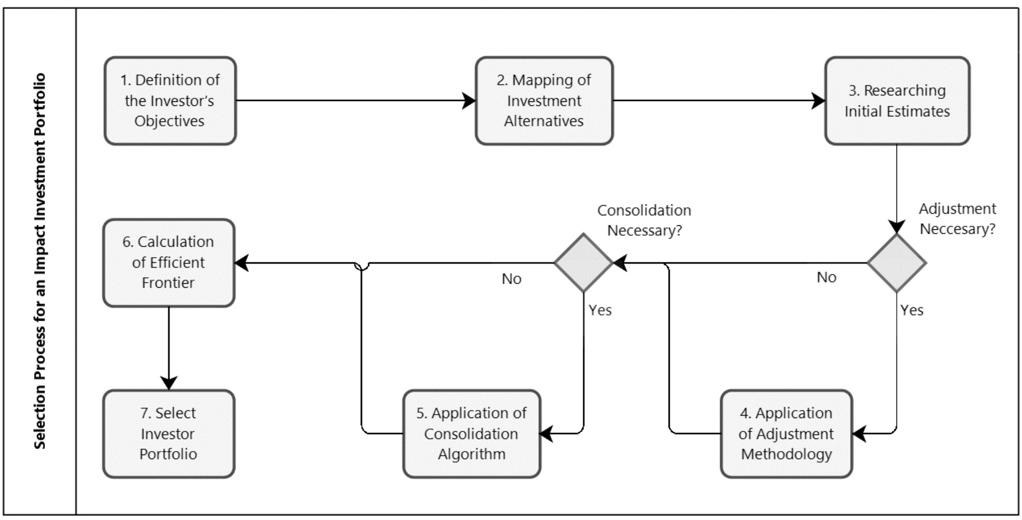

Figure 1 – Steps for selecting an Impact Investment Portfolio.

The details of each step are presented below:

1. Definition of the intended objectives: In this step, an explanation of the financial and socio-environmental objectives intended by the investor and the decision variables that need to be measured must occur.

2. Mapping of investment alternatives: In this step, a mapping of the allocation possibilities in investment programs with the potential to reach the objectives defined in the previous step should be carried out.

3. Survey of information on estimates: This step aims to identify academic research and expert opinions on the estimates of the impacts of investments, programs or policies that use instruments with purposes similar to those intended by the investor.

4. Adjustment of estimates: The purpose of this phase of assembling the portfolio is to seek a conservative adjustment of the impacts identified in the previous step. It is based on the quality of the sources of information and the adequacy of the context of the original study to the reality of the program to be implemented, ensuring the transportability of the impact results.

5. Consolidation of estimates: After adjusting the estimates of the studies and individual opinions, the objective of this phase is to apply a predefined technique for consolidating impact estimates for each decision variable of each investment alternative.

6. Calculation of the efficient frontier: This stage seeks, through the application of the multi-objective optimization technique, to eliminate the allocation possibilities from the dominated portfolios, and to explain the existing trade-offs between the objectives for the decision maker.

7. Selection of the final portfolio: The last stage of the modeling deals with the definition of the final portfolio, based on the individual preferences of the decision makers and the available allocation options present in the efficient frontier.

Obviously, each of these steps deserves an in-depth discussion about the multiple possible approaches. Some of the techniques already introduced in the Literature Review can be applied directly to address some of the listed steps, such as the use of impact assessment techniques to measure the effect of a particular investment in the information gathering stage, or the use of meta-analysis in the consolidation stage of studies and multi-objective optimization techniques in the calculation stage of the efficient frontier. However, in others, subjective opinions will be necessary, requiring some degree of discretion on the part of the decision maker.

In the adjustment stage of the estimates, as already discussed in the Literature Review it is known that there are differences in quality and reliability between the various possible methodologies used in assessing impact. In addition, there is also the problem of the transportability of the results. Thus, in order to reflect this qualitative information in the estimates, a kind of function can be adopted to relate the quality of the information source and the type of extrapolation to the quantification of the necessary adjustment to the estimate. In this sense, it is suggested that a modeler adopts a scheme similar to that presented in Table 2, in which seven possible degrees of reliability (maximum, very high, high, medium, low, very low and minimum) are related to

Table 2 – Degree of reliability of the impact estimates depending on the source of information and the type of extrapolation.

Information Source	Type of extrapolation	Degree of reliability in return and socio-environmental risk
Experimental method	- Consensus between multiple studies and appropriateness of the Theory of Change context proven by measurements.	- Maximum
	- Context validation of the Theory of Change by subjective analysis.	- High
	- Lack of consensus between studies and/or lack of context validation in the Theory of Change.	- Very low
Quasi-experiment	- Consensus between multiple studies and appropriateness of the Theory of Change context proven by measurements.	- Very high
	- Context validation of the Theory of Change by subjective analysis.	- Medium
	- Lack of consensus between studies and / or lack of context validation in the Theory of Change.	- Very low
Economic Theory/ Expert Opinion	- Consensus between multiple opinions / theories.	- Low
	- Lack of consensus between multiple opinions/theories.	- Very low
Pioneering	- Choices based on intuition with little theoretical support	- Minimum

an adjustment value in the estimates, which can be obtained in a discretionary way or through a Bayesian model.

After the adjustment stage of the individual estimates, one can use the Bayesian estimates suggested by equations (5) and (6) for the consolidation of the results. Therefore, with the estimates of return and risk of each of the possible investments in the financial and socio-environmental dimensions, the creation of a model of the Multi-objective Mean-Variance type is suggested, as presented below, to determine the efficient frontier and the selected final portfolio:

$$\begin{aligned}
&\textit{maximize} && \boldsymbol{Z}(\boldsymbol{x}) = w_1\boldsymbol{x}'\boldsymbol{r}^1 - w_2\boldsymbol{x}'\boldsymbol{\Sigma}^1\boldsymbol{x} + w_3\boldsymbol{x}'\boldsymbol{r}^2 - w_4\boldsymbol{x}'\boldsymbol{\Sigma}^2\boldsymbol{x} + \dots \\
& && + w_{(n\times 2-1)}\boldsymbol{x}'\boldsymbol{r}^n - w_{(n\times 2)}\boldsymbol{x}'\boldsymbol{\Sigma}^n\boldsymbol{x} \\
&\textit{subject to} && g_i(\boldsymbol{x}) \leq 0, i = 1, 2, \dots, m \\
& && \boldsymbol{x} \geq 0 \\
& && \boldsymbol{x}'\mathbf{1} = 1
\end{aligned} \tag{11}$$

where $\boldsymbol{x}$ is the vector with k decision variables, $x_1, x_2, \ldots, x_k$, containing the percentage invested in each k-th investment alternative; n, the total number of impact dimensions observed with the action, each containing two objectives, one for return and the other for risk; $\boldsymbol{r}^j$, the vector with the expected returns in impact dimension j for each of the investment alternatives (for example, $\boldsymbol{r}^1$ can represent the expected financial returns from investments, $\boldsymbol{r}^2$ can represent the returns in terms of the number of jobs generated by investments, etc.); and $\boldsymbol{\Sigma}^j$, the matrix with the variances and covariance estimated in impact dimension j for each of the investment alternatives (for example, $\boldsymbol{\Sigma}^1$ can represent the variance and covariance matrix of financial returns, $\boldsymbol{\Sigma}^2$ can represent the matrix of variances and the covariance of the number of jobs generated by investments).

In order to illustrate the application of the impact portfolio selection model presented in this section, the protocol will be applied to a hypothetical sample problem, presented below:

Sample problem: Forming a portfolio of impact investments

Suppose that an impact investor faces three investment possibilities: i) the creation of a micro-credit line for urban entrepreneurs (Program UM), ii) the offer of an agricultural credit line to promote family farming for rural workers (Program AC); and iii) a funding program exclusively dedicated to the purchase of new machinery (Program MF). In addition to obtaining the financial return, the investor aims, as a result of his investment, that, on average, there will be an increase in the family income of the individuals who accept the financing and that the programs will create jobs.

In the survey conducted by the investor, he identified a series of studies, interviewed some experts and analyzed historical records to get the estimates of returns (or policy effects) and the degree of uncertainty related to each of these estimates (Table 3). Furthermore, he considers that there is a correlation in the program's financial returns of $\rho_{UM,AC} = 0.3$, $\rho_{UM,MF} = 0.5$, and $\rho_{AC,MF} = 0.6$ and that the financial returns decrease exponentially as a function of the proportion allocated to each one of them.

4 RESULTS

This section aims to present the results and analysis of the sample problem described in the previous section. As several stages of the protocol described in the methodology have already been defined in the presentation of the problem, the derivation of the result will focus on the last four stages of the protocol, as shown in Figure 1.

The first step suggested for solving the problem is derived directly from the investor's individual beliefs about the collected inputs to gauge the effects of each investment. Thus, according to the problem definition, the impact estimates of the credit programs are adjusted individually using the information source adjustment factors. After calculating the individual estimates, the Bayesian estimates suggested in Rubin (1981), used by equations (5) and (6), are used to consolidate the estimates, as shown in Table 4.

Table 3 – Risk and return estimates for credit programs for each of the objective categories.

Investment	Objective Category	Source of Information	Estimated Return (%)	Estimated Risk (%)
Urban Microcredit (UM)				
	Financial			
		Historical Data	10.0	5.0
	Income			
		Quasi-experiment	12.5	12.8
		Expert Opinion	40.0	12.5
	Employment			
		RCT	5.0	5.2
		RCT	3.5	3.7
Agricultural Credit (AC)				
	Financial			
		Historical Data	8.0	4.0
	Income			
		RCT 1	0.0	8.0
		RCT 2	20.0	10.0
	Employment			
		Quasi-experiment	6.3	2.4
Machinery Financing (MF)				
	Financial			
		Historical Data	8.0	3.0
	Income			
		RCT	18.0	12.0
		Expert Opinion	14.5	6.7
	Employment			
		Expert Opinion	-1.7	4.8

Note: The data presented in this table was arbitrarily generated for the sample problem.

Table 4 – Consolidation of average income and employment impact estimates (in standard deviations) and financial returns by credit program.

Program	Financial Return (%)	Income Impact (%)	Employment Impact (%)
Urban Microcredit			
Policy effect	10.0	13.9	4.0
Standard error	5.0	12.5	3.0
Agricultural Credit			
Policy effect	8.0	7.8	5.0
Standard error	4.0	6.2	3.0
Machinery Financing			
Policy effect	8.0	12.0	-2.0
Standard error	3.0	8.0	6.0

It may be observed that the weighting scheme favors studies with lower standard errors in the estimates for the final average estimate. For example, the consolidation of the agricultural credit studies presents a final estimate of 7.8%, despite the simple average of the estimates being 10%, since the standard error of Study 1 is less than that of Study 2, which pulls the value down.

Also, in relation to Table 4, it may be observed that credit programs are not dominant in relation to each other. This does not mean, however, that if a dominance relationship were found, an allocation would not be justified in both programs, considering that there could be a gain in this strategy as a result of the diversification principle.

The next step is the estimation of the efficient frontier (Pareto frontier) given by the application of equation (11) with the problem parameters in order to compute the non-dominated vectors for the six objectives implicit in the problem (i.e., maximization of financial returns, maximization of impact on income, maximization of impact on employment, minimization of financial risk, minimization of uncertainty about the impact on income and minimization of uncertainty about the impact on employment).

In the case of this study, the problem was formulated and solved using PYMOO, a multi-objective optimization package in Python. The algorithm used for solution was the NSGA-II, with a 1,000 population size that evolves over 1,000 generations. The choice of this algorithm to solve the problem is that it is suitable to deal with nonlinear problems and it has great generality to handle more complex problems, which would normally be required in real problems in this research field.

To compare the performance of the algorithm, the problem was also solved using a grid search, making each decision variable vary in the interval [0.1] by a step of 0.01. It is worth noting that this type of algorithm is very inefficient for high-dimensionality problems, but for the specific case it is useful to help measure the convergence of the multi-objective genetic algorithm.

The Figure 2 presents the non-dominated solutions considering decision makers who only care about one of the three objective categories, considering their dimensions of risk and return. In all categories, returns follow a concave relationship on risk, as observed in the Pareto Frontier of Markowitz's Mean-Variance Portfolio. This means that the diversification of investments produces greater impacts for a given level of risk than predicted by the simple linear combination of the individual investment parameters. Therefore, like in Markowitz's (1952) conclusion, this fact emphasizes the importance of considering the portfolio as a whole also for the achievement of socio-environmental objectives, rather than the simple individual investment choices.

Due to the high dimensionality of the objective space, Deb (2001) suggests that the Pareto Frontier visualization could be done in pairs. Figure 3 presents the non-dominated solutions for all 15 possible pairs of different objectives. To improve the visualization of the data, the objectives were rescaled between 0 and 100, with 100 being the most desirable (that is, the one with the highest financial return, the greatest social impact, the least financial risk or less uncertainty about the social impact) and 0 being the least desirable (less financial return, less social impact, greater risk or greater uncertainty about social impact).

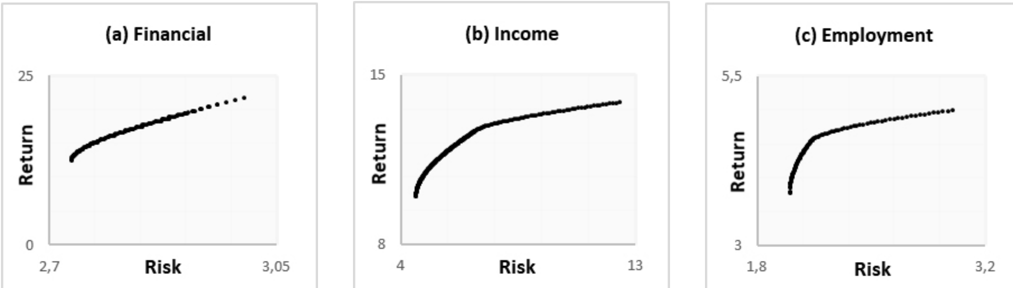

Figure 2 – Risk and return (in annual %) depending on the allocation among the investment alternatives regarding the three objective categories.

In Figure 3, the rightmost and uppermost points are the dominant solutions when the decision maker considers only the pairs of objectives presented in the axis labels of subplots (a) to (o). However, considering the six objectives at once, it is observed that there are a series of interior points that are also efficient when the decision maker cares about at least one of the other four objectives not presented in each of the figure's subplots.

Another metric used to measure the performance of the algorithm is the percentage of solutions wrongly evaluated as belonging to the Pareto Frontier, as the error ratio presented in Veldhuizen (1999). Using the grid search method solutions as a proxy for the true Pareto Frontier, it was estimated that 1.1% (11 out of 1,000) of the solutions presented by the NSGA-II algorithm are actually dominated solutions. On the other hand, 0.9% (35 out of 4,291) of the grid method solutions are dominated by the NSGA-II solutions. Therefore, using the definition of Zitzler et al. (2003), the approximation of the Pareto-optimal set produced by the two algorithms are incomparable, since neither the grid method solutions weakly dominates the NSGA-II solutions nor the NSGA-II solutions weakly dominates the grid method solutions (a set of solutions A weakly dominates a set of solutions B, if any objective vector in B is weakly dominated by a vector in A).

Figure 3 is also useful to assess the degree of diversity of genetic algorithm solutions, since it is desirable that solutions found span the entire Pareto-optimal region uniformly (Deb, 2001). However, the analysis of subplots shown in the Figure indicates that there are some regions with empty spaces of solutions.

Finally, a single frontier portfolio can be selected, considering the individual preferences of the decision maker. For example, using the pseudo-weights method proposed in Deb (2001), one can calculate the normalized distance to the worst solution regarding each objective, providing a relative importance factor for each objective corresponding to the solution.

Taking, for instance, an investor who values a balance between all objectives, he would choose the investment portfolio $\boldsymbol{x} = [x_{UM}, x_{AC}, x_{FM}] = [0.33, 0.44, 0.23]$ with pseudo-weights $\boldsymbol{w} =$ [*Financial Return, Income Return, Employment Return, Financial Certainty, Income Certainty, Employment Certainty*] $= [1,1,1,1,1,1]$ to opt for the scaled objective vector **Z**=[21, 49, 72,

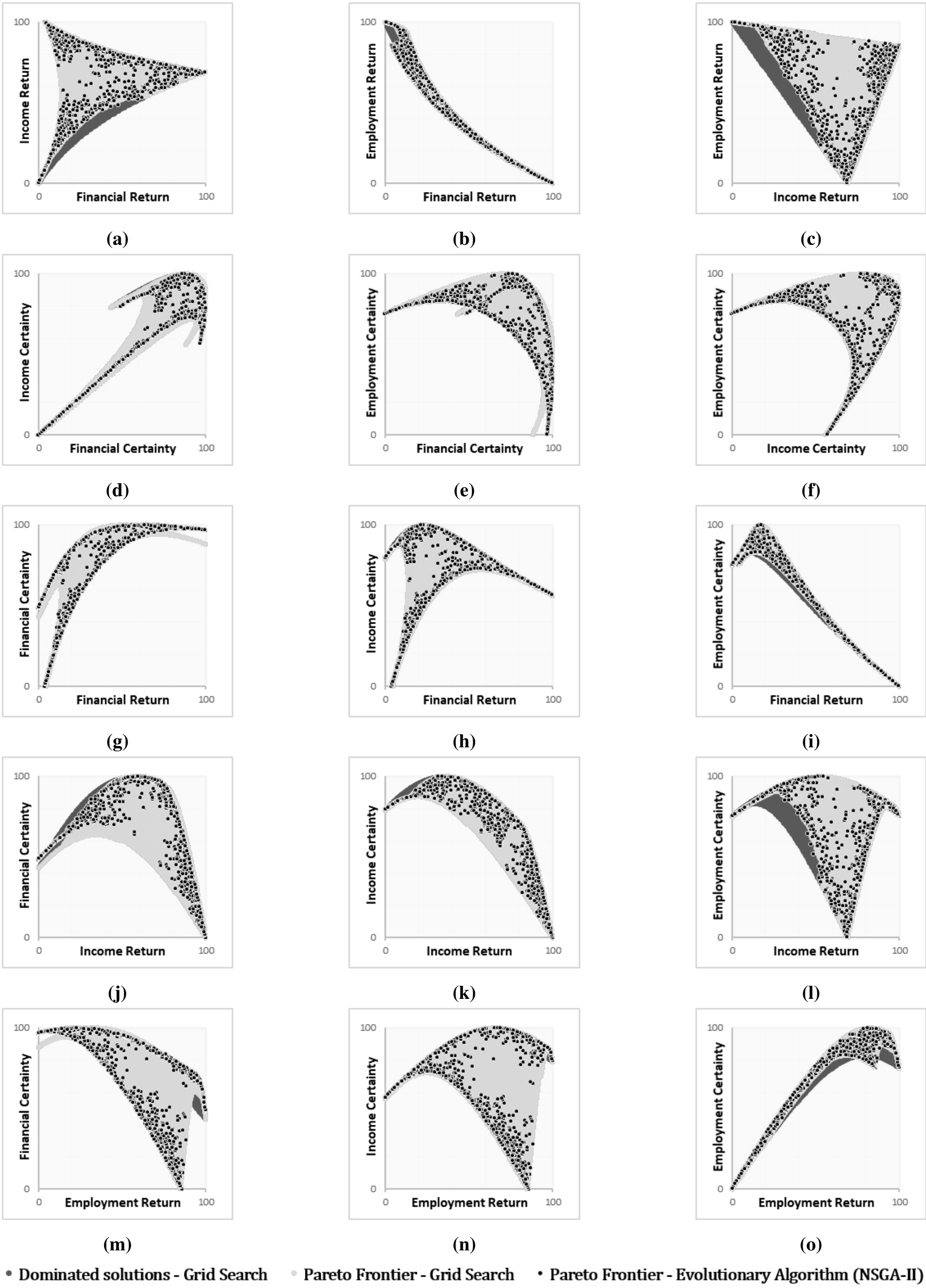

Figure 3 – Objective Space and Pareto Frontier approximation according to the Grid Optimization Method and the Evolutionary Algorithm for each pair of objectives.

84, 91, 96] (Figure 4-a). On the other hand, an investor who values the financial dimension a little more, adopting the weights $\boldsymbol{w} = [2,1,1,2,1,1]$, would be better off choosing the portfolio $\boldsymbol{x} = [0.20, 0.26, 0.54]$ to get the vector of objectives $\boldsymbol{Z} = [42,57,43,97,65]$ (Figure 4-b). Such choices of optimal solutions would be left basically unchanged regardless of the chosen optimization method (i.e., grid or evolutionary optimizations).

Therefore, based on the numerical results shown here, it is easy to realize that it is possible to extend the ideas present in the efficient Markowitz frontier to manage portfolios with multiple objectives. It may be observed that gains from diversification in allocation can also be used to reduce the risks associated with the intended effects of socio-environmental programs.

(a) Optimal choice for $w = [1,1,1,1,1,1]$ **(b)** Optimal choice for $w = [2,1,1,2,1,1]$

Grid Optimization **Evolutionary Algorithm (NSGA-II)**

Figure 4 – Pseudo-weights optimal choices according to the Grid Optimization and the Evolutionary Algorithm optimal sets.

5 FINAL CONSIDERATIONS

This paper has investigated the use of a protocol in the selection of impact portfolios. Such a protocol requires the investor to carry out a prior survey to collect evidence about the impact estimates of investments, and promote a reassessment of the estimates according to the quality of the available information sources and the appropriateness of the context in which the previous surveys were carried out, consolidating impact estimates in a systematic way, eliminating dominated portfolios from the analysis, and deciding on the portfolio, belonging to the efficient frontier, that meets the investor's preferences for financial and socio-environmental objectives.

In particular, this methodology is innovative in that it extends the gains of diversification, already contemplated in traditional portfolio management, to socio-environmental objectives. This fact allows investors to build more efficient portfolios through allocations in various programs that aim at the same objective, but that have little relation between their transmission channels and the effect pursued in the variable of interest.

The implementation of this method has been illustrated using a numerical example, in which the perspective of an investor who intends to allocate his resources in three credit programs was studied in order to seek, in addition to a financial return, the increase in the average income of families that joined the financing program, as well as the increase in the number of jobs generated by the program. In the case in question, the efficient frontier was derived, and the existing trade-offs made explicit in each of the possible allocations. Despite the fact that the study is based on a hypothetical case, the method can be applied to real cases directly, entailing additional difficulties such as the search for pre-existing impact assessments, and a robust analysis in relation to the validity of the studies found in the new context of the program that is intended to be implemented.

A limitation of this work, which can be addressed in future studies, is the disregard of intertemporal preferences in decision making. For example, it may be more rational for an investor to choose financial gains in an initial phase, and then divide the accumulated amount at a later time. Another example, also related to the timeless dimension of the analysis, is the possibility of taking advantage of experience gained from pilot projects to reduce the uncertainty associated with the large-scale implementation of a given program.

Finally, it should be noted that the adoption of a protocol that values scientific knowledge for decision making is a great advantage in relation to the trial-and-error method generally adopted by the actors who seek to promote socio-environmental gains with their actions.

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