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An RBC model of the Brazilian economy with stylized fiscal shocks*

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Contents

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Abstract · Resumo

This paper builds a real business cycle (RBC) model with government, aiming to replicate key features of the Brazilian economy. I first calibrate and then I use Bayesian methods to estimate the model for Brazil, with 20 years of quarterly aggregate data. Contrary to the conventional knowledge, I find mixed evidence on the pro-cyclicality of the fiscal policy in Brazil. Moreover, the results suggest that tax rate changes have been used to counter changes in the level of government indebtedness, however in a small degree if compared to other international evidence.

1. Introduction

In the last five years there has been a steady decline in fiscal positions of the national and a number of sub-national governments in Brazil. Major culprits are the large economic crisis that hit Brazil in 2015/2016, which slashed revenues, along with the extremely imbalanced public pension systems, that only worsens with the recent trend of population ageing. Additionally, the share of mandatory, non-discretionary, government expenditures keeps increasing, giving administrations less room for a contractionary policy. All of these factors combined are making debt sustainability a source of concern in Brazil. Even though most of the public debt is denominated in local currency, held by domestic residents, the issue is increasingly affecting the

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country's economic performance. With that scenario, understanding how fiscal policy has been managed in the last decades in Brazil is a key piece of information. This paper enhances that understanding, by using the most recent data available in order to estimate a fiscal real business cycle (RBC) model to Brazil.

RBC models are perhaps the most important tool in modern macroeconomics. The seminal work of Kydland and Prescott (1982) offered macroeconomists a framework based on rational expectations and strong microeconomic foundation, with a wide range of applications. From the early days until the mid-2000's the main approach was to calibrate parameters to levels observed in a certain economy, and then analyze whether the model was able to replicate major stylized facts. In case the replication failed a puzzle would be stablished. By tweaking the models, these puzzles ended up eventually being solved, typically with the introduction of nominal and real frictions (habit formation, capital utilization, investment adjustment costs, sticky prices, non-ricardian consumers, etc). The real business cycle framework begun to include a class of models that were no longer real, and it was rebranded a "dynamic stochastic general equilibrium" (DSGE) model. Models grew in size, and with the evolution of computational capacity, and more specifically, with the creation of softwares such as Dynare, frameworks with more than one hundred equations and variables became a common sight. That same evolution allowed RBC/DSGE models to be estimated, using computing-intensive Bayesian methods.

Some recent studies use the RBC framework to tackle the effects of different fiscal instruments on the economic cycle. Stähler and Thomas (2012) simulate the effects of different alternatives of fiscal consolidation. They show that the reduction of public sector wages is the least damaging option in terms of lost output and employment. Alternatively, slashing public investment is the most damaging one. Bhattarai and Trzeciakiewicz (2017) estimate a fiscal RBC model for the UK showing the effectiveness of different fiscal instruments in terms of their GDP multiplier.

This paper builds a small-sized RBC model for the Brazilian economy focused on the fiscal policy. I assume a closed economy, and a government that levies taxes, issues bonds and sells them to domestic residents, in order to pay for its expenses. There is no balanced budget, but the government follows a couple of fiscal rules that are consistent with long run solvency. Initially the model is just calibrated and its predictions are compared with the real data. The model's small size implies that it will not replicate with precision some characteristics of the economy. So the model will predict an investment that is a bit too volatile and a private consumption that is a bit too smooth. Because there is no monetary side in the model, predictions regarding the interest rate will be a bit inaccurate. On the other hand the model predicts relatively well the fiscal variables, such as government expenditures and the primary budget. It successfully predicts the government expenditures volatility relative to output of 1.3, as well as the highly volatile primary budget.

Using Bayesian methods, I estimate the model for Brazil using 20 years of quarterly aggregate data. The model predicts that the effects of technological and government expenditure shocks are highly persistent. The tax rate shocks, on the other hand are a lot less persistent. So the effect of a temporary tax rate shock fades away a lot faster if compared to the other two shocks. The model also predicts a slightly negative response of current government expenditures to lagged output, suggesting that, over this 20 year period, government attempts to react on a countercyclical fashion moderately prevailed over a well-known procyclicality of government expenditures. This result challenges the notion that developing economies are invariably bound to a course where government expenditures end up enhancing the cycle rather than mitigating it. Interestingly, however, when current instead of lagged output is considered, a positive and high correlation between output and government consumption emerges. The estimation also suggests that tax rate changes are used in Brazil to counter changes in the level of government indebtedness. But the magnitude of that response is relatively small if compared to similar estimates for other countries (for example Bi & Traum, 2014).

This paper is organized as follows. Section 2 presents some stylized facts of the Brazilian economy. Section 3 describes the proposed RBC model. Section 4 calibrates the model, and then estimates it. Some concluding remarks are then drawn in section 5.

2. Stylized facts

This section investigates some of the key stylized facts of the Brazilian economy, with specific interest on variables related to the fiscal policy. The idea is to confront these stylized facts with the predictions generated by the model. Appendix B highlights in more detail the data used in this section. We are particularly interested in how the fiscal variables relate with the cycle in Brazil.

Table 1 presents some characteristics of the economic cycle in Brazil. The standard deviation of output of 2.88 suggests a relatively volatile economy. It is two to three times more volatile than the US economy, for example. Consumption is almost as volatile as output. Interestingly, consumption more volatile than output is a well documented empirical regularity of developing economies (Neumeyer & Perri, 2005). A number of previous studies for Brazil have reported consumption being more volatile than output. For example, Ellery and Gomes (2005) use data from the Brazilian economy in the twentieth century to find a relative volatility of consumption to output of 1.1. That feature is typically related to credit constraints, in which some families behave as "rule-of-thumbs" that simply spend their current incomes at each point in time. The early 2000's witnessed a large expansion of household credit in Brazil, which may account for a reduction in consumption volatility, with a figure such as the one displayed in Table 1.

Table 1. Standard deviations, correlations with output and autocorrelations of macroeconomic variables in Brazil

	Standard Deviations		Correlation with Output	Autocorrelation
	%	Relative to Output		
Output	2.878	1.000	1.000	0.749
Consuption	2.870	0.997	0.877	0.755
Government Consumption	3.938	1.368	0.660	0.290
Investment	8.990	3.123	0.744	0.532
Hours	3.101	1.077	0.384	0.577
Primary Surplus	134.256	46.642	0.303	0.026
Nominal Interest Rates	18.850	6.549	-−0.328	0.699

Note: Based on quarterly data (1998:Q1 to 2018:Q1). All series were Hodrick-Prescott filtered and seasonally adjusted. Monetary values are expressed in Brazilian Reais of June 2018. National Account data comes from IBGE, the index for hours worked in the industry comes from CNI, the primary surplus data comes from the National Treasury Department, and the interest rates are the CDI/over.

Investment, government purchases and the primary surplus are 3.12, 1.37, and 46.6 times more volatile than output, respectively. The inability of using fiscal policy in a countercyclical fashion is another stylized fact of developing economies. Indeed, the correlation of government consumption with output of 0.66 is relatively high. However, the correlation of lagged output and government expenditures (not reported in Table 1) is low in Brazil, around 0.1. So, the procyclicality or close neutrality of government expenditures depends upon which variable they are being correlated with. The government budget is highly volatile but has a much smaller correlation with the cycle.[1] In this case, even procyclical expenditures may be partly neutralized by procyclical revenues.

Another feature of the Brazilian economic cycle reported in Table 1 is its small persistence. The autocorrelation of output is 0.75. To put it in perspective, in the US economy the output autocorrelation is around 0.9. That means that output and other macroeconomic variables in the Brazilian economy are less dependent upon their previous levels, and hence the effects of shocks tend to fade away much faster.

3. The model

I consider here a basic model of a closed economy with government. The key elements are preferences, technologies and resource constraints for the private and public agents. As a standard practice in this class of models, I also introduce some public

[1] The data on the primary surplus refers to the central government, while the data on government purchases, from the national accounts, refers to the entire government, which includes subnational governments.

finance rules. There is a representative consumer/producer household who derives utility from the consumption of a single good, and from leisure time. The production of this single good requires capital and labor through a standard Cobb-Douglass technology with an exogenous technological shock. A share τ of the total output is taxed, and finances the only government expenditure G. This public expenditure is assumed to be just a wasteful absorption of resources, not affecting the utility of consumption goods, nor the productivity of production factors. The government may run deficits, which are financed by issuing bonds that pay an interest rate r, and are purchased by the representative household.

3.1 Households

The representative household lives infinitely, and chooses consumption C_t and labor N_t to maximize his expected flow of lifetime utility

$$U = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{N_t^{1+\gamma}}{1+\gamma} \right), \tag{1}$$

where $0 < \beta < 1$ is the time discount factor. He is restricted by the following budget constraint:

$$B_{t+1} = (1 + r_t)B_t - C_t - I_t + (1 - \tau_t)Y_t\,, \tag{2}$$

where τ_t is the tax rate on income (output) Y_t, and B_t is the amount of public bonds held by the household. Since the model does not have money or prices, everything is measured in terms of units of the single representative good.

The household is also the producer of the good according to a standard Cobb-Douglass technology:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{3}$$

where A_t is the exogenous technology level, and K_t, the capital stock, which is a predetermined variable that depreciates at the rate δ and evolves according to a standard capital accumulation law $K_{t+1} = (1 - \delta)K_t + I_t$. So, the representative household also has to choose the current amount of investment that will define next period's capital stock.

Technology is assumed to have an autoregressive component, as well as an iid random shock:

$$\widehat{A}_{t+1} = \rho^A \widehat{A}_t + \varepsilon_{t+1}^A \quad \text{with} \quad \varepsilon_t^A \sim \mathcal{N}\!\left(0, \sigma_A^2\right), \tag{4}$$

where the hat denotes percentage deviations from steady-state levels.

3.2 Government

The government consumes part of the production of the representative good in the amount of G_t, and taxes income at the rate of τ_t. Government consumption does not increase the representative individual's utility nor the productivity of inputs.[2] The primary budget, $\tau_t Y_t - G_t$, excludes debt services, and is likely to be imbalanced at each point in time. Eventual deficits or surpluses imply that the government is either issuing more bonds or buying them back, such that the government intertemporal budget constraint is always satisfied:

$$B_t = (1 + r_t)B_t + G_t - \tau_t Y_t. \tag{5}$$

Equations (2) and (5) together define the overall resource constraint of the closed economy, namely

$$Y_t = C_t + I_t + G_t. \tag{6}$$

The government also abides by two fiscal rules. The first one stablishes that the government adjusts the tax rate to keep debt sustainability:

$$\hat{\tau}_{t+1} = \rho^\tau \hat{\tau}_t + \xi^\tau \left(\widehat{B}_t - \widehat{Y}_t\right) + \varepsilon^\tau_{t+1} \quad \text{with} \quad \xi^\tau > 0;\ \varepsilon^\tau_t \sim \mathcal{N}\left(0, \sigma^2_\tau\right), \tag{7}$$

where the hat denotes percentage deviations from the steady-state levels. Parameter ξ^τ captures the response of taxes to lagged debt. It is one of the key parameters in the Bayesian estimation of in section 4. It's magnitude relates to fiscal responsibility. If it is close to zero (or negative) than it indicates a lack of governments commitment to fix fiscal imbalances. If ξ^τ is high, on the other hand, then it indicates a better, more responsible fiscal behavior.

The second fiscal rule allows the model to test for Brazil a particular characteristic of the cycle in developing economies, which is a procyclical fiscal policy (e.g., Gavin & Perotti, 1997, and Bi, Shen, & Yang, 2016). The rule is given by

$$\widehat{G}_{t+1} = \rho^g \widehat{G}_t + \xi^g \widehat{Y}_t + \varepsilon^g_{t+1} \quad \text{with} \quad \varepsilon^g_t \sim \mathcal{N}\left(0, \sigma^2_g\right). \tag{8}$$

The government's primary surplus, $\tau_t Y_t - G_t$ has its three elements constantly affected by random shocks. Technology shocks in the case of output, and the two shocks included in the fiscal rules in the case of τ_t and G_t. The variation in the debt level B_t in equation (5) is determined residually, in order to satisfy the government's budget constraint. At each point in time there is a specific level of debt that balances that intertemporal budget constraint. The question then is what mechanism in the model makes that level of debt the same level the household is willing to hold, such that the market for bonds clears at each point in time (assuming no other foreign or domestic agent can buy public bonds)? The mechanism is instant adjustments in the attractiveness of those bonds, captured by the interest rate r_t.

[2]Even if individuals care for G_t, their optimal intertemporal choices would not be changed if G_t is additive in the utility function. In this case, for simplicity, models typically omit the G_t term.

3.3 Steady-state equilibrium

The model's steady-state is the point where all variables are constant. I assume that this point has a positive level of government debt ($\overline{B} > 0$ where the upper bar denotes steady-state values). The implication is that in steady-state the primary budget must attain a particular level of surplus just enough to service the debt, and keep it constant.

A few further assumptions are required to fully characterize the steady-state. Specifically, I need to choose the equilibrium values of the tax rate variable, and the debt/GDP ratio. The values chosen are $\overline{\tau} = 0.32$, and $s_B = \overline{B}/\overline{Y} = 2$. So, in steady-state the government taxes 32 % of income/production, and the debt is 50 % of annual GDP (or 2 quarters of GDP).

4. Calibration and estimation

This section presents the main predictions of the model under two different approaches. First I calibrate the model parameters to the Brazilian economy. Second, I estimate the parameters using Bayesian techniques.

4.1 Calibration

The model is calibrated here in order to reproduce the key elements of the Brazilian economy. Parameter values are set in a way to be consistent with recent macroeconomic trends observed in Brazil. I also set some values based on other similar models in the literature. Table 2 summarizes the parameter values used for calibration. The discount factor β is set at 0.988, consistent with a quarterly interest rate of 1.2 % (4.95 % per year), which is very close to the recent historical average of real interest rates practiced in the Brazilian economy.

Capital is a lot more expensive and labor is a lot cheaper in Brazil, as compared to developed economies. The usual division of aggregate income of 30 % accruing to capital and 70 % to labor is typically not valid in developing economies, where a 50 %–50 % split seems more plausible. I use a value of 0.448 for the capital share, which is parameter α in the model. The capital depreciation parameter δ is set at 0.025, representing an annual depreciation of capital stock slightly lower than 10 %. The parameter γ, which captures the inverse of the labor supply elasticity is set to zero. When γ is set to zero, then we are in Hansen's (1985) set up of labor indivisibility. A small γ means a more indivisible labor supply, which is a reasonable assumption. Brazil's labor market is known for being a lot less flexible than, say, the US labor market. That includes a more rigid structure of labor hours.

Brazil's tax burden ranges around 32 % of GDP. So, the steady-state tax rate $\overline{\tau}$ is set to 0.32. Likewise, the steady-state debt/GDP ratio is set to 2. By that means, it would take two quarters of GDP to fully pay the public debt, or roughly half the

Table 2. Calibrated parameters

β	discount factor	0.988
α	capital income share	0.448
δ	capital depreciation rate	0.025
γ	inverse of the labor supply elasticity	0
τ	steady-state tax rate	0.32
s_b	steady-state debt/GDP ratio	2
ρ^a	AR(1) coefficient of A_t	0.84
ρ^g	AR(1) coefficient of gov. purchases	0.47
ρ^τ	AR(1) coefficient of the tax rate	0.8
ξ^τ	tax rate response to gov. indebtedness	0.047
ξ^g	gov. spending response to lagged output	0.12
σ^a	standard deviation of ϵ^a	0.608
σ^g	standard deviation of ϵ^g	3,421
σ^τ	standard deviation of ϵ^τ	1,581

annual GDP. The gross debt / annual GDP ratio remained around 50 % for a long period of time in Brazil. So I picked that to represent a steady-state level. That ratio has gone up to 75 % in recent years amidst a serious fiscal crisis, but it can hardly be considered a long-term equilibrium level. The parameters ρ^a, ρ^g , and ρ^τ capture the degree of persistence of technology, government consumption and taxation. The later two are the fiscal policy instruments in the model. The values for these parameters are 0.84, 0.47, and 0.8, respectively. They are taken from Bi et al. (2016). Also from the same author, I take the tax rate response to government indebtedness, ξ^τ, and the government spending response to lagged output, ξ^g, which are set to 0.047 and 0.12, respectively.

With the aforementioned parameter values, steady-state ratios are very consistent with the Brazilian economy. Private consumption, investment and government consumption shares of income are 0.5, 0.2, and 0.3, approximately. The capital/output ratio is 8.2 in steady state.

The standard deviations of shocks were set up on the following way. First, the standard deviation of the technology shock, σ^a, was set in order to match output volatility with that observed in Brazilian data. With only a technological shock, the standard deviation of government purchases given by the model is several times lower than the volatility observed in the data.[3] Then naturally, a key feature of the introduction of government purchase shocks in the model is to increase the volatility of that variable. Hence, σ^g was increased until the volatility of government purchases within the model would roughly match the data (readjusting σ^a down in the process). The introduction of tax revenue shocks help to bring the volatility of hours and the primary surplus closer to what is observed in the data. Therefore, σ^τ was set in order to improve the volatility of these two variables up to a point where it would not disrupt the relative volatilities found before the introduction of the tax shock, and that already were reasonable approximations with the data.

The main theoretical predictions of the model are summarized in Table 3. It presents the standard deviation of a set of macroeconomic variables, their correlation with output, and autocorrelation. The model is driven by technology shocks, government expenditure shocks, and tax rate shocks. Some features of the Brazilian economy are successfully reproduced by the model. Consumption is smoother and investment is more volatile than output. However, the model predicts a consumption a bit too smooth (0.76 of output's volatility against 0.99 in the data) and an investment a bit too volatile (3.83 of output's volatility against 3.12 in the data). That is a well-known feature of this class of models, and can usually be corrected with the introduction of capital adjustment costs (see, for example Groth & Khan, 2010).

By gauging the magnitudes of the two fiscal shocks, I manage to get relative volatilities of government consumption and labor hours that are fairly consistent with the data. Government purchases are about 37 % more volatile than output, just as in the data. Working hours are just about as volatile as output, both in the model

Table 3. Standard deviations, correlations with output and autocorrelations of macroeconomic variables in Brazil

	Standard Deviations		Correlation with Output	Autocorrelation
	%	Relative to Output		
Output	2.863	1.000	1.000	0.883
Consuption	2.179	0.761	0.712	0.990
Government Consumption	3,934	1.374	0.189	0.487
Investment	10,976	3.834	0.827	0.758
Hours	2,911	1.017	0.743	0.807
Primary Surplus	53,369	18.641	0.082	0.573
Nominal Interest Rates	106.5	37.192	−0.019	0.009

Note: Based on the model with $\alpha = 0.448$, $\beta = 0.988$, $\delta = 0.025$, $\rho^a = 0.84$, $\rho^g = 0.47$, $\rho^t = 0.8$, $\gamma = 0.0001$, $\xi^g = 0.12$, $\xi^\tau = 0.047$.

[3]It is 3.9 in the data against 0.6 in the model, with a standard deviation of the technology shock set to 0.728.

and in the data. The model also correctly predicts an extremely volatile government primary budget, about 19 times more volatile than output (46 times in the data). In addition, the model successfully predicts procyclical consumption, investment, and government purchases. The later, however, has a higher correlation with output in the data than in the model. (0.66 against 0.19). This is so because in the model most of the driving mechanism behind fluctuations in the government consumption is in its own shock, rather than in fluctuations coming from output. That explains the correlation mismatch. In fact, when the model is driven only by technology shocks, it generates a government consumption that is about three times less volatile than in the data. The introduction of a government consumption shock with a relatively large standard deviation fixes that, but to the expense of having a larger portion of the movements in government consumption uncorrelated with the cycle. The model also reports a positive correlation between hours and output, but higher than the one observed in the data (0.74 against 0.38).[4]

The model predicts a primary surplus and an interest rate that are roughly uncorrelated with the cycle. In the data, however, we observe the primary surplus being slightly procyclical and the interest rate being slightly countercyclical (correlations with output of 0.3 and −0.32, respectively). The model predicts a higher persistence in most variables than what is observed in the data, except for the nominal interest rates.

The model's impulse responses to a one time technology shock with the magnitude of one standard deviation are presented in Figure 1. There is an instantaneous jump in output, and a slow decay afterwards, due to the autoregressive component of the technology A_t. The increase in productivity leads to increases in the use of both inputs, labor hours and investment. There is a jump in consumption followed by a well-known hump-shaped trajectory. The pro-cyclical government expenditure rises with lagged output. The interest rate trajectory is governed by the dynamics embedded in the Euler equation (19). Government indebtedness is reduced with a fall in the level of the debt and in the debt/GDP ratio, which in turn reduces the tax burden, given the tax fiscal rule (7). There is also a jump in the primary surplus, which tells us that the increase in output and in revenues more than compensates the decrease in the tax rate and increase in government purchases.

Figure 2 depicts the impulse responses to a one standard deviation shock in government purchases. As government expenditures go up, the primary surplus falls, and the government debt grows. Private investment is crowded out, and falls, along with private consumption. With higher government expenditures a ricardian household expects higher taxes in the future, and therefore drops his instantaneous consumption in order to seek a smooth future consumption path. To compensate for an expected lower disposable income, the same household is willing to increase

[4]That is a well-known feature of these classes of models, related to what is known as the productivity-hours puzzle.

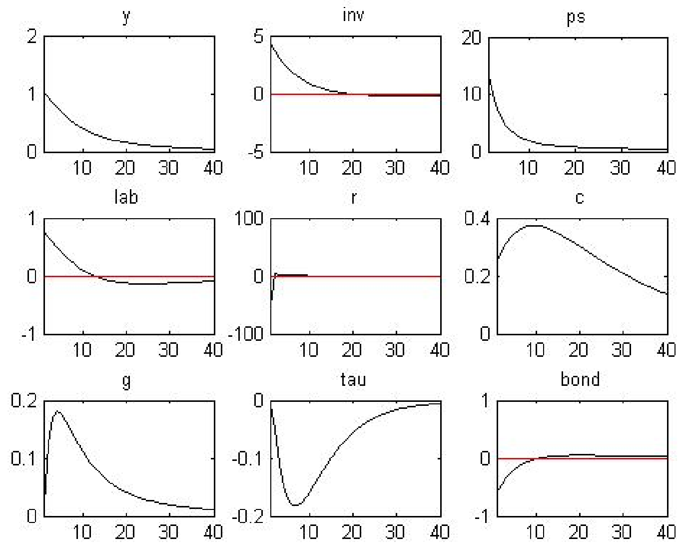

Figure 1. Impulse responses to a shock in Technology

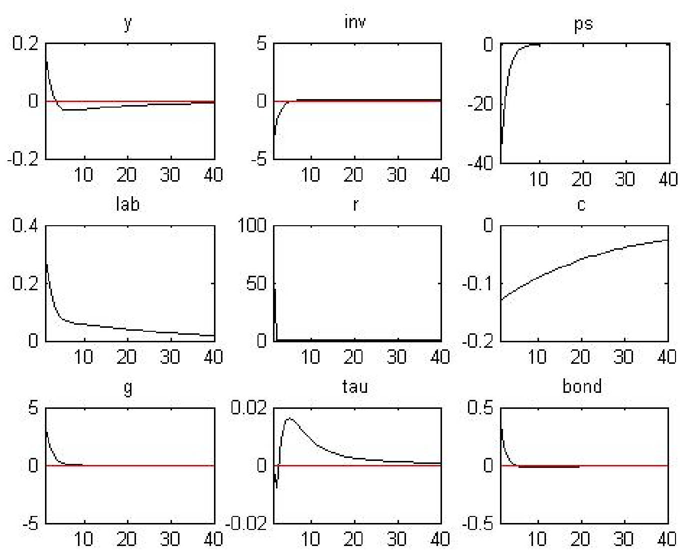

Figure 2. Impulse responses to a shock in Government Purchases

his labor supply, along the lines of the jump observed in labor hours in Figure 2. The path of the debt/GDP ratio defines the path of the tax rate, which initially falls because of an instantaneous jump of GDP that is more intense than the one of the debt. But as the GDP convergence is more sluggish than the debt, the tax rate rebounds, and exhibits a hump shaped path on the positive quadrant.

The impulse responses of a one standard deviation shock in the tax rate are presented in Figure 3. An increase in the tax rate causes a jump in the primary surplus, and a drop in the debt. A higher tax rate implies a lower disposable income. That means less incentive to produce and to consume. It shifts the relative price of consumption vis-à-vis leisure, and the household ends up dropping consumption and labor supply in exchange for more leisure time. Less output requires less capital, which leads to a drop in investment. The fall in output is followed by a fall in government purchases, given its pro-cyclical characteristic, defined by the fiscal rule (8). That can be seen as an upside down hump shape in Figure 3.

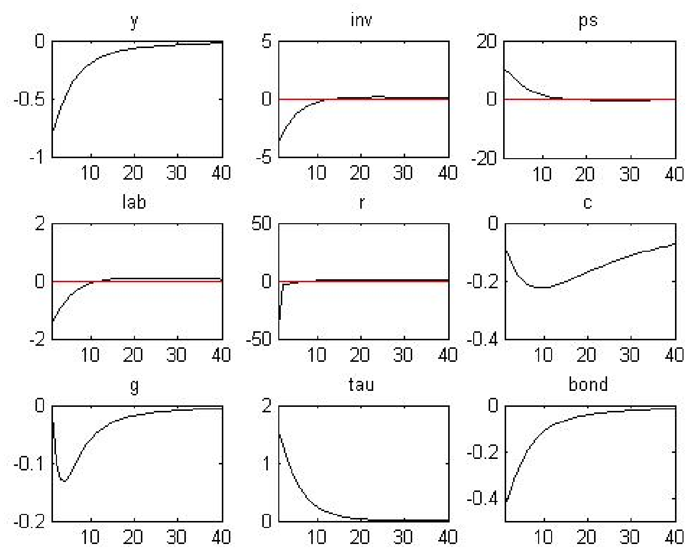

Figure 3. Impulse responses to a shock in the Tax Burden

4.2 Parameter Estimates

This section presents the estimation of the fiscal RBC model outlined in section 3. The Bayesian maximum likelihood method is used to estimate a vector θ of parameters in two steps. First I specify prior distributions for the parameters. Second, I combine these prior distributions with the likelihood function of the model to obtain the posterior distributions. The product of these two terms, the prior and the likelihood function, stablishes the posterior distribution $\pi(\theta|y_T)\alpha f(y_T \vee \theta)\pi(\theta)$, as defined by the Bayes Theorem. As the data keeps being updated new priors are formed, with the property of convergence to the true distribution $\pi(\theta)$.[5]

The model is estimated for Brazil, through the period 1998:Q1 / 2018:Q1. The estimation is based in three observables, namely, GDP, aggregate investment, and government's primary budget. For each observable I take the percentage deviations from its sample mean. Then I use simple ordinary least squares regressions to detrend and to remove seasonal effects. Figure 4 presents the transformed data used for the Bayesian estimation. The raw data is the same described in Appendix B, and used to construct Table 1 in section 2.[6]

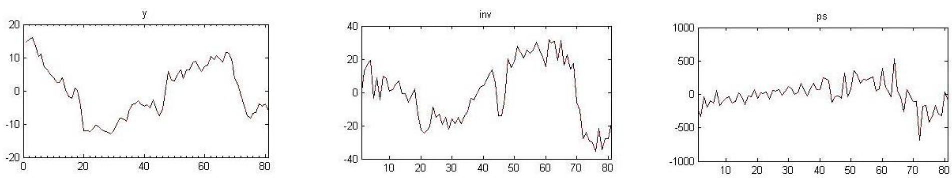

Figure 4. Data used in estimation (transformed)

4.3 Prior and posterior distributions

There are eight parameters of the model suited for estimation, namely, the autoregressive parameter in equation (4), the two parameters of fiscal rule (7), the two parameters of fiscal rule (8), and the standard deviation of the shock terms of equations (4), (7) and (8). Some parameters in DSGE models are known for being difficult to estimate. For those, the calibrated values presented in Table 2 are considered.

The prior distributions have been chosen in a way to preserve theoretical restrictions on parameter values. The range of values for a certain parameter must lie well within the prior support. So, for example, gamma and inverse gamma distributions are assigned to parameters that have to be positive. The beta distribution is used as a prior of parameters with values that presumably are within the interval [0, 1]. As for the means and standard deviations of priors I use values that have been suggested by other authors, such as Bi et al. (2016).

The results of the estimation are presented in Table 4, with the means of the posterior distributions and the 90 % confidence intervals, along with the priors. Beginning with the autoregressive terms, the estimation of the model suggests a high persistence of technology and government expenditure shocks (0.99 and

[5]This is done through a Monte Carlo Markov Chain (MCMC) method with the Metropolis–Hastings algorithm, as is the standard practice for this class of models.

[6]The Metropolis–Hastings algorithm was set up to run a total of 100,000 draws. The acceptance rate was 33 %. The multivariate diagnostic indicated convergence.

Table 4. Bayesian estimation of the parameters

	Prior			Posterior		
Parameter	Distribution	Mean	Std. Dev.	Mean	Confidence Interval	
ρ^{α}	Beta	0.5	0.25	0.9923	0.9852	0.9995
ρ^{g}	Beta	0.5	0.25	0.9409	0.9101	0.9723
ρ^{τ}	Beta	0.5	0.25	0.1376	0.0035	0.2601
σ_{ϵ}^{a}	Inverse Gamma	1.0	inf	5.4596	4.7519	6.1588
σ_{ϵ}^{g}	Inverse Gamma	1.0	inf	10.0837	8.1529	11.8917
σ_{ϵ}^{τ}	Inverse Gamma	1.0	inf	4.7172	3.9056	5.5154
ξ^{g}	Normal	0.1	0.05	−0.0246	−0.0719	0.0244
ξ^{τ}	Gamma	0.03	0.05	0.0201	0.0101	0.0296

0.94, respectively), and a low persistence of tax rate shocks, with an autoregressive parameter of only 0.14. Figure 5 in Appendix C depicts the posterior distributions sided by their respective priors. The posterior distribution is very tight for the technology autoregressive parameter, a bit less tight for the government expenditure parameter, and relatively loose for the tax rate parameter.

As for the effect of economic activity on government expenditure (parameter ξ^{g}), our estimation suggests a rather unexpected result, with a negative posterior mean of −0.025. The negative value contradicts the widespread notion that developing economies don't have the ability to perform the much desired countercyclical fiscal policy. That is due to the fact that during economic crisis government solvency is usually put to test, with falls in output and tax revenues and increases in the debt/GDP ratio. A countercyclical expenditure increase is usually not feasible under those circumstances. That holds true for the most part of Brazilian economic history. However, within the time span of the data used for estimation, there was one distinctive episode in which Brazilian policymakers managed to successfully adopt a strong countercyclical fiscal policy. That was in the aftermath of the 2008 US subprime crisis. Back then, the Brazilian government had sound fiscal indicators, which afforded the adoption of that policy. However, a very different scenario was in place when the 2015/2016 crisis erupted. The government had no choice other than slashing expenditures. The combination of these different episodes of countercyclical and procyclical government expenditures is probably behind an estimate of ξ^{g} very close to zero. To put that number in perspective, the same parameter was estimated for the Greek economy by Bi and Traum (2014). These authors found posterior medians for ξ^g in the vicinity of 1.20.[7]

The posterior mean for the parameter ξ^τ is 0.02. The positive signal is expected, so that the government increases the tax rate when the public debt grows as a share of GDP. However, the small magnitude indicates a low responsiveness. Bi and Traum (2014) estimate a similar parameter for the Greek economy, relating the tax rate responsiveness to government indebtedness. They found a posterior median of 0.22. The lower estimate for the Brazilian economy might suggest that its governments are less willing, or less able to respond to a higher debt by raising the tax rate.

5. Conclusion

This paper builds a stylized DSGE model for the Brazilian economy that focuses on fiscal policy. It is a closed economy, with a government that finances its consumption by levying taxes and by issuing bonds, and follows two fiscal rules that are consistent with long run solvency.

The model is first calibrated to the Brazilian economy, successfully replicating some of its characteristics. Its predictions match relatively well the data in what refers to volatilities. So, consumption is smoother than output (unlike most developing economies), investment is more than three times more volatile than output, labor hours is about just as volatile as output, government expenditures are about 1.3 times as volatile as output, and the government primary budget is extremely fickle. Its proportional standard deviation is 46 times higher than output's (although the model predicts that figure to be only 18 times the equivalent value for output). Given the simplicity of the model it fails to replicate some other features, most notably related to the interest rate variable.

Using Bayesian methods, I estimate the model for Brazil. The model predicts that the effects of technological and government expenditure shocks are highly persistent, in the sense that their effects dissipate very slowly. The effects of tax rate shocks, on the other hand are a lot less persistent. So the effect of a temporary tax rate shock fades away a lot faster. The model also predicts a fiscal policy that is slightly countercyclical in Brazil. That prediction is based on a fiscal rule that relates current government expenditure with lagged output. Indeed when we look to the data, the correlation between these two variables is rather weak, close to zero. Nevertheless, that contrasts with a large positive correlation observed between current output and government expenditures, which points toward a procyclical fiscal policy. Additionally, the estimation suggests that tax rate changes are used in

[7] They did not report the mean, but since they use a gamma distribution as a prior, which is skewed to the right, and since the reported posterior roughly resembles the prior, the mean tends to be larger than the median.

Brazil to counter changes in the level of government indebtedness. However the magnitude of that response is relatively small if compared to a similar estimates for other economies.

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Appendix A.

This appendix analyzes the model's dynamics in more details. The representative household chooses how much to consume C_t; how many hours to work N_t; how much to invest I_t, which defines the capital stock K_{t+1}; and how many bonds to buy today B_t, which defines next period bond holdings B_{t+1}. The four first-order optimality conditions are:

$$\frac{1}{C_t} = \mu_t \,, \tag{9}$$

$$N_t^{\gamma} = (1-\tau_t)(1-\alpha)A_t K_t^{\alpha} N_t^{-\alpha}\mu_t \,, \tag{10}$$

$$\mu_t = \mathbb{E}_t\, \beta\mu_{t+1}\left[(1-\tau_t)A_{t+1}\alpha K_{t+1}^{\alpha-1}N_{t+1}^{1-\alpha} + 1 - \delta\right], \tag{11}$$

$$\mu_t = \mathbb{E}_t\, \beta\mu_{t+1}\left(1 + r_{t+1}\right), \tag{12}$$

where μ_t denotes the budget constraint's shadow price. Since the model is mostly non-linear, instead of pursuing an analytical solution, the standard procedure in these types of models is to linearize it, and try numerical solutions. In steady-state, equations (11), (12), and the government budget constraint (5) become:

$$\frac{\overline{K}}{\overline{Y}} = \frac{\alpha\beta\left(1-\overline{\tau}\right)}{1-\beta(1-\delta)}, \tag{13}$$

$$\overline{r} = \frac{1-\beta}{\beta}, \tag{14}$$

$$s_g = \overline{\tau} - \overline{r} s_B \,, \tag{15}$$

where the upper bar denotes steady-state values and s_g and s_B denote the share of public consumption on output and the debt/output ratio, respectively. The steady-state equilibrium is fully characterized with the law of motion of capital and the resource constraint that become $s_i = \overline{K}/\overline{Y}$ and $s_c = 1 - s_i - s_c$, where s_i and s_c are the share of investment and private consumption on output, respectively. We calibrate $\overline{\tau}$ and s_B to the Brazilian economy in order to get a reasonable steady-state, in line with historical data.

The log-linearized version of the model is given by the four optimality conditions:

$$\widehat{C}_t = -\widehat{\mu}_t \,, \tag{16}$$

$$\widehat{N}_t(\gamma+\alpha) = \widehat{A}_t + \alpha\widehat{K}_t - \widehat{C}_t - \frac{\overline{\tau}}{1-\overline{\tau}}\hat{\tau}_t \,, \tag{17}$$

$$\widehat{\mu}_t = \widehat{\mu}_{t+1} + \eta_A\widehat{A}_{t+1} + \eta_K\widehat{K}_{t+1} + \eta_N\widehat{N}_{t+1} - \eta_\tau\hat{\tau}_{t+1} \tag{18}$$

$$\widehat{\mu}_t = \widehat{\mu}_{t+1} + \frac{\overline{r}}{1+\overline{r}}\hat{r}_{t+1} \,. \tag{19}$$

The technology and the law of motion for the capital stock

$$\widehat{Y}_t = \widehat{A}_t + \alpha \widehat{K}_t + (1-\alpha)\widehat{N}_t\,, \tag{20}$$

$$\hat{I}_t = \frac{1}{\delta}\widehat{K}_{t+1} - \frac{1-\delta}{\delta}\widehat{K}_t\,. \tag{21}$$

The resource constraint, the government budget constraint and the primary surplus equation

$$\widehat{Y}_t = s_C\widehat{C}_t + s_I\hat{I}_t + s_g\widehat{G}_t\,, \tag{22}$$

$$\frac{-\widehat{B}_{t+1}}{\bar{r}} + \frac{1+\bar{r}}{\bar{r}}\widehat{B}_t + \hat{r}_t = \frac{-\bar{\tau}}{\bar{r}s_B}\hat{\tau}_t - \frac{\bar{\tau}}{\bar{r}s_B}\widehat{Y}_t + \frac{s_g}{\bar{r}s_B}\widehat{G}_t\,, \tag{23}$$

$$\widehat{PS}_t = \frac{\bar{\tau}}{\bar{r}s_B}\hat{\tau}_t + \frac{\bar{\tau}}{\bar{r}s_B}\widehat{Y}_t - \frac{s_g}{\bar{r}s_B}\widehat{G}_t\,, \tag{24}$$

where $\eta_A = 1-\beta(1-\delta)$; $\eta_K = (\alpha-1)\eta_A$; $\eta_N = -\eta_K$; and $\eta_\tau = \frac{-\bar{\tau}}{1-\bar{\tau}}\eta_A$. There are twelve equations—nine equations, (16)–(24), along with the three exogenous rules (4), (7) and (8) that define the dynamics of twelve variables, namely consumption, labor, technology, investment, capital stock, output, debt, tax rate, government consumption, interest rate, primary surplus, and the lagrangean multiplier.

Appendix B.

This appendix is about the data used in the paper. I use data over the period 1998Q1–2018Q1 on seven macroeconomic variables: output, consumption, investment, government purchases, government primary surplus, total hours worked, and nominal interest rates. The national account data on GDP, aggregate consumption, aggregate investment and government consumption are obtained from the Brazilian Institute of Geography and Statistics (IBGE). Investment is the gross fixed capital formation plus change in inventories. The data is in millions of Brazilian Reais of June 2018, deflated using the IGP-M index, which is a general price index (retail and wholesale) computed by Getulio Vargas Foundation (FGV). The primary surplus series for the central government ("above the line" concept; excludes states and municipalities) is obtained from the Brazilian national treasury department. It is originally a monthly series, which I deflated using the IGP-M index and then just added the values of every three months to define quarterly values. For the total hours worked I took an index from the National Industry Confederation (CNI) of hours worked in the industry. It is a monthly index that was transformed to a quarterly index just by considering the observation of the last month of each quarter. For the nominal interest rate I use the series of CDI/over rates, available, for instance, at the Brazilian Central Bank website. This is a good proxy of the actual rates that are in effect in the Brazilian economy. Again, it is originally a monthly series, and in this case I transform it into a quarterly series by taking a simple average of the monthly rates for every quarter.

To eliminate trends all the series are Hodrick-Prescott filtered (with $\lambda = 1600$), and only the cyclical component is taken. To obtain the proportional standard deviations I take the standard deviation of the HP-filtered series and divide it by the mean of the unfiltered series.

Appendix C.

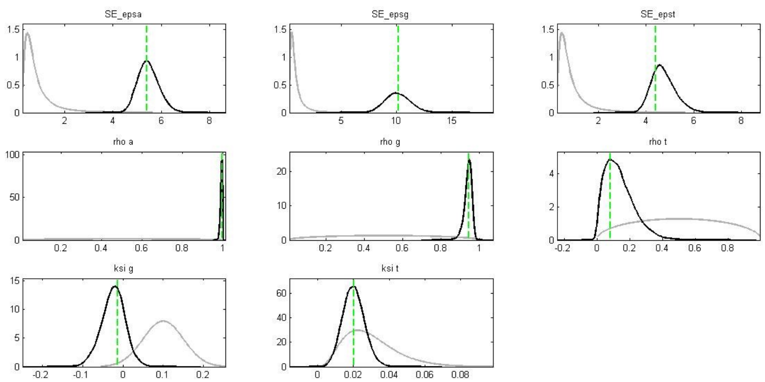

Figure 5. Prior and posterior distributions of the parameters

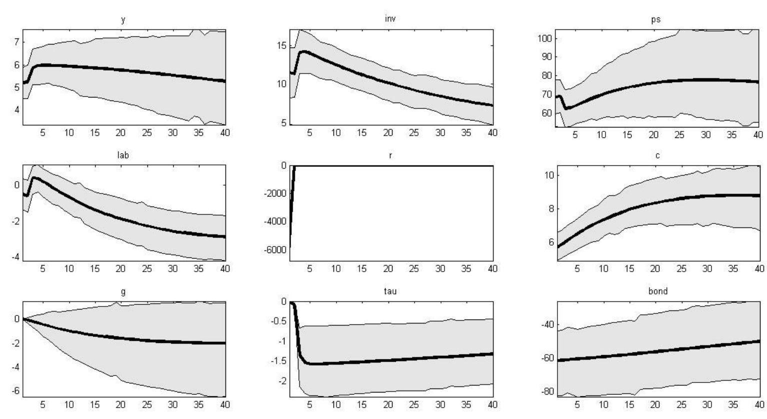

Figure 6. Bayesian impulse responses to a technology shock

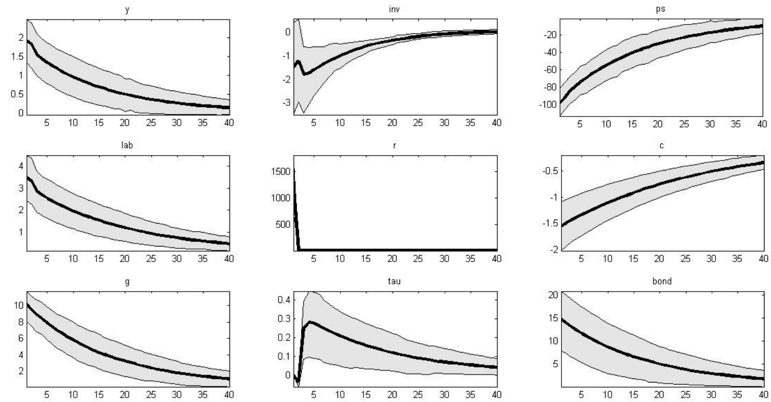

Figure 7. Bayesian impulse responses to a government expenditure shock

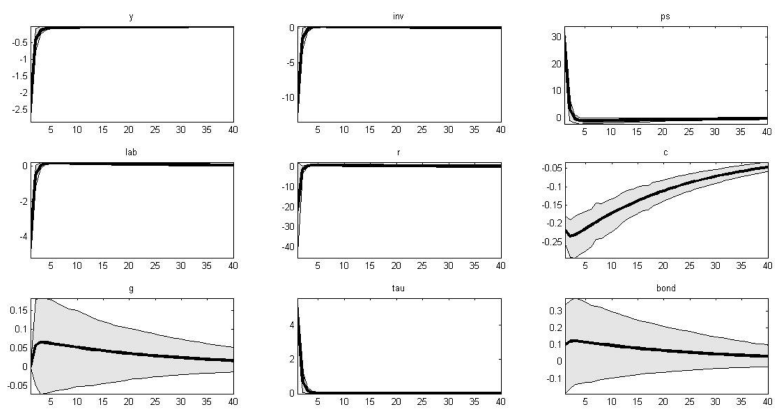

Figure 8. Bayesian impulse responses to a tax rate shock