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Economic valuation of a toll road concession with traffic guarantees and the abandonment option

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Abstract

Governments worldwide have been encouraging private participation in transportation infrastructure. To increase the feasibility of a project, public–private partnership (PPP) may include guarantees or other support to reduce the risks for private investors. It is necessary to value these opportunities under a real options framework and thereby analyze the project's economic feasibility and risk allocation. However, within this structure, sponsors have an implicit option to abandon the project that should be simultaneously valued. Thus, this article proposes a hypothetical toll road concession in Brazil with a minimum traffic guarantee, a maximum traffic ceiling, and an implicit abandonment option. Different combinations of the minimum and maximum levels are presented, resulting in very high or even negative value added to the net present value (NPV). The abandonment option impacts the level of guarantee to be given. Governments should calibrate an optimal level of guarantees to avoid unnecessarily high costs, protect the returns of the sponsor, and lower the probability of abandonment.

Keywords

Real options. Economic evaluation. Public–private partnership. Toll road concession. Government guarantees.

1. Introduction

Private participation in transportation infrastructure projects has been sought by governments globally because of scarcity of public resources, economic expansion, increased demand for better services, and deterioration of transportation infrastructure – particularly in emerging economies. Public–private partnership (PPP) agreements have been used as an important financial engineering alternative to attract private investment to infrastructure projects. PPP can be defined as an arrangement in which private parties participate in or provide support for the provision of infrastructure-based services (Grimsey & Lewis, 2004). In this context, project finance structures have been used to finance public infrastructure particularly based on concession agreements (Yescombe, 2002). PPP can take different forms; one popular arrangement used in transportation projects is the build-operate-transfer (BOT) approach, in which a private investor has the right to finance, develop, and operate a project for a defined period and then transfer it to the government.

Both private- and public-sector parties are concerned with the viability of infrastructure projects; however, regardless of public party's objectives, a project must be financially attractive to private investors. Profitability and viability are subject to the risks associated with infrastructure sectors, including construction, political, currency, and force majeure risks (Fishbein & Babbar, 1996); thus, a key success factor is the optimal allocation of risks among the participants. Besides these challenges, transportation projects are subject to greater uncertainties pertaining to future demand or traffic, which are difficult to estimate and may impact revenue levels. Under real options theory, the uncertainties pertaining to the future rewards of an investment opportunity are taken into account

in the valuation process. A firm that chooses to make an investment holds an "option" analogous to a financial option. According to Dixit & Pindyck (1994), the option gives

> the right (which we need not exercise) to make an investment expenditure (the exercise price of the option) and receive a project (a share of stock) the value of which fluctuates stochastically.

Therefore, flexibilities in the decision process should be viewed as "real options." The central objective of PPP is to create conditions that encourage the private sector to participate in the construction and operation of infrastructure projects that may initially seem infeasible. When the profitability of the project is weak, governments can use mechanisms to mitigate risks that adversely impact the return to the private sector; some of these mechanisms also exhibit real options characteristics.

Considering transportation infrastructure projects, this study models traffic guarantees and an implicit right of abandonment under real options theory, and proposes the simultaneous valuation in a hypothetical toll road project. The traffic guarantees are modeled as a composition of a minimum traffic guarantee and a maximum traffic ceiling. The composition can be designed for different levels of protection. The minimum traffic guarantee can make the project more attractive to private investors since it guarantees a minimum level of revenue. On the other hand, the maximum traffic ceiling works like a cap for traffic, allowing the government to control for higher-than-expected returns; it is worth mentioning that this traffic ceiling is designed to limit the revenue received by the concessionaire and it does not restrict the real traffic level in the road.

The contribution of this paper is twofold. A more sophisticated combination of minimum traffic guarantee and maximum traffic ceiling level for revenues is proposed, based on the demand guarantees designed for the Fourth Line of the São Paulo Metro in Brazil (detailed in the Appendix), in order to evaluate a hypothetical toll road concession. Besides the definition of the minimum and maximum traffic levels calculated from the expected traffic level for each year of concession, we propose to use different percentages of the portion of revenue to be received or paid by the concessionaire. Both mechanisms are treated as real options; two different methods to calculate their values are analyzed, one based on analytical concepts – as used by Galera & Soliño (2010) –, and another based on simulation – as used by Brandão & Saraiva (2008). Second, although some authors have analyzed traffic or demand guarantees,

this paper additionally proposes the possibility of abandonment of the project by the sponsors and values this right as a real option. The concept of abandonment option is broadly used in real options analysis. Usually, management can abandon current operations permanently if market conditions are adverse, limiting the losses in adverse scenarios (Trigeorgis, 1996). In this study, the abandonment option is valued based on concepts presented by Pollio (1998) related to project finance structures. The interaction between the abandonment option and the traffic guarantees should be analyzed to understand how this additional option impacts the level of guarantees to be given. The optimal combination of minimum and maximum traffic levels should be selected on the basis of three objectives: to avoid an unnecessarily high guarantee, to protect the expected returns to sponsors, and to lower the probability of abandonment.

The rest of this paper is structured as follows. Section 2 presents a literature review. Section 3 introduces the methodology used to value the proposed traffic guarantees and the right of abandonment in a hypothetical toll road. The parameters and results are presented and interpreted in Section 4. Section 5 concludes the study.

2. Literature review

Real options theory has been broadly used in the literature on transportation infrastructure projects to value different embedded flexibilities not specially related to mechanisms used to mitigate risk. Wooldridge et al. (2002) applied a real options valuation method to Dulles Greenway project, a toll road in Virginia, USA, to incorporate the option of waiting up to five years before building the highway. Bowe & Lee (2004) analyzed the Taiwan High-Speed Rail Project, in which the construction and operation of the rail system embodied multiple interacting flexibilities and involved an option to defer or postpone construction, an option to abandon early in the construction phase, options to expand or contract, and an option to abandon or switch use at any time. Zhao et al. (2004) modeled a highway system focusing on the real options of expansion and rehabilitation. They used quantitative models of uncertainties pertaining to demand, costs, and land availability. Wei-hua & Da-shuang (2006) proposed a concession decision model with three real options embedded: the option to adjust concession price, the option to develop the surrounding land, and the option to expand capacity.

Regarding the incentive instruments used by governments to mitigate risks and attract

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private investors in transportation concessions, the mechanisms can be classified according to the following three criteria, based on a taxonomy proposed by Vassallo (2006): 1) the chosen trigger variable – which can be traffic, revenues, or internal rate of return (IRR) – used as a reference point for initiating either the implementation of a guarantee or the modification of contract conditions, for example; 2) risk allocation between the parties, which sometimes involves minimum and maximum target levels for the trigger variable; or 3) the compensation mechanism adopted, including a subsidy or a change in contract length. Given these criteria, the author observes that, in practice, three main approaches are primarily adopted globally. The first approach emphasizes the economic balance of the concession through the IRR and the establishment of acceptable levels for this variable. The second approach is based on guarantees of traffic or revenues; in this approach, the risk is shared between the government and the concessionaire because minimum and maximum bands are usually considered. The third approach is related to the contract's length, whose endpoint should correspond to the moment when a target variable is achieved in the form of least present value revenue (LPVR) mechanisms (Vassallo, 2010).

When incentive instruments exhibit the characteristics of real options, traditional economic valuation techniques cannot correctly quantify them. In order to analyze economic feasibility and risk allocation, projects involving such mechanisms must be valued under a real options theory framework. Examples of incentives are shown by Rose (1998) and Alonso-Conde et al. (2007)—who analyzed the Melbourne CityLink Project, a toll road in Australia. In this project, two agreements—based on the investor's IRR for terminating the project before the end concession term or deferring the payment of concession fees—can be identified as interacting options embedded in this project. Other types of incentives are also proposed by Wibowo (2004), who offered a case study of an Indonesian toll road project to analyze the financial impact of guarantees provided by the government, such as tariff adjustment according to inflation rate or an equivalent amount compensation; a ceiling for interest rate during the debt service period with an equivalent amount compensation if the rate turns out to be higher than a specific value; and minimum revenue and minimum traffic guarantees. This paper will first focus on the benefit of the minimum demand guarantee. In a toll road project, this guarantee is the minimum traffic or revenue guarantee. If the incentive is not correctly quantified, sponsors may

disregard the real expected return and decide not to invest; or governments may provide guarantees that are larger than necessary to afford a fair rate of return, incurring in high costs. Irwin (2003) examined some types of support provided by governments—including guarantees of risks that the government cannot control, such as the risk of future demand for public services. Such guarantees are similar to put options and should be correctly valued using option-pricing techniques. Chiara et al. (2007) proposed a new approach for revenue guarantees based on exercise dates determined during the operational phase. Cheah & Liu (2006) analyzed the minimum revenue guarantee in the Malaysia-Singapore Second Crossing. Brandão & Saraiva (2008) proposed a hybrid model for BR-163, a Brazilian toll road. They used a Monte Carlo simulation to incorporate a minimum traffic guarantee and the payment of excess revenue if the traffic was above a certain level. Galera & Soliño (2010) used an analytical methodology to analyze a minimum traffic guarantee in a highway concession in Spain. In both papers, Brandão & Saraiva (2008), and Galera & Soliño (2010) used the concept of the market price of risk in risk-neutral stochastic processes. This methodology was also employed by Irwin (2003). In Brazil, as a real case of government guarantees implementation, the PPP contract of the Fourth Line of the São Paulo Metro, an expansion of the major Brazilian state's subway, describes mechanisms to mitigate currency, construction, and demand risks. Regarding demand risks, the proposed mechanism is based on bands of minimum and maximum demand levels with different levels of adjustment. The details are described in the Appendix.

In addition, our paper proposes the existence of the sponsors' abandonment right and values it as a real option; this approach is based on the concepts presented by Pollio (1998) in relation to project finance structures. As defined by Yescombe (2002), project finance is a method of raising long-term debt financing based on lending against cash flow generated by a special purpose company which represents the project alone. In this study, the project is the toll road concession, operated by the sponsor, or the concessionaire, through concession agreements with the government. There is a high ratio of debt to equity with a high percentage of the initial investment being financed by third parties, or lending banks. Pollio (1998) presents a broad review about project finance and its participants focusing on the motivation for choosing this structure over other debt options. Within an options-theoretic framework, the author arguments that risk management features that are inherited in such structures are the core

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motivation. A central feature of project finance is that upon completion project risks are partially transferred from sponsors to lenders. If conditions that affect project values change, sponsors have an implicit decision of either continuing to repay the loan or else default. Under a real options framework, the additional flexibility provided by an implicit abandonment option interacts with the government guarantees present in the project of this study and affects the project value.

3. Toll road concession with traffic guarantee, traffic ceiling and an abandonment option

PPP in Brazil is defined by a federal law as a supported concession (Brasil, 2004) that allows the government to grant monetary support to concessionaires. On the basis of the demand guarantees designed for the Fourth Line of the São Paulo Metro in Brazil (São Paulo, 2006), in this study we analyze a hypothetical project involving a PPP for a 25-year toll road concession. To make the concession attractive to the private sector in terms of traffic risk, the government offers a minimum traffic guarantee; however, a traffic ceiling is also considered as a means for the government to avoid returns that may be much higher than expected. In this case, if the traffic lies above the traffic ceiling, the concessionaire pays the excess revenue to the government, and if it lies below the traffic floor, the concessionaire receives additional revenue guaranteed by the government.

Moreover, the concession dictates the project finance structure, very common in infrastructure projects. Based on the methodology proposed by Pollio (1998), the third option embedded in the project is the concessionaire's implicit right of abandonment.

The stochastic variable is traffic. In the literature on toll road projects, many authors have modeled demand as the risk variable based on a geometric Brownian movement (GBM) (Galera & Soliño, 2010; Irwin, 2003, 2007; Rose, 1998; Wei-hua & Da-shuang, 2006), even though others claim that the movement may be more complex (Brandão & Saraiva, 2008; Chiara et al., 2007; Garvin & Cheah, 2004; Zhao et al., 2004). In this study, the traffic is modeled as a GBM, which allows different analysis methods, including the analytical one based on Black & Scholes (1973) formulas. Furthermore, using simulation methods, the analysis can be extended to other movements. GBM can be an appropriate choice to model traffic in emerging countries since the expected demand tends to present exponential

growth rate during the period of interest. On the other hand, the use of GBM imposes a limitation since the process does not take into account the road maximum traffic capacity. In this case, the analysis could be extended to incorporate an expansion option for the road or the process could be modeled as a GBM with a reflection upper barrier. In developed countries, other processes with mean reversion characteristic could be more suitable to describe the demand long-term level.

The traffic stochastic process is given as

$$\frac{d\theta}{\theta} = \alpha dt + \sigma dz \tag{1}$$

where θ is the traffic, α is the expected drift, σ is the volatility, and dz is the Wiener process. The correspondent risk-neutral process can be represented as

$$\frac{d\theta}{\theta} = (\alpha - \lambda\sigma)dt + \sigma dz^* \tag{2}$$

where dz^* is the Wiener increment under risk-neutral probability. The parameter λ is the market price of the traffic risk, which can be calculated as (Hull, 2006)

$$\lambda = \frac{\rho_{\theta,m}}{\sigma_m}(\mu_m - r) \tag{3}$$

where $\rho_{\theta,m}$ is the correlation between traffic changes and market index returns, μ_m is the expected return of a market index, σ_m is the volatility of the market index, and r is the risk-free rate.

3.1. *Minimum and maximum traffic level options*

Figure 1 represents the options for the minimum traffic guarantee and maximum traffic limit in each year of the concession term, considering only one level of minimum and maximum traffic.

Within a real options theory framework, the minimum traffic guarantee and maximum traffic ceiling can be treated as put and call options, respectively. Let θ_i be the real traffic and $\bar{\theta}_i$ be the expected traffic in year i in equivalent vehicles per day. Let a_i be a percentage below 100% and b_i a percentage above 100%, based on the expected traffic and representing the minimum and maximum traffic levels, respectively. Let y_i be a percentage corresponding to the portion of revenue that will be received or paid by the concessionaire. Let τ be the direct revenue tax fee and p be the toll fee. Considering continuous operation (365 days per year), the payoffs for the put and call options for
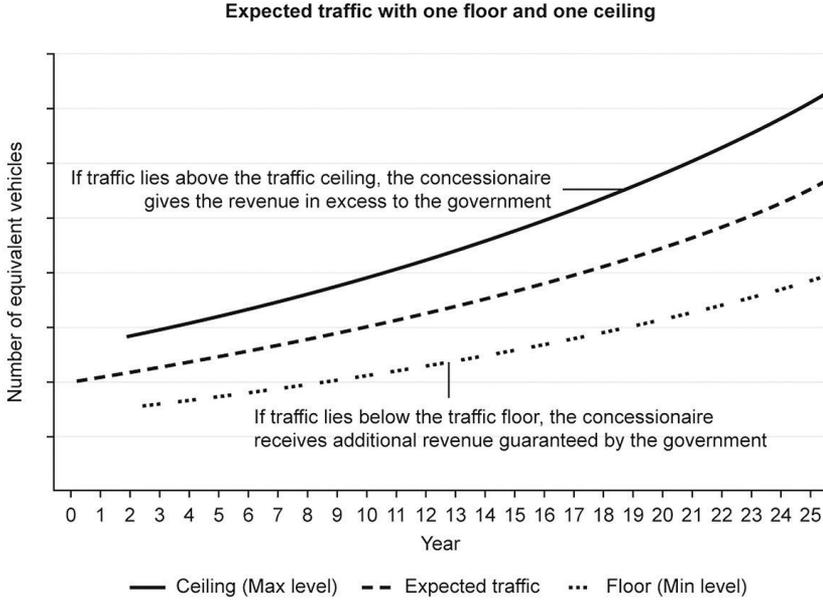
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Expected traffic with one floor and one ceiling

Figure 1. Project with one floor and one ceiling.

each year i during the concession term can be defined as follows:

$$Put : P_i = \left[y_1 \max\left(a_1\overline{\theta}_i - \theta_i, 0 \right) \right].365.(1-\tau).p \quad (4)$$

$$Call : C_i = -\left[y_1 \max\left(\theta_i - b_1\overline{\theta}_i, 0 \right) \right].365.(1-\tau).p \quad (5)$$

The put payoff corresponds to the amount to be received by the concessionaire, and the call payoff corresponds to the excess revenue to be paid. The options' values are calculated for different symmetric combinations of minimum and maximum traffic levels, and for different percentages of protection based on the parameters presented in section 4 (Data and results). Both options are modeled directly on the same underlying asset, the traffic level. They are mutually exclusive, but exist simultaneously at each period of the concession term. We compare the following two methods: an analytical method and a Monte Carlo simulation method.

3.1.1. Analytical method

The analytical method used by Galera & Soliño (2010) to value a minimum traffic guarantee is based on Black & Scholes (1973) and Merton's (1973) formulas. Designating the traffic θ as the underlying asset, the partial differential equation that must be solved for the derivative $F(\theta)$ is

$$\frac{1}{2}\frac{\partial^2 F}{\partial \theta^2}\sigma^2\theta^2 + (\alpha - \lambda\sigma)\theta\frac{\partial F}{\partial \theta} + \frac{\partial F}{\partial t} - rF = 0 \quad (6)$$

Black & Scholes (1973) differential equation relates the value of a derivative F contingent on the underlying asset, in this case, represented by θ. The partial differential equation (PDE) can be obtained using contingent claims approach, by building a risk-free portfolio $\Phi = F - n\theta$, where n is known as the delta hedge, and imposing that this portfolio must return the risk-free rate r. This PDE is parabolic and similar to the heat PDE in Physics.

Because the traffic is assumed to be a GBM, we can use the payoff given by Equation 4 to calculate the revenue to be received in each period. For the minimum traffic guarantee, the derivative $F(\theta)$ is a put option and the present value for the option of year i is

$$P_i(t=0) = 365.(1-\tau).p.y_1\left[a_1\overline{\theta}_i e^{-rt} N(-d_2) - \theta_0 e^{(\alpha-\lambda\sigma-r)t} N(-d_1) \right]$$

$$where\ d_1 = \frac{ln\left(\frac{\theta_0}{a_1\overline{\theta}_i}\right) + \left(\alpha - \lambda\sigma + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}\ and\ d_2 = \frac{ln\left(\frac{\theta_0}{a_1\overline{\theta}_i}\right) + \left(\alpha - \lambda\sigma - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \quad (7)$$

Here, r is the risk-free rate, $\overline{\theta}_i$ is the daily average traffic level in year i, and θ_0 is the initial expected daily average traffic level. Analogously, we can use the payoff given by Equation 5 to calculate the excess revenue to be paid in each period. For the maximum traffic ceiling, the derivative $F(\theta)$ is a call option and the present value for the option of year i is

$$C_i(t=0) = -365.(1-\tau).p.y_1\left[\theta_0 e^{(\alpha-\lambda\sigma-r)t} N(d_1) - b_1\overline{\theta}_i e^{-rt} N(d_2) \right]$$

$$where\ d_1 = \frac{ln\left(\frac{\theta_0}{b_1\overline{\theta}_i}\right) + \left(\alpha - \lambda\sigma + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}\ and\ d_2 = \frac{ln\left(\frac{\theta_0}{b_1\overline{\theta}_i}\right) + \left(\alpha - \lambda\sigma - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \quad (8)$$

Let n be the concession term given in years. The value added by the compounded options to

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NPV, including excess revenue to be paid from the concessionaire to the government and that to be received by the concessionaire from the government, would be given by

$$Value\ Added = \sum_{i=1}^{n} P_i(t=0) + C_i(t=0) \qquad (9)$$

3.1.2. Monte Carlo simulation method

In this alternative method, a risk-neutral Monte Carlo simulation was used to evaluate the options. The GBM discretization is given as follows:

$$\theta_{t+\Delta t} = \theta_t e^{\left(\alpha - \lambda\sigma - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}} \qquad \varepsilon \sim N(0,1) \qquad (10)$$

Simulating the traffic and cash flows for each year, we can calculate the project's original NPV without any option and the project's NPV in the presence of guarantees in each year of the concession. The cash flow in each period can be calculated as follows, assuming that the working capital effect is null on average:

$$CF_t = (Revenues_t - Operational\ Costs_t - Maintenance\ Costs_t - Depreciation_t - Interest_t)$$
$$(1 - Income\ Tax_t) + Depreciation_t - Amortization_t - Investment + \Delta Working\ Capital_t \qquad (11)$$

The original revenues at each year t are given by $Revenues = \theta_t p(1-\tau)D$, where p is the toll rate, θ_t is the average daily traffic, τ is the direct tax, and D is the number of operating days of the road during each year (i.e., 365 days per year in continuous operation). When the minimum and maximum traffic level options are considered, the total revenues in each period in the previous equation will be

$$Revenues_t = Original\ Revenues_t + Additional\ Revenue_t\ from\ minimum\ traffic\ guarantee$$
$$- Revenue_t\ in\ Excess\ from\ maximum\ traffic\ level \qquad (12)$$

The project original NPV is computed discounting the cash flows calculated only with the original revenues. It is assumed that the working capital effect is null on average. The NPV with traffic options is obtained from the cash flows calculated with the revenues as shown in Equation 12. The value added by the options is then given by

$$Value\ Added = NPV\ with\ traffic\ options - original\ NPV \qquad (13)$$

3.2. Abandonment option

Pollio (1998) proposed a real options approach for the strategic analysis of project finance structures with limited recourse. In this structure, the flexibility would be provided by an implicit right of abandonment at each repayment date. Because

the project finance structure results in limited liability for the sponsors, the borrower—or the sponsors—will exercise the option only if the project equity is nil or negative.

The value of the project with the abandonment option is given by

$$V = \max(\text{continuation value, abandonment exercise value}) \qquad (14)$$

where the continuation value includes the expected operational revenues and costs of the project and the costs to repay the debt. Considering the toll road project, to value the abandonment option, we build a traffic threshold curve based on backward optimization. The curve can be used to value the option, and to calculate the probability of abandonment and the average time of abandonment.

To obtain the traffic threshold curve, a 100-quarter-step binomial tree is built to represent traffic evolution during the 25 years of the concession's life. The higher the number of time-steps, the better the accuracy of the model to approximate the stochastic process of the traffic and, in this case, the 100-quarter-step tree was chosen, which we consider accurate enough for this valuation. The traffic follows a GBM: for each node, the traffic θ_i^s in period i and state s can increase to $u\theta_i^s$ or decrease to $d\theta_i^s$ in the following period. Thus, the parameters u, d, and q (the risk-neutral probability that the traffic will increase) are

$$u = e^{\sigma\sqrt{\Delta t}} \qquad (15)$$

$$d = \frac{1}{u} = e^{-\sigma\sqrt{\Delta t}} \qquad (16)$$

And

$$q = \frac{e^{(\alpha - \lambda\sigma)\Delta t} - d}{u - d} \qquad (17)$$

We build a cash flow tree based on the traffic tree. Thus, we can calculate the project value going backward from the last period of the cash flow tree ($t=T$). For each node of the binomial tree, which corresponds to each state s in period $t = i$, considering the implicit right of abandonment, the value of the project with the abandonment option given by Equation 14 can be written as

$$V_i^s = max\left(CF_i^s + \frac{1}{(1+r)}\left[q.V_{i+1}^u + (1-q)V_{i+1}^d\right], 0\right) \qquad (18)$$

where V_i^s is the present value of the project in period $t = i$ and state s considering the abandonment option, CF_i^s is the cash flow in period $t = i$ and state s, V_{i+1}^u is present value in period $t = i + 1$ and state

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u, and V^d_{i+1} is the present value in period $t = i + 1$ and state *d*. In our model, the abandonment cost is zero, but additional exit costs can be considered. The concessionaire may examine the backward calculations and choose to abandon the project at each node if the continuation value is negative.

The binomial tree also enables us to identify an abandonment region that includes a set of nodes where the abandonment is optimal, i.e., the result of Equation 18 is equal to zero. The decision rule can be represented by a traffic value that corresponds approximately to the first state node in which the abandonment option is exercised for each period. This value is given by the highest traffic for which the abandonment exercise is the optimal decision in each period; this paper refers to this value as the traffic threshold. The binomial tree algorithm starts at the end date of the concession ($t=7$) and works backward until the start date ($t=0$). This means that Equation 18 is calculated recursively and, for each *t*, the threshold θ^*_t is the maximum θ_t where it is optimal to exercise the abandonment option.

With the traffic threshold for each period, the threshold curve is complete and defines the abandonment region during the entire concession period. Using the threshold curve, the value added by the abandonment option is calculated using a risk-neutral Monte Carlo simulation because the project is abandoned each time the stochastic traffic path meets the threshold curve (first exit time). Again, the project original NPV is computed discounting the cash flows calculated using the original revenues, while the NPV with abandonment option is obtained from the discounted cash flows considering the exercise of the abandonment whenever the traffic hits the threshold curve.

$$Value\ Added = NPV\ with\ abandonment\ option - original\ NPV \quad (19)$$

When the traffic guarantees are also considered in the project, the interaction between the options causes the threshold curve to change. In this case, the cash flow trees must be rebuilt because there is additional revenue to be received, or excess revenue to be paid, in each node. When a project contains multiple real options, the interaction among these options influences their values (Trigeorgis, 1996). The implicit abandonment option may lose value when the minimum and maximum traffic level options are considered.

Given the traffic threshold curves, the probability and average time of abandonment are calculated in each situation (with or without the minimum and maximum traffic level options) using a real Monte Carlo simulation. Real simulation uses the stochastic process real drift, which is represented here as α. Risk-neutral simulation is used to price derivatives, but real simulation is required to calculate the option exercise probability. The results are given by the following.

$$Probability\ of\ Abandonment =$$
$$= \frac{number\ of\ interations\ where\ abandonment\ is\ exercised}{total\ number\ of\ iterations} \quad (20)$$

$$Average\ Time\ of\ Abandonment =$$
$$= \frac{\sum periods\ when\ abandonment\ is\ exercised}{number\ of\ iterations\ where\ abandonment\ is\ exercised} \quad (21)$$

The average time of abandonment is the average moment correspondent to the abandonment exercise during the concession. When abandonment is considered, the guarantee option contains an additional important benefit. This option becomes interesting from both the sponsors' and lenders' viewpoints. Besides increasing the expected project value and decreasing the sponsors' risk, the guarantee can be designed to reduce the probability of abandonment, and consequently reduce the lenders' risk.

4. Data and results

The relevant parameters for the project, their descriptions, and values for the hypothetical project analyzed in this study are listed in Table 1.

The symmetric percentages a_1/b_1 representing the minimum and maximum traffic levels over the expected traffic range from 50%/150% to 90%/110%, respectively. The protection percentage y_1 corresponding to the portion of revenue to be received or paid by the concessionaire ranges from 50% to 100%.

Using @Risk® software to simulate cash flows with 5,000 iterations, the expected NPV for the original project without any option was R$ 70.5 MM. The value added by symmetric combinations of minimum and maximum traffic levels and different protection percentages using both methods are presented below in Tables 2 and 3, based on Equations 9 and 13, respectively.

The difference between the values obtained using the two different methods can be explained by how income tax is treated. In the analytical method, the options' premium is calculated as net revenue and is directly added to the original project NPV; alternatively, in the simulation method, the options' premium is based on net profit after income tax for each year. Regarding the proposed project, the income tax treatment in the simulation method is more realistic because the additional and exceeded revenue (from minimum and maximum traffic levels,

respectively) impacts the profit, and consequently, the income tax to be paid and the final cash flow in each period. If the income tax is zero, the simulation results converge to the analytical results.

The results of using the simulation method to calculate the value added in each year, considering different symmetric combinations of minimum and maximum traffic level options are presented in

Table 1. Parameters.

Parameters	Description	Values
p	Tariff (R$)	5.50
τ	Direct taxes	14%
n	Concession term (years)	25
θ_0	Initial expected daily average traffic level[a]	100,000
α	Traffic drift (p.a.)	4%
σ	Traffic volatility (p.a.)[b]	10%
Inv	Initial investment: 50% in year 0 and 50% in year 1 (R$)	1,000 MM
Loan	Loan principal: 50% in year 0 and 50% in year 1, with 2-years of delayed payment (R$)	700 MM
r	Risk-free rate (p.a.)	6%
i	Loan rate (p.a.)	8%
n_2	Loan term (years)	15
OC_1	Annual operating costs in year 1 (R$)	30 MM
OC_2	Annual operating costs from year 2 to year 25 (R$)	60 MM
MC_1	Annual maintenance costs from year 2 to year 9 (R$)	50 MM
MC_2	Annual maintenance costs from year 10 to year 18 (R$)	70 MM
MC_3	Annual maintenance costs from year 19 to year 25 (R$)	90 MM
n_3	Investment depreciation term (years)	15
IT	Income Tax	34%
$\rho_{\theta,m}$	Correlation between IBovespa returns[c] and ABCR Index changes[d]	0.40
μ_m	IBovespa expected return (p.a.)	12%
σ_m	IBovespa returns volatility (p.a.)	25%
λ	market price of risk of traffic[e]	0.096
u	traffic increase factor (binomial tree)	1.1052
d	traffic decrease factor (binomial tree)	0.9048
q	risk-neutral probability (binomial tree)	69.74%

In this hypothetical example, the terminal value of the assets is null; [a] There is no traffic in year 0 and year 1. θ_0 is a reference value to estimate traffic in the following years. The expected traffic values for each year were calculated using GBM, i.e., $\theta_i = \theta_0 e^{\alpha i}$; [b] Drift and volatility were estimated on the basis of several Brazilian toll roads; [c] IBovespa (São Paulo Stock Exchange Index) is used as the market index; [d] The ABCR Index was chosen to represent the traffic (http://www.abcr.org.br). This index is calculated by the Brazilian Roads Concessionaires Association and a consulting firm in Brazil on the basis of flows of light and heavy vehicles on highways stretches under concession. It is used in this study to calculate the correlation between traffic changes and IBovespa returns; [e] The market price of risk of traffic was calculated based on Equation 3.

Table 2. Value added by min/max traffic options using analytical method (R$ 000).

a_i/b_i Min(%)/Max (%) traffic levels as percentages of expected traffic	y_i (Percentage of revenue to be paid or received)					
	50	60	70	80	90	100
50/150	−4.914	−5.896	−6.879	−7.862	−8.845	−9.827
60/140	7.837	9.404	10.971	12.539	14.106	15.674
70/130	32.088	38.506	44.923	51.341	57.759	64.176
80/120	69.600	83.520	97.440	111.360	125.280	139.200
90/110	119.764	143.717	167.670	191.623	215.576	239.529

Table 3. Value added by min/max traffic options using Monte Carlo simulation method (R$ 000).

a_i/b_i Min(%)/ Max (%) traffic levels as percentages of expected traffic	y_i (Percentage of revenue to be paid or received)					
	50	60	70	80	90	100
50/150	−2.285	−698	−2.986	1.251	−2.023	−2.991
60/140	8.990	12.166	12.647	15.089	17.722	17.970
70/130	29.567	37.118	41.322	50.364	53.823	59.581
80/120	63.394	74.735	86.058	98.282	109.855	118.942
90/110	107.739	126.364	146.176	163.975	182.744	199.310

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Figure 2. This value may be negative during some years of the concession depending on the minimum and maximum traffic levels. A lower minimum guaranteed level and higher symmetric maximum traffic level result in a longer period of negative premiums. In the first several years, the maximum level options exceed the minimum level options.

The total value added by the minimum and maximum traffic options to the expected NPV can be very high or even negative depending on the minimum guaranteed traffic level and the corresponding maximum traffic level. The government should choose an optimal combination regarding the return to sponsors and its own risk exposure.

Under the project finance structure, when the implicit abandonment option is considered, other factors may influence the government's decision regarding guarantee options. In this case, the sponsor will decide optimally between continuing to manage the project and abandoning it at each repayment date. This option adds value to the project and interacts with the minimum and maximum traffic level options previously analyzed. According to the methodology, in the absence of traffic guarantees, the original threshold curve and the abandonment region can be graphically represented as shown in Figure 3.

The solid line that limits the original abandonment region is the original traffic threshold curve (when no other option is considered in the project). In addition, dashed curves represent random paths of the stochastic traffic. When any path meets the threshold curve, the process stops and the project is abandoned. The non-smooth thresholds are related to discrete cash flows modeled for the 25-year project.

When minimum and maximum traffic level options are added to the model, new threshold curves are obtained. Different situations may be proposed to analyze the interaction of the options. The threshold curves considering the symmetric combinations of minimum and maximum traffic levels (given by the percentages a_1/b_1 over the expected traffic in each period) and 100% protection ($y_1 = 100\%$) are graphically represented in Figure 4. In this case, if the traffic floor is 80% or 90% of the expected traffic, there is no threshold curve, and consequently, abandonment is never optimal. Considering the other floors of 50%, 60%, and 70% of the expected traffic, the corresponding traffic threshold curves involve only a few years in the beginning of the concession term.

Figure 5 presents the threshold curves in the presence of 50% protection. In this case, the threshold curves exist for all symmetric combinations of floors and ceilings. However, as the floor becomes lower, abandonment becomes possible in the last years of the concession term. In addition, the threshold traffic values for the first few years become higher and the probability of abandonment increases, as expected.

When the abandonment option is considered (without minimum and maximum traffic level options), the @Risk® software calculates the expected project NPV as R$ 104.2 MM. Comparing
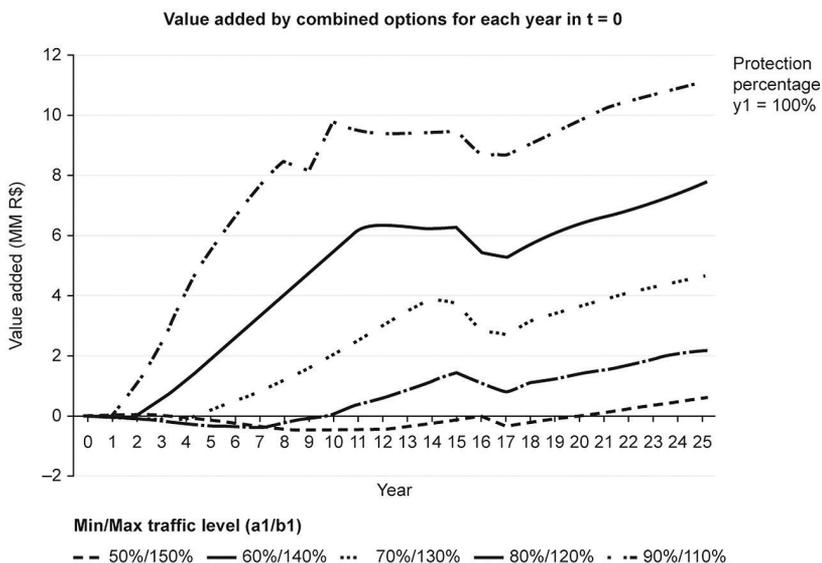

Figure 2. Value added by combined options for each year in t = 0.

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this value with the original expected NPV, the value added by the abandonment option is R$ 33.7 MM. When the minimum and maximum traffic level options are also included in the model, the options interact and their values change. For example, Table 4 presents comparative results for different symmetric options of minimum and maximum traffic with 100% of protection (y1=100%)°.

For higher levels of guaranteed traffic, such as 80% or 90% of the expected traffic, the abandonment option is insignificant because it is never exercised—as predicted in Figure 5. The value added by all options in those highest guarantee cases, with or without abandonment, is the same, reflecting that the abandonment option has no impact. As the guarantee decreases, the abandonment option
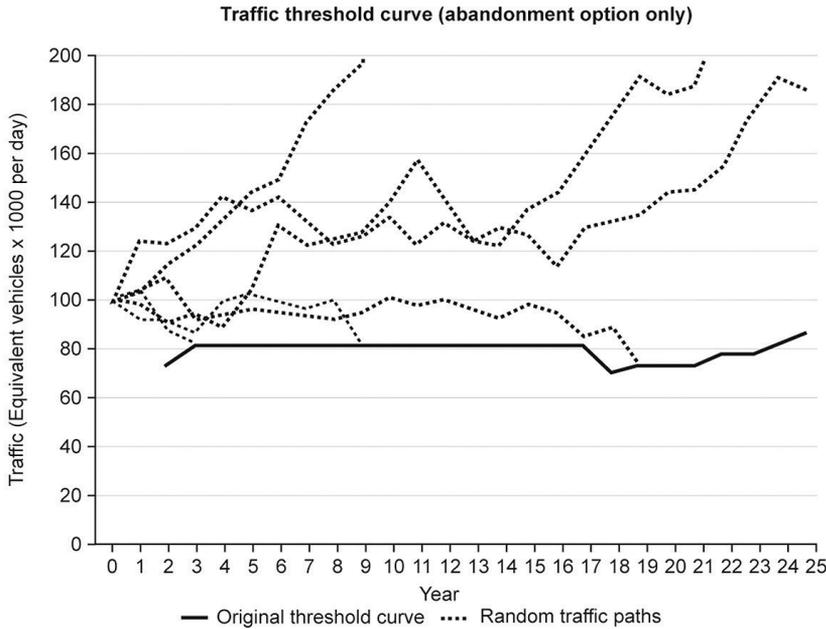

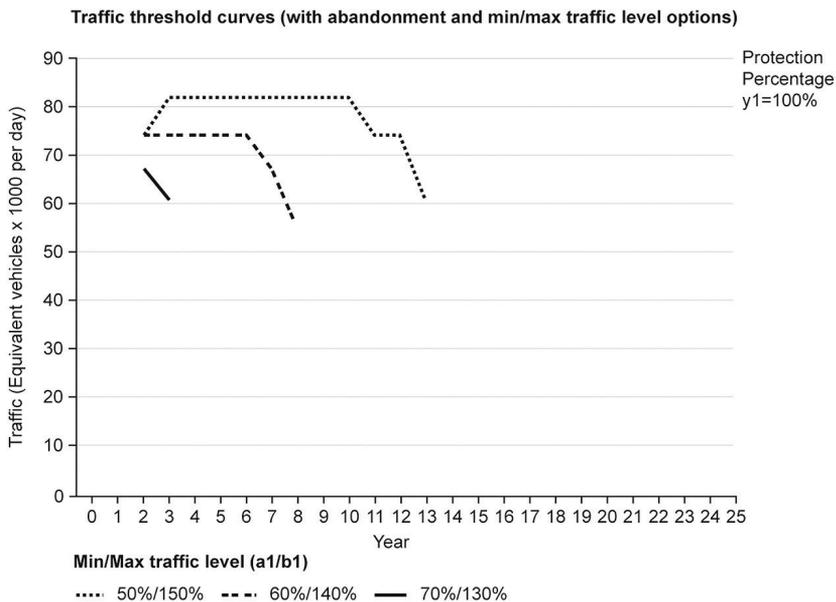Figure 3. Original traffic threshold curve.

Figure 4. Traffic threshold curves (with min/max traffic level options and 100% protection).

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Traffic threshold curves (with abandonment and min/max traffic level options)
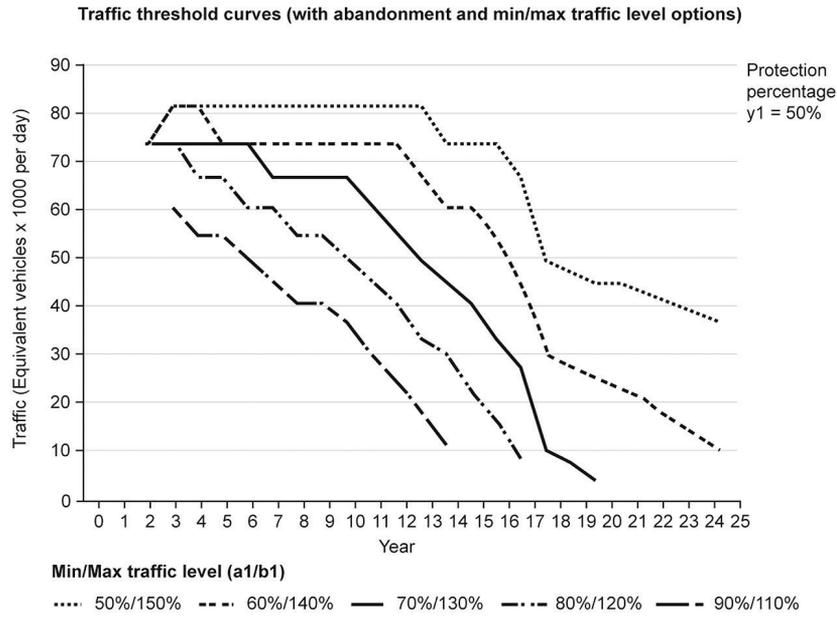

Figure 5. Traffic threshold curves (with min/max traffic level options and 50% protection).

Table 4. NPV and value added with and without abandonment option.

Min(%)/Max(%) traffic level (a_1/b_1) with 100% of protection ($y_{1=}$100%)	Without abandonment option (R$ 000)		With abandonment option (R$ 000)	
	NPV	Value added by all options	NPV	Value added by all options
50/150	66.987	−2.991	84.232	14.254
60/140	87.891	17.970	92.729	22.808
70/130	129.766	59.581	129.662	59.477
80/120	189.535	118.942	189.535	118.942
90/110	269.629	199.310	269.629	199.310

Probability of abandonment (cases with min/max traffic level options)

Figure 6. Probability of abandonment.

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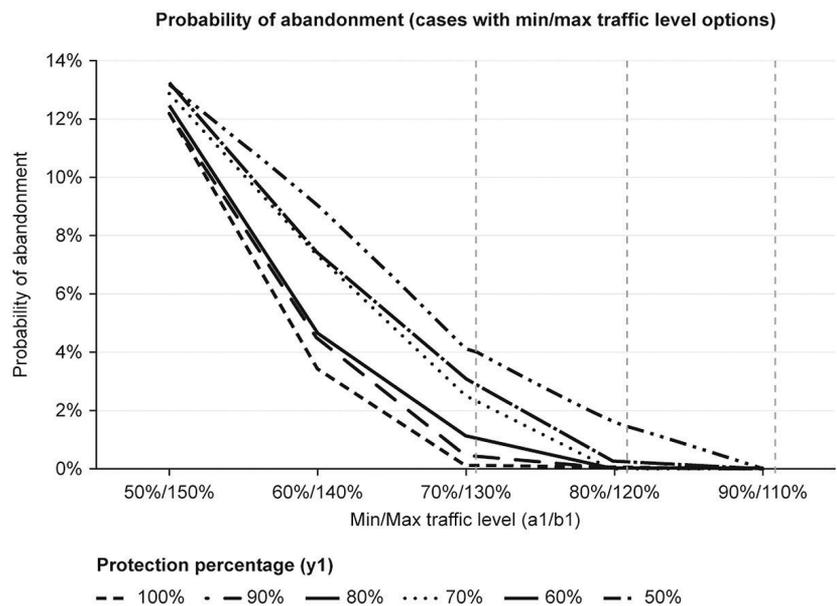
Average time of abandonment (cases with min/max traffic level options)
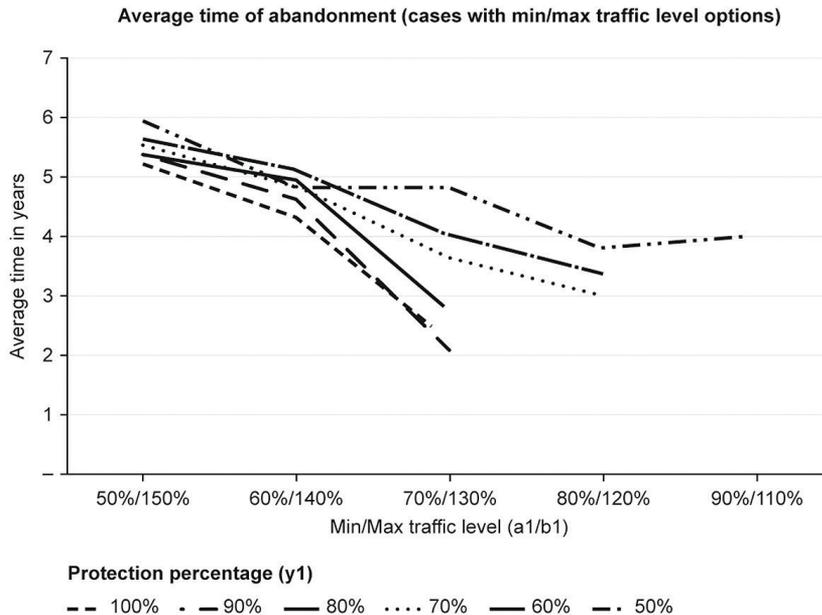

Figure 7. Average time of abandonment.

becomes more relevant, and the total value added by the existing options is higher.

However, when considered together with the abandonment option, the guarantee options are strategically important. From the government's viewpoint, it is possible to design a guarantee that minimizes the probability of abandonment and thus political and social problems. On the other hand, the guarantees lower the default risk for lenders. Thus, loan interest rates may be reduced and the project may become more attractive.

Based on the threshold curves, it is possible to calculate the probability of abandonment. In the original project, when only the implicit abandonment option is considered, the probability of abandonment is 14.93% and the average time is 7.22 years. Figures 6 and 7 present the results when the minimum and maximum traffic level options, respectively, are also considered. As the protection percentage increases, the probability of abandonment decreases for all guaranteed traffic levels. Considering the floor level from 70% to 90% (and the respective symmetric ceilings), the probability is much lower than the original 14.93% for all protection percentages analyzed. For example, for the symmetric combination of 70%/130%, the probability of abandonment ranges from 0.09% to 4.12%, depending on the protection percentage. For the 90%/110% case, it ranges from 0.00% to 0.06%.

The average time of abandonment occurs by the 6th year in all situations. Because abandonment is more likely in the initial years of the concession term, the government could review the traffic projections and limit the guarantees' payments. The lenders would be injured because of the default; this injury would prompt renegotiation.

5. Conclusions

In PPP agreements, the support mechanisms applied to public infrastructure projects in order to attract private capital can be very sophisticated. These mechanisms should be designed based on the embedded benefits and the risk exposure of the participants; correct valuation requires financial tools such as real options theory. By incorporating the options characteristics of different forms of support, or of the flexibilities identified in a project, these instruments can add value and mitigate and reallocate the risks involved. The risk to private investors can be reduced, making the project more attractive.

Considering transportation infrastructure projects, based on a hypothetical toll road concession, this study aimed to model traffic guarantees and an implicit right of abandonment under real options theory in order to analyze the impact and the interaction of such flexibilities. A more sophisticated combination of minimum traffic guarantee and

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maximum traffic ceiling level was proposed. To value the combination of those traffic guarantees, two methods were analyzed , and the simulation method was revealed to be more realistic than the analytical one. In the analytical method, the present payoffs' values are added directly to the original project's NPV (without options), and the effect of income tax is disregarded. However, the simulation method bypasses this problem.

The results show that the value added by considering different symmetric combinations of the minimum and maximum traffic levels may be negative during some years of the concession. A lower minimum guaranteed level and higher symmetric maximum traffic level result in a longer period of negative premiums. In the first several years, for most combinations, the maximum level options exceed the minimum level options. The total value added by the minimum and maximum traffic options to the expected NPV for the whole concession term can be very high or even negative depending on the minimum guaranteed traffic level and the corresponding maximum traffic level.

However, when considering the right of abandonment, which is implicit in project finance structures, it is necessary to value all options simultaneously. The concessionaire may decide to pay the debt service or abandon the project in each period. Using a combination of floor, ceiling, and protection percentages, the government can choose an optimal guarantee level. Three objectives should be considered: the concession should be attractive to private capital; the probability of abandonment may be limited to a desired level; and the government can minimize its risk exposure. Non-symmetric combinations of traffic floors and ceilings and their multiple levels may also be studied and compared. The relevance of the abandonment option depends on the level of the guarantees. Thus, the support mechanisms confer the additional benefit of minimizing the probability of abandonment. In this scenario, the government should additionally examine the guarantee option to minimize the probability of abandonment due to the social problems that abandonment may cause. Governments should choose an optimal combination of minimum and maximum traffic levels to avoid guarantees that are too high, to protect sponsors' expected returns, and to lower the probability of abandonment. In addition, when abandonment is considered, the guarantee becomes significant to lenders because it reduces the risk of default.

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Appendix

In Brazil, the relevance of PPP is related to the deteriorating infrastructure and scarcity of public resources for investment. In 1995, Brazilian Federal Law 8987 was created to broadly regulate public concessions, including infrastructure-based services. In 2004, Brazilian Federal Law 11079 defined a PPP as a supported concession that allows the government to grant monetary support to concessionaires. Individual states' legislation also regulates PPP locally.

The Fourth Line of the São Paulo Metro is the first example of PPP implementation in Brazil. A contract was signed in November 2006 with a consortium led by Companhia de Concessões Rodoviárias (CCR), a toll road company in Brazil and a major private toll road concession groups in Latin America. The project involves a 30-year concession to operate a 12.8 km stretch of subway in São Paulo, the largest city in Brazil. The investment by the consortium will be US$ 340 million.

The mechanism used to mitigate the demand risk in the abovementioned PPP is based on the minimum and maximum levels of demand. There is a range of demand without protection (up to $\pm 10\%$ of the projected demand). There are also two bands of protection (the first between $\pm 10\%$ and $\pm 20\%$ of the projected demand and the second after $\pm 20\%$ of the projected demand, limited to $\pm 40\%$ of the projected demand). Thus, there are two lower levels

(floors) and two upper levels (ceilings) for the traffic, which involve payments from the government to the concessionaire or from the concessionaire to the government.

Considering the same fee for all consumers, the mechanism can be described as follows. Let D_i be the real demand in period i, $\overline{D}_i$ the projected demand in period i, and p the tariff for the consumers.

- If the real demand lies between 90% and 110% of the projected demand, there will be neither subsidy nor taxation.

- If the real demand lies between 80% and 90% of the projected demand, the revenue will be adjusted by the following formula:

$$Md = \left[0.6(0.9\overline{D}_i - D_i) \right] p \qquad (A.1)$$

In this range, the government provides 60% protection. The revenue will be complemented by 60% of what it lacks for 90% of the projected demand.

- If the real demand lies below 80% of the projected demand, the revenue will be adjusted by the following formula:

$$Md = \left\{ 0.06\overline{D}_i + \left[0.9\left(0.8\overline{D}_i - D_i \right) \right] \right\} p \qquad (A.2)$$

In this range, the government provides 90% protection. The revenue will be complemented

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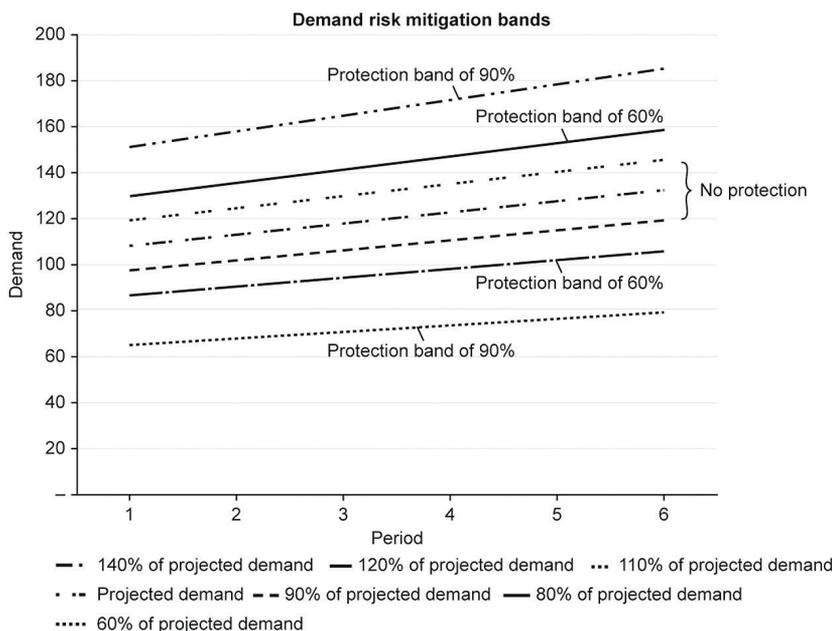Demand risk mitigation bands

Figure 1A. Demand risk mitigation bands.

by 90% of what it lacks for 80% of the projected demand, based on the previous level.

- If the real demand lies between 110% and 120% of the projected demand, the revenue will be adjusted by the following formula:

$$Md = -\left[0.6\left(D_i - 1.1\bar{D}_i\right)\right]p \qquad (A.3)$$

In this range, the concessionaire pays the government 60% of what exceeds 110% of the projected demand.

- If the real demand lies above 120% of the projected demand, the revenue will be adjusted by the following formula:

$$Md = -\left\{0.06\bar{D}_i + \left[0.9\left(D_i - 1.2\bar{D}_i\right)\right]\right\}p \qquad (A.4)$$

In this range, the concessionaire pays the government 90% of what exceeds 120% of the projected demand, based on the previous level.

- If the real demand lies below 60% or above 140% of the projected demand, the economic balance should be re-established.

Figure 1A represents the situation of a hypothetical demand.

Such conditions can be modeled as a composition of put and call options. If demand lies below 90% of the projected demand, the concessionaire has two puts that can be simultaneously exercised depending on the real demand. The payoffs in each period are as follows:

$$Put1: \ Md_1 = \left[0.6\max\left(0.9\bar{D}_i - D_i, 0\right)\right]p \qquad (A.5)$$

$$Put2: \ Md_2 = \left[0.3\max\left(0.8\bar{D}_i - D_i, 0\right)\right]p \qquad (A.6)$$

If demand lies above 110% of the projected demand, the government has two calls that can be simultaneously exercised depending on the real demand. The payoffs in each period are as follows:

$$Call1: \ Md_1 = -\left[0.6\max\left(D_i - 1.1\bar{D}_i, 0\right)\right]p \qquad (A.7)$$

$$Call2: \ Md_2 = -\left[0.3\max\left(D_i - 1.2\bar{D}_i, 0\right)\right]p \qquad (A.8)$$