

The Naive Central Banker

Marcelo de Carvalho Griebeler*

Contents: 1. Introduction; 2. The Benchmark Case: The Stabilizer Central Banker; 3. The naive central banker; 4. Optimal monetary policy; 5. The gain of deviation under commitment; 6. Concluding Remarks; Appendix A. The stabilizer central banker's optimal monetary policy; Appendix B. The naive central banker's optimal monetary policy; Appendix C. Omitted proofs; Appendix D. Graphics.

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There has been in some countries a trend of assigning other functions to central banks besides price stability. The most suggested function to be added to monetary authority's obligations is to pursue economic growth or full employment. In this paper we characterize the behavior and analyse the optimal monetary policy of, what we call, a naive central banker. We describe the naive behavior as one that does face the inflation-unemployment trade-off, but it tries to minimize both variables simultaneously. Our findings, both under discretion and commitment, indicate that the naive central banker delivers lower expected inflation and inflation variance than the benchmark behavior whenever the economy is rigid enough. However, the degree of conservativeness also affects this result, such that the less conservative the naive policymaker, the more rigidity is necessary.

Existe, em alguns países, uma tendência de atribuir outras funções ao banco central além da estabilização de preços. A função mais sugerida a ser incorporada nas obrigações da autoridade monetária é a perseguição do crescimento econômico ou do pleno emprego. Neste artigo caracterizamos o comportamento e analisamos a política monetária ótima do chamado banqueiro central ingênuo. Descrevemos o comportamento ingênuo como aquele que enfrenta o trade-off inflação-desemprego, mas tenta minimizar ambas variáveis simultaneamente. Nossos resultados, tanto sob discricção quanto sob commitment, indicam que o banqueiro central ingênuo gera inflação esperada e variância inflacionária menores do que o comportamento de referência, sempre que a economia apresentar rigidez suficiente. Entretanto, o grau de conservadorismo também afeta este resultado, tal que quanto menos conservador for o banqueiro ingênuo, mais rigidez é necessária.

*Universidade Federal do Rio Grande do Sul, Faculdade de Ciências Econômicas. Av. João Pessoa, 52, sala 33, 3º andar, Centro, Porto Alegre-RS, Brasil. CEP 90040-000. E-mail: marcelo.griebeler@ufrgs.br



1. INTRODUCTION

There has been in some countries a trend of assigning other functions to central banks besides price stability. This is particularly true in countries where the central banks are not independent and thus are potential targets of political pressure. The most suggested function to be added to monetary authority's obligations is to pursue economic growth or full employment. The effects of the recent financial crisis, mainly the decreasing of GDP and the increasing of unemployment, may be considered a possible explanation for this suggestion. Based on the Keynesian idea of stimulating the economy by using fiscal and monetary policy in period of crisis, some policymakers in those countries have cogitated to use the central bank as an "agent of development".¹

The idea of central banks having to pursue economic growth is not new. While we can find the theoretical roots of such a controversy on the debate about the neutrality of money and whether monetary policy should be used to affect output, the history of central banking tells how practice has evolved over time.² Epstein (2005), for example, presents an historical account of central bank's action both in developed and developing countries. His conclusion is that the dominant "best practice" approach to central banking is dramatically different from the historically dominant theory and practice. In fact, it is argued all central banks have engaged in some kind of industrial policy or "selective targeting" in order to promote growth, and the only difference lies in which sector they have incentivized.

It is worthy to note the above historical view is not consentaneous. Orphanides (2012) is one of the authors which claim that throughout the existence of central banks their main objective has remained the same: stability. It is argued that what has been evolved over time is not central bank's function or objectives but the central banker's understanding on how to achieve and maintain stability. Even in financial crisis periods, the activist approach to monetary policy—one which wants to guide the economy towards attainment of its ideal output level—has not been the one chosen by central bankers, according to the author. Although the referred stability covers price, financial and economic dimensions, the point made is clear, namely the central bank's behavior has changed too little over time.

However, the new trend of expanding central bank's function is not just about making monetary policy more or less active. Instead the idea is to assign more *statutory* functions or goals for the central bank. This implies, for example, that promoting full employment would be in the same level of priority than pursuing price stability. One can say that a central banker who tries to achieve both targets at the same time, although he is aware about the role of the inflation-unemployment trade-off expressed by the Phillips Curve, presents a naive behavior.

But what are the economic effects of the monetary policy of a central bank which must fight inflation and stimulate output simultaneously? Literature both empirical and theoretical have not studied optimal monetary policy under those circumstances. In fact, there is a plenty of researchers exploring policies of the so called stabilizer central banker (about the nomenclature, see Griebeler, 2015), whose preferences are represented by a loss function quadratic both in inflation and output (e.g. Backus & Driffill, 1985; Ball, 1995; Canzoneri, 1985; Chari & Kehoe, 1990; Cukierman & Liviatan, 1991), in line with the seminal studies by Barro & Gordon (1983a, 1983b). But, to the best of our knowledge, none has considered a monetary authority which takes price and output stability as complementary targets.

In this paper we characterize the behavior and analyse the optimal monetary policy of, what we call, a naive central banker. We describe the naive behavior as one that does face the inflation-unemployment trade-off, but it tries to minimize both variables simultaneously. Specifically, it is defined as one that considers price and output stabilization as complementary "goods". In particular, we are interested in investigating the effects on the inflationary bias under both discretion and commitment. In order to

¹Observe that the trend we have mentioned is different from the controversy about the adoption of NGDP targeting versus price level targeting. While we are referring to an increase in the central bank's functions—pursuing both targets, for instance—, this controversy is about choosing one of the two regimes.

²For more on central banking and its evolution throughout history, see Blinder (2008, 2010); Capie, Fischer, Goodhart, & Schnadt (1995); Goodhart (2011).

do so, we use the simple Barro–Gordon’s framework, in which the central bank must minimize its loss function subject to a Lucas’s supply curve and the rational expectations assumption. The novelty that we introduce in this context is a loss function which represents that kind of complementarity relationship. However, since our model is not dynamic, it is just a first step toward studying the effects of the naive behavior. In fact, reputation issues are out of our framework, such that we cannot study how central banker is penalized according to his choices over time.

Our main findings concern to the comparison between the results of the naive and the stabilizer central banker, our benchmark, who considers price and output stabilization as substitute goals. First, both behaviors present similarities: there is an inflationary bias, and this bias depend primarily on how conservative is the central banker. In fact, when his level of conservativeness is maximum, the bias vanishes. Second, when both policymakers act with discretion, for given level of conservativeness of both central bankers, a necessary and sufficient condition for the naive one delivers higher both expected inflation and inflation variance is that output be inelastic enough with relation to inflation surprise. Third, this result is also true under non-binding commitment, but now we have a difference, namely as the elasticity goes to infinity, inflation delivered by both central bankers converges. Finally, we compare the incentives to cheat the commitment that both face and found that, for the same 1% of deviation from the inflation target, the percentage gain in terms of welfare of the stabilizer central banker is twice as high as the gain of the naive one.

The above result indicates there is a case for the naive central banker. In fact, the naive policymaker delivers lower both expected inflation and inflation variance whenever the economy’s rigidity is high enough. By rigidity we mean mainly informational one, given we choose the Lucas supply curve, but we conjecture our results would change too little if we used a model with price or wage stickiness. Therefore, one could conclude that, in terms of inflation, it is preferable to have a naive individual conducting the monetary policy than a stabilizer one whenever such a rigidity is present in the economy. The drawback of such a conclusion is that how enough must be the economy’s rigidity depend on how conservative both central bankers are. Empirical evidence seems to indicate that a naive central banker would be a easier target of political pressure, what would reflect on a more populist behavior. Thus, the threshold of rigidity would be larger, decreasing the range in which there is a case for a naive central banker. In short, given the stabilizer central banker’s level of conservativeness, the less conservative the naive central banker, the more rigid must be the economy.

Our findings are related to all the literature about inflationary bias and the potential solutions for this problem. Besides the studies which followed the seminal model of Barro & Gordon (1983a, 1983b), many of them mentioned in one the above paragraphs, there are several slightly modified versions of the original one which are close to our analysis. Examples are works which consider persistence in output and asymmetric preferences for the central bank (e.g. Jonsson, 1997; Nobay & Peel, 2003; Ruge-Murcia, 2003). Another branch in this literature is the modern approach provided by the New Keynesian models (e.g. Clarida, Gali, & Gertler, 1999; Damjanovic, Damjanovic, & Nolan, 2008; Sauer, 2010). However, by analysing the behavior of a specific type of central banker, our results are more close to those of Guzzo & Velasco (1999); Jerger (2002); Lippi (2002), which found there is a case for a populist central banker, and Herrendorf & Lockwood (1997); Rogoff (1985), which focus on the behavior of the conservative central banker. A good survey on the inflationary bias problem may be found in Walsh (2010).

This paper is organized as following. In section 2 we present the benchmark behavior, namely the stabilizer central banker’s one. We analyse his optimal monetary policy under discretion and commitment, as well as the resulting inflationary bias and its determinants. The naive central banker is characterized in section 3. We also investigate the optimal choice under discretion and commitment in this section and in the next. Thus, in section 4, inflation and output delivered by both policymakers are compared. Section 5 presents a further comparison between the naive behavior and the benchmark by analysing the elasticity of their welfare with respect to deviations from the commitment of delivering the inflation target. We make final comments regarding the limitations of our model as well as to potential extensions in section 6. Details of the results’ derivation can be found in Appendix A, for the stabilizer



central banker, and Appendix B, for the naive one. Finally, Appendix C presents the propositions' proofs omitted throughout the text, and Appendix D shows some graphics of our findings.

2. THE BENCHMARK CASE: THE STABILIZER CENTRAL BANKER

We follow literature by assuming that central banker has preferences over only two variables, namely inflation, π , and output, y . Let Π and Y be the subsets of \mathbb{R} , then $(\pi, y) \in \Pi \times Y \subseteq \mathbb{R} \times \mathbb{R}$.³ Before we introduce the naive central banker, we define the benchmark case for future comparisons. Our choice is the so called stabilizer central banker, the most common behavior used in the literature. This kind of central banker wishes to minimize deviations of inflation from its target and of output from its social optimal level. These two goal are considered substitutes and the central banker gives different weights to each one in its loss function. Formally, we can use the definition suggested by Griebeler (2015):

Definition 1. Let λ be a parameter that measures the relative weight given to output y , y^* be the output target, π be the inflation, and π^* be the inflation target. We define preferences of a stabilizer central banker as $(\pi_1, y_1) \succ_{STA} (\pi_2, y_2)$ if and only if $|\pi_1 - \pi^*| + \lambda|y_1 - y^*| \leq |\pi_2 - \pi^*| + \lambda|y_2 - y^*|$.

It is possible to show (Griebeler, 2015) that preferences like those defined above have an utility representation. Due to the non-smoothness of the absolute value function, it is useful to adopt the following well known functional form to represent them:

$$L_S(\pi, y) = \frac{1}{2} [(\pi - \pi^*)^2 + \lambda(y - y^*)^2].$$

As usual, throughout the paper, we assume policymaker's choices are constrained by a Lucas supply curve, given by

$$y = \bar{y} + a(\pi - \pi^e) + \epsilon, \quad a > 0, \tag{1}$$

where \bar{y} is the potential level of output (of long run), $\pi^e = \mathbb{E}[\pi]$ is the inflation expectation, ϵ is a supply shock, with mean and variance given by $\mathbb{E}[\epsilon] = 0$ and $\mathbb{V}[\epsilon] = \sigma_\epsilon^2$, respectively. The parameter a is a measure of output's sensitiveness with respect to inflation surprise. Assume we are considering variables in log, then a is the output's elasticity with respect to $(\pi - \pi^e)$. Furthermore, in order to have a source of inflationary bias in our model, we assume that $y^* > \bar{y}$, by the reasons suggested by Kydland & Prescott (1977), for example. With those assumption we are able to analyse optimal monetary policy.

2.1. Discretion

There is no uncertainty in our model, such that we can think of policymaker as choosing inflation directly. Thus, when central banker chooses inflation taking inflation expectation as given, his optimal choice results in

$$\mathbb{E}[\pi_S^D] = \pi^* + \lambda a(y^* - \bar{y}) \tag{2}$$

$$\mathbb{V}[\pi_S^D] = a^2 \lambda^2 \sigma_\epsilon^2. \tag{3}$$

Some conclusions may be drawn from (2) and (3). First, under discretion, there is an inflationary bias and its magnitude is given by $\mathbb{E}[\pi_S^D] - \pi^* = \lambda a(y^* - \bar{y})$. Therefore, the bias is determined by three factors: how high is the social optimal output in relation to the potential level, the degree of conservativeness of the policymaker, and the output's elasticity with respect to inflation surprise. Still, an ultra conservative central banker ($\lambda = 0$) is a sufficient condition to vanish the bias. Second, the

³Our results may be easily generalized to choice sets of any finite dimension. For example, they may apply to a central banker with preferences over inflation, output and exchange rate.

higher the value of a , the higher the volatility of inflation is. The same is true about parameter λ . Once again, an ultra conservative central banker yields inflation's variance equal to zero. Finally, by substituting π_S^D into (1), we find that $\mathbb{E}[y_S^D] = \bar{y}$ and $\mathbb{V}[y_S^D] = \sigma_\epsilon^2$, that is, there is no gain in terms of output by implementing π_S^D .

The value of the stabilizer central banker's expected loss under discretion is

$$\mathbb{E}[L_S(\pi_S^D, \bar{y} + \epsilon)] = \lambda(1 + \lambda a^2) \frac{[(y^* - \bar{y})^2 + \sigma_\epsilon^2]}{2}.$$

2.2. Commitment

Suppose now that the stabilizer central banker makes a binding commitment about what inflation will be before expected inflation is determined. Since the commitment is binding, $\pi^e = \pi$, which implies $y = \bar{y} + \epsilon$ and thus the policymaker's problem is to minimize $[(\pi - \pi^*)^2 + \lambda(\bar{y} + \epsilon - y^*)^2]/2$ by choosing π . The solution is simply $\mathbb{E}[\pi_S] = \pi^*$, with $\mathbb{V}[\pi_S] = 0$. However, the central banker could make an announcement of pursuing a determined level of inflation and then does not fulfill his promise. The reason for such a behavior is to create an inflation surprise and hence to increase output above its potential level.

Lets investigate what the stabilizer central banker gains by cheating the public. Suppose that his announcement makes $\pi^e = \pi^*$, such that now its optimal expected inflation and variance are given by

$$\mathbb{E}[\pi_S^C] = \pi^* + \frac{\lambda a}{1 + \lambda a^2} (y^* - \bar{y}) \quad (4)$$

$$\mathbb{V}[\pi_S^C] = \frac{\lambda^2 a^2}{(1 + \lambda a^2)^2} \sigma_\epsilon^2. \quad (5)$$

We can use the expressions above to find the expected output and its variance:

$$\mathbb{E}[y_S^C] = \frac{(\bar{y} + \lambda a^2 y^*)}{1 + \lambda a^2} \quad (6)$$

$$\mathbb{V}[y_S^C] = \frac{\sigma_\epsilon^2}{(1 + \lambda a^2)^2}.$$

The gain of cheating can be seen in terms of welfare,

$$\mathbb{E}[L_S(\pi_S^C, y_S^C)] = \frac{\lambda}{(1 + \lambda a^2)} \frac{[(y^* - \bar{y})^2 + \sigma_\epsilon^2]}{2},$$

because $\mathbb{E}[L_S(\pi_S^D, \bar{y} + \epsilon)] > \mathbb{E}[L_S(\pi_S^C, y_S^C)]$. So, there exists an incentive to the stabilizer central banker to create inflation surprise. In fact, it is possible to see that $\mathbb{E}[y_S^C] > \bar{y}$, which yields the improvement in his welfare. We will return to this topic later, when this incentive will be compared to one of the naive central banker. Also, observe the role of the degree of conservativeness of the stabilizer central banker in the output's behavior: the lower λ (more conservative) is, the lower the difference $\mathbb{E}[y_S^C] - \bar{y}$ and the higher the output's variance are.

3. THE NAIVE CENTRAL BANKER

We call a naive behavior one which considers stabilization of price and output complementary goals of policy. The reason for this name is that a central banker with such a behavior, even though knows the trade-off between the two goals, he tries to achieve them simultaneously. A naive central banker therefore prefers a state of the economy, defined by a pair (π, y) , to another if and only if both inflation and output are closer to their targets. Formally:



Definition 2. Let π^* and y^* be the inflation and the output desired by society, respectively. We define the preference of a naive central banker, \succeq_N , by: $(\pi_1, y_1) \succeq_N (\pi_2, y_2)$ if and only if $\max\{|\pi_1 - \pi^*|, \phi|y_1 - y^*|\} \leq \max\{|\pi_2 - \pi^*|, \phi|y_2 - y^*|\}$, where ϕ is a constant which indicates the degree of complementarity between price and output stabilizations.

Definition 2 clearly satisfies transitivity, completeness and continuity such that there exists a continuous utility function which represent such a preference. In fact, the behavior described above is similar to that of consumer choosing his bundle with two complementary goods. Thus, a function of Leontief's type may be a good choice to represent it.⁴ Observe, however, that, since there is a point of satiety, (π^*, y^*) , the indifference curves will be rectangles rather than the traditional "L" shape. Following the literature, we study the minimization problem of the central banker, such that \succeq_N may be represented by a loss (disutility) function:

$$L_N(\pi, y) = \max\{|\pi - \pi^*|, \phi|y - y^*|\}. \tag{7}$$

In some examples in the next sections we will assume $\phi = 1$, what we consider the most intuitive case and the nearest of the behavior described in the introduction, because the central banker gives the same weight to both targets. Nevertheless, our general form (7) allows different degrees of complementarity between inflation and output targets. In the rest of the paper we will say the higher the value of ϕ , the less conservative (the more populist) the naive central banker is. The reason for adopting such an informal definition is explained by an example. Consider two different naive central bankers, one with $\phi = 1$ and other with $\phi' = 2$. For the same pair (π, y) , the banker with $\phi' = 2$ always earns twice as much disutility than the one with $\phi = 1$. In fact, in the example they will achieve the same welfare only if output deviation of ϕ' is the half of the deviation of ϕ . In short, the higher the value of ϕ , the more weight is given to output stabilization by the naive central banker.

It is trivial to see that (7) does formally represent \succeq_N . An example is helpful to show intuitively that the naive behavior described in the introduction is well described by \succeq_N (and then by (7)). Suppose that $\pi^* = y^* = 2$, $\phi = 1$, and there are two available bundles, $(1, 1)$ and $(1, 1.5)$. By applying the definition of \succeq_N , we have $(1, 1) \sim_N (1, 1.5)$, which means that the naive central banker is indifferent between a pair of inflation and output in which both deviate 1 from their targets and a pair in which inflation deviates 1 and output deviates 0.5. This is exactly the essence of the behavior which "ignores" inflation-output trade-off: his welfare only increases when both aims approach their targets simultaneously, such that there is no opportunity of substitution between them. In contrast, $(1, 1.5) \succeq_S (1, 1)$, because the stabilizer behavior does consider price and output stabilization as substitutes. In short, \succeq_N is a suitable way of capturing such a naive behavior. However, we can also show that the representation by (7) is not unique, such that $\hat{L}_N(\pi, y) = \max\{(\pi - \pi^*)^2, \phi(y - y^*)^2\}$ represents \succeq_N as well. Our choice by (7) is due to its straightforward interpretation, since absolute value is a measure of deviation. Moreover, the advantage in terms of differentiability presented by \hat{L}_N is needless, because the maximum function is already non-smooth.

Given that central banker's preferences are represented by a loss function rather than an utility one, his problem is a minimization:

$$\min_{\pi} L(\pi, y) \quad \text{subject to (1)}.$$

To learn more about the behavior of the naive central banker, lets show that he faces an incentive to choose inflation higher than π^* in order to try to cause inflation surprise and then to affect positively the output. Suppose that people believe in the central banker's announcement that inflation target will be surely achieved, such that their inflation expectation is $\pi^e = \pi^*$. Further, suppose that the central banker does have discretion. Thus the Lucas supply curve (1) becomes $y = \bar{y} + a(\pi - \pi^*) + \epsilon$, such that the central banker's problem as function only of π is $L_N(\pi, y(\pi)) = \max\{|\pi - \pi^*|, \phi|\bar{y} - y^* + a(\pi - \pi^*) + \epsilon|\}$.

⁴More on preferences that can be representad by a Leontief utility function, see Voorneveld (2014).

The optimal inflation satisfies $|\pi - \pi^*| = \phi|\bar{y} - y^* + a(\pi - \pi^*) + \epsilon|$ and then we would have to consider two cases: $\pi - \pi^* = \phi[\bar{y} - y^* + a(\pi - \pi^*) + \epsilon]$ and $\pi - \pi^* = -\phi[\bar{y} - y^* + a(\pi - \pi^*) + \epsilon]$.

Figure 1 plots the rectangle of indifferences of the naive central banker and the Lucas supply curve. First, observe that the “smaller” is the rectangle—namely, its area—the lower is the loss, such that the minimum of $L(\pi, y)$ occurs in (π^*, y^*) . In addition, the degree of complementarity affects both height and width of the rectangles. Since ϕ multiplies the term of output stabilization in (7), for given π^* and y^* , the higher ϕ , the lower the height of the rectangles is. We can see that by observing that, for a fixed π_0 , the height of the rectangle is $h(\pi_0) = 2(\pi^* - \pi_0)/\phi$, such that $\lim_{\phi \rightarrow \infty} h(\pi_0) = 0$. On the other hand, the higher ϕ , the higher the width is: for a fixed y_0 , the width of the rectangle is $w(y_0) = 2\phi(y^* - y_0)$, such that $\lim_{\phi \rightarrow \infty} w(y_0) = \infty$ (assuming $y^* > y_0$).

By determining the perimeter of the rectangles of indifference, its height and width determine also how many different pairs of inflation and output yield the same welfare for the banker. A higher perimeter offers more options for a given level of disutility. Therefore, it is fundamental to know how conservative is the naive central banker in order to study his available choices. For example, for a central banker with $\phi = 2$, 1% of inflation deviation from its target yields the same loss that any value up to 0.5% of output deviation from its desirable level. Alternatively, if $\phi = \frac{1}{2}$, the loss yielded by 1% of deviation in inflation is the same of any deviation in output up to 2%.

Second, as the supply curve is upward-sloping, one can wonder whether the solution of the constrained problem may be any of four corners of the rectangles. The two cases we have to analyse are:

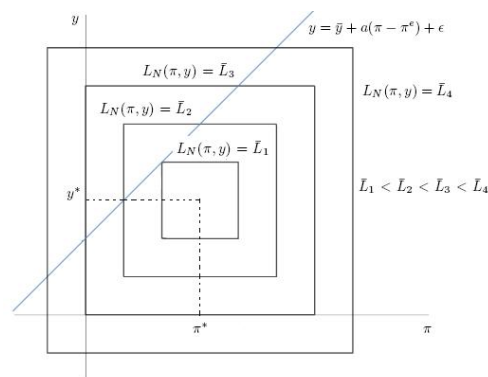
- (i) $\pi > \pi^*$ and $y > y^*$ or $\pi < \pi^*$ and $y < y^*$;
- (ii) $\pi > \pi^*$ and $y < y^*$ or $\pi < \pi^*$ and $y > y^*$.

In the next section we explore both cases and show which one will occur (see Proposition 2 below). For now, assume the case (ii) as true, such that $\pi - \pi^* = -\phi[\bar{y} - y^* + a(\pi - \pi^*) + \epsilon]$. Then, the optimal (expected) inflation and its variance are given by

$$\mathbb{E}[\pi_N^C] = \pi^* + \frac{\phi}{1 + a\phi}(y^* - \bar{y}) \tag{8}$$

$$\mathbb{V}[\pi_N^C] = \frac{\phi^2}{(1 + a\phi)^2} \sigma_\epsilon^2, \tag{9}$$

Figure 1. Rectangles of indifference of the naive central banker.





where one can see that there exists inflationary bias, because $y^* > \bar{y}$. This inflation surprise makes (expected) output be above its potential level:

$$\begin{aligned} \mathbb{E}[y_N^C] &= \bar{y} + \frac{a\phi}{1+a\phi}y^* \\ \mathbb{V}[y_N^C] &= (1-a\phi)^2\sigma_\epsilon^2. \end{aligned} \tag{10}$$

The expected central banker’s loss in this setting is then⁵

$$\begin{aligned} \mathbb{E}[L_N(\pi_N^C, y_N^C)] &= \mathbb{E}\left[\max\left\{\left|\frac{\phi}{1+a\phi}(y^* - \bar{y} - \epsilon)\right|, \phi\left|\frac{\bar{y} - y^* + \epsilon}{1+a\phi}\right|\right\}\right] \\ &= \frac{\phi}{1+a\phi}\mathbb{E}[|y^* - \bar{y} - \epsilon|]. \end{aligned}$$

The above results can be compared to those of the stabilizer central banker under the same conditions (non-binding commitment). For example, by comparing expected inflation and its variance, we find that $\mathbb{E}[\pi_N^C] \geq \mathbb{E}[\pi_S^C]$ and $\mathbb{V}[\pi_N^C] \geq \mathbb{V}[\pi_S^C]$ if and only if $\phi - \lambda a \geq 0$. This same condition also plays an important role when we compare the results related to output and its variability. Proposition 1 below brings more details about such a comparison.

Proposition 1. *Under non-binding commitment, the following statements are true:*

- (i) $\mathbb{E}[\pi_N^C] \geq \mathbb{E}[\pi_S^C]$ if and only if $a \leq \frac{\phi}{\lambda}$;
- (ii) $\mathbb{V}[\pi_N^C] \geq \mathbb{V}[\pi_S^C]$ if and only if $a \leq \frac{\phi}{\lambda}$;
- (iii) if $a \leq \frac{\phi}{\lambda}$, then $\mathbb{E}[y_N^C] \geq \mathbb{E}[y_S^C]$;
- (iv) as $a \rightarrow +\infty$, *ceteris paribus*, $\mathbb{E}[\pi_N^C] \rightarrow \mathbb{E}[\pi_S^C]$, $\mathbb{V}[\pi_N^C] \rightarrow \mathbb{V}[\pi_S^C]$, and $\mathbb{E}[y_N^C] > \mathbb{E}[y_S^C]$.

One way to interpret the results of Proposition 1 is by assuming that both levels of conservativeness, λ and ϕ , are fixed, such that relevant parameter for the analysis is the output’s elasticity with respect to inflation surprise, a . Thus, a necessary and sufficient condition for the naive central banker to deliver expected inflation and inflation variance higher than the stabilizer one is that the elasticity a be low enough. If, for instance, both central bankers have the same degree of conservativeness—even though their measures are not perfectly comparable—, such that $\phi/\lambda = 1$, then such a condition becomes: output has to be inelastic with relation to inflation surprise. Note also that the same condition is only sufficient for the expected output delivered by the naive central banker be higher than one delivered by the stabilizer one.

Another important point to note in the above results is that, despite the fact of high values of a implies both $\mathbb{E}[\pi_N^C] \leq \mathbb{E}[\pi_S^C]$ and $\mathbb{V}[\pi_N^C] \leq \mathbb{V}[\pi_S^C]$, as a increases the difference between the inflation results of both policymakers tends to zero. This means that, as the benefit of inflation surprise becomes greater, the behavior of both central bankers related to inflation converges. The same is not true about output: in many cases, for low enough values of a , the naive delivers higher output; for “intermediate” values, the stabilizer is who delivers higher output; and, as a increases (and exceeds a given value \bar{a}) the

⁵There is no closed form for $\mathbb{E}[|y^* - \bar{y} - \epsilon|]$. However, given that the absolute value is a convex function, we can apply the Jensen inequality and obtain a lower bound for $\mathbb{E}[L_N(\pi_N^C, y_N^C)]$. By doing so, we have $\mathbb{E}[|y^* - \bar{y} - \epsilon|] \geq |\mathbb{E}[y^* - \bar{y} - \epsilon]| = |y^* - \bar{y}| = y^* - \bar{y}$. Thus, $\mathbb{E}[L_N(\pi_N^C, y_N^C)] \geq \phi(y^* - \bar{y})/(1+a\phi)$.

naive again delivers higher output.⁶ One can visualize those results in figures D-1a, D-1b, D-1c, and D-1d in Appendix D, where are shown the differences $\mathbb{E}[\pi_N^C] - \mathbb{E}[\pi_S^C]$, $\mathbb{V}[\pi_N^C] - \mathbb{V}[\pi_S^C]$, $\mathbb{E}[y_N^C] - \mathbb{E}[y_S^C]$, and $\mathbb{V}[y_N^C] - \mathbb{V}[y_S^C]$, respectively, as a function of a . We also consider three possible cases: $\phi/\lambda = 1$, $\phi/\lambda > 1$ and $\phi/\lambda < 1$.

4. OPTIMAL MONETARY POLICY

4.1. Discretion

Under discretion, central banker chooses inflation taking expectations of inflation as given. Substituting (1) into (7), we have the unconstrained problem of minimization $L(\pi) = \max\{|\pi - \pi^*|, \phi|\bar{y} - y^* + a(\pi - \pi^e) + \epsilon|\}$. The solution is given by the points that satisfy $|\pi - \pi^*| = \phi|\bar{y} - y^* + a(\pi - \pi^e) + \epsilon|$. There are two possibilities to consider (cases (i) and (ii) in section 3). Proposition 2 states that the optimum always occurs either $\pi > \pi^*$ and $y < y^*$ or $\pi < \pi^*$ and $y > y^*$. Undoubtedly, the most common situation in practice is inflation higher than its target and output lower than its potential level, which henceforth we assume as true.

Proposition 2. *Consider the two possible cases of solution of $|\pi - \pi^*| = \phi|\bar{y} - y^* + a(\pi - \pi^e) + \epsilon|$, namely (i) $\pi - \pi^* = \phi|\bar{y} - y^* + a(\pi - \pi^e) + \epsilon|$ and (ii) $\pi - \pi^* = -\phi|\bar{y} - y^* + a(\pi - \pi^e) + \epsilon|$. Then, case (ii) always gives the minimum loss for the naive central banker.*

By using the result above, we may obtain the inflation as a function of public's expectation,

$$\pi = \pi^* + \frac{\phi a(\pi^e - \pi^*) + \phi(y^* - \bar{y}) - \phi\epsilon}{1 + \phi a}. \quad (11)$$

With rational expectations, people form their expectations by knowing central banker's incentive to stimulate inflation surprise, such that $\pi^e = \pi$. By substituting this into (11), we obtain the (expected) inflation of equilibrium and its variance:

$$\mathbb{E}[\pi_N^D] = \pi^* + \phi(y^* - \bar{y}) \quad (12)$$

$$\mathbb{V}[\pi_N^D] = \phi^2 \sigma_\epsilon^2. \quad (13)$$

Still, the expected output of equilibrium under discretion is equal to its potential level, $\mathbb{E}[y_N^D] = \bar{y}$, and its variance is $\mathbb{V}[y_N^D] = \sigma_\epsilon^2$.

The results of the naive central banker under discretion are quite similar to those yielded by the stabilizer one. First, observe that there is an inflationary bias as well, and its magnitude also depends on both how higher is the social optimal level output compared to the long run level and the degree of conservativeness. The difference is in the absence of the output elasticity with respect to inflation surprise. Still, an ultra conservative central banker ($\phi = 0$) delivers the expected inflation equal to the inflation target. Second, the same can be said about the determinants of the inflation variance. Finally, given the assumption of rational expectations, inflation is completely anticipated by the agents, and the expected output is equal to its level of long run.

⁶To be more precise, the behavior of $\mathbb{E}[y_N^C] - \mathbb{E}[y_S^C]$ depends on the values of all parameters in the model. In fact, $\mathbb{E}[y_N^C] - \mathbb{E}[y_S^C] = a[a^2\bar{y}\lambda\phi - a\lambda(y^* - \bar{y}) + y^*\phi]$, such that its roots are $a = 0$ and $a = \left[\lambda(y^* - \bar{y}) \pm \sqrt{\lambda^2(y^* - \bar{y})^2 - 4\bar{y}y^*\lambda\phi}\right] / 2\bar{y}\lambda\phi$, which can be either real (either a single or two different) or complex. In Figure D-1c, for example, we have the case of two different real roots ($\phi/\lambda = 1$ and $\phi/\lambda < 1$) and the case of complex roots ($\phi/\lambda > 1$).



We can compute the welfare of the naive central banker under discretion, which is given by

$$\begin{aligned} \mathbb{E} \left[L_N(\pi_N^D, \bar{y} + \epsilon) \right] &= \mathbb{E} \left[\max \{ |\phi(y^* - \bar{y}) - \phi\epsilon|, \phi|\bar{y} - y^* + \epsilon| \} \right] \\ &= \phi \mathbb{E} [|y^* - \bar{y} - \epsilon|]. \end{aligned}$$

Clearly, we have $\mathbb{E} \left[L_N(\pi_N^D, \bar{y} + \epsilon) \right] > \mathbb{E} \left[L_N(\pi_N^C, y_N^C) \right]$, which implies that there is a worsening in the policymaker's welfare with relation to the setting under ono-binding commitment.

Proposition 3. *Under discretion, the following statements are true:*

- (i) $\mathbb{E} \left[\pi_N^D \right] \geq \mathbb{E} \left[\pi_S^D \right]$ if and only if $\frac{\phi}{\lambda} \geq a$;
- (ii) $\mathbb{V} \left[\pi_N^D \right] \geq \mathbb{V} \left[\pi_S^D \right]$ if and only if $\frac{\phi}{\lambda} \geq a$.

It is possible to see Proposition 3 is based on the same condition of Proposition 1 and presents similar results. Therefore, the interpretation of itens (i) and (ii) of both are almost the same. The only difference is the fact of inflation and output now are yielded when policymakers act with discretion rather than with non-binding commitment. However, Proposition 3 no longer states convergence between expected inflations and inflation variances. In fact, as we can see in Figure D-1e in Appendix D, as a goes to infinity, $\mathbb{E} \left[\pi_N^C \right] - \mathbb{E} \left[\pi_S^C \right]$ goes to $-\infty$, that is, the inflation delivered by the stabilizer central banker becomes infinitely higher than one delivered by the naive one. The same is true about the variance. Mathematically, this difference is evident, because now we have monotonic functions while with commitment they present increasing and decreasing intervals.

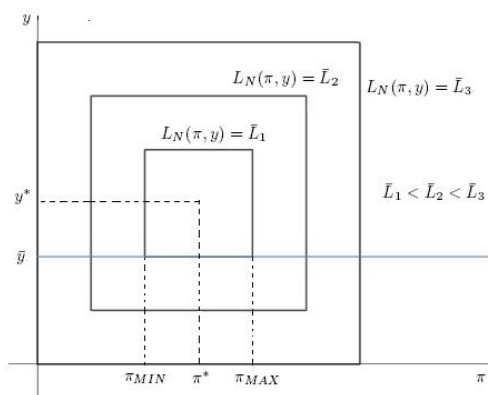
The reason for the absence of convergence between inflations under discretion is on the features of both central bankers' behavior. Recall that under discretion the expected inflation delivered by the stabilizer policymaker, given by (2), depends on the output's elasticity with respect to inflation surprise. As he chooses inflation taking the inflation expectation as given, the higher the elasticity, the more the marginal gain of the inflation surprise in terms of output. Thus, as a increases, his inflation is higher with the aim of creating inflation surprise. On the other hand, the inflation delivered by the naive policymaker does not depend on a , what we can see in (12). As this central banker does not improve his welfare by substituting price level by output, a higher elasticity would improve y but also would not changed π , what would let his welfare unchanged. Therefore, his incentive to create inflation surprise is not affected by changes in the elasticity.

4.2. Commitment

We have already seen the results of both central bankers under commitment, but it is worthy to take a deeper look in the naive behavior in this setting, given its singular features. Recall that, under commitment, the central banker's promise of pursuing the inflation target is credible, such that $\pi^e = \pi = \pi^*$ and there is no inflationary bias. In this case, the expected output is equal to its potential level as well. By keeping its promise, the monetary authority's expected loss is then $\mathbb{E} \left[L_N(\pi^*, \bar{y} + \epsilon) \right] = \phi \mathbb{E} [|y^* - \bar{y} + \epsilon|]$. In addition, the characteristics of the naive behavior make this same welfare be achieved for infinitely many other pairs (π, y) . In order to see this, observe that now the supply curve is just $y = \bar{y} + \epsilon$, such that it is represented by a horizontal line in the plane $\pi - y$.

As Figura 2 shows, the fact of the naive central banker has rectangles of indifference implies there is a line segment in which the supply curve coincides with the loss function's level curve. Therefore, there is a continuum of inflation levels which yields the same loss that the inflation target does. In fact, notice that inflation target is the center of the interval $[\pi_{MIN}, \pi_{MAX}]$. We must characterize it, especially with respect to its width and its determinants. In order to do so, we first have to find the value of its

Figure 2. Naive behavior under commitment.



endpoints. Note that $\pi_{\text{MIN}} < \pi$ and $\pi_{\text{MAX}} > \pi$, such that the first one fits item (i) and the second fits item (ii) of Proposition 2. Therefore, they must satisfy

$$\begin{aligned} \pi_{\text{MIN}} &= \pi^* - \phi(y^* - \bar{y} + \epsilon) \\ \pi_{\text{MAX}} &= \pi^* + \phi(y^* - \bar{y} + \epsilon) = \pi^D, \end{aligned}$$

where we use the fact of $y = \bar{y} < y^*$.

This first analysis, however, may be misleading, because it might indicate that the naive central banker can choose any inflation in the range $[\pi_{\text{MIN}}, \pi_{\text{MAX}}]$ without changing the slope of the supply curve and thus letting his welfare be unchanged. Recall that his optimization problem is constrained by (1), such that its slope changes whenever $\pi^e \neq \pi$. Therefore, if the central banker chooses $\pi \neq \pi^*$ and public believes that the commitment is binding, the supply curve satisfies (11), which is no longer a horizontal line. This implies that every deviation of his announcement has gain (or cost) in terms of welfare. In fact, any policy deviation makes the analysis of section 3 the most suitable, as we have seen. Nevertheless, the shape of the indifference curves of the naive central banker seems to indicate that small deviations might cause little changes in $L_N(\pi, y)$, given that the slope of (11) would change little as well. The next section studies the gain of the deviation and compares the results of the naive central banker with our benchmark case.

5. THE GAIN OF DEVIATION UNDER COMMITMENT

As we have seen, the shape of indifference curves of the naive central banker seems to indicate that small deviations from the commitment $\pi = \pi^*$ causes small changes in his welfare. However, it is necessary to use our benchmark case in order to define what we are calling “small changes”. In this section we use two measures to evaluate the incentives that both policymakers face to cheat the public. The first one is the optimal deviation: how much the central banker would deviate from the inflation target, once he has assumed the commitment of pursuing it, if he made his choice by maximizing his welfare.

Let $d_i = \pi - \pi^*$ be the deviation from the inflation target under commitment for $i = S, N$. Observe that we can write L_i in terms of d , which gives us the following expression for the stabilizer:

$$L_S(d) = \frac{1}{2} \left[d^2 + \lambda(\bar{y} - y^* + ad + \epsilon)^2 \right].$$



This quadratic function reaches its minimum when $d_S = a\lambda[(y^* - \bar{y}) - \epsilon]/(1 + \lambda a^2)$, the same value of $\pi_S^C - \pi^*$. In other words, the optimal deviation for the stabilizer central banker is just the value of the inflationary bias yielded when he adopts π_S^C . Similarly, for the naive one we have $L_N(d) = \max\{|d|, \phi|\bar{y} - y^* + ad + \epsilon|\}$, which reaches its minimum when $d_N = \phi[(y^* - \bar{y}) - \epsilon]/(1 + \phi a)$. Once again, the optimal deviation is given by the bias obtained when the policymaker adopts his optimal policy under (non-binding) commitment, π_N^C .

There is no novelty in the above analysis: the naive central banker will choose an inflation which yields a higher deviation (bias) than one adopted by the stabilizer if and only if $\phi - \lambda a > 0$. We have already obtained such a result in section 3, which may make us to think that, whenever the mentioned condition is satisfied, the naive central banker has more incentive to deviate than the stabilizer. However, this measure accounts for the absolute magnitude of the central bankers' welfare. As the functional forms of the loss functions are different from each other, it is not possible to compare the values of L_S with L_N . We need a measure which considers welfare but not in absolute terms.

We can use the idea of elasticity in order to overcome the aforementioned difficult. We must answer the following question: under (non-binding) commitment, what is the percentage change in the central banker's welfare if inflation increases one percent? When we evaluate this elasticity at the point $\pi = \pi^*$, we have a measure of the incentive to deviate from the inflation target, independent of the absolute magnitude of the central banker's loss. Then, define

$$\eta_i = \left. \frac{\partial L_i}{\partial \pi} \frac{\pi_i^C}{L_i} \right|_{\pi_i^C = \pi^*}$$

as the elasticity, for $i = S, N$. Recall that the derivative is evaluated in $\pi^e = \pi^*$ and π_i^C is given by either (4) or (8).

Our results indicate that the stabilizer central banker has a welfare improvement higher than the naive one for the same 1% deviation from the inflation target:

$$\eta_S = -\frac{2a\pi^*}{y^* - \bar{y} - \epsilon}$$

$$\eta_N = -\frac{a\pi^*}{y^* - \bar{y} - \epsilon}.$$

Observe that the magnitude of the η_i is primarily determined by a and π^* : the more elastic the output to inflation surprise, the higher η_i is; similarly, the higher the inflation target, the higher η_i is. Furthermore, one can see that those elasticities do not depend on the conservativeness of the policymakers.

The conclusion of this section confirms what the graphical analysis of section 4 suggests, namely that the incentives for the naive central banker deviates from the commitment are lower than those of the stabilizer one. In fact, the result obtained here is even stronger: it states that, for the same 1% of deviation, the response (in percentage) of the welfare of the former is twice higher than the response of the latter. Moreover, this finding is independent of how conservative both policymakers are. Once again, the main reason for such a difference is on the nature of the naive behavior, since that it does not substitute price by output stabilization, which implies in low gains of creating inflation surprise.

6. CONCLUDING REMARKS

There is a case for a naive central banker. Whenever the supply curve's slope is large enough, the naive policymaker delivers both expected inflation and inflation variance lower than the stabilizer one. If both types present the same level of conservativeness ($\phi/\lambda = 1$), for example, such a condition is just that the output be inelastic with respect to inflation surprise. In order to have more economic intuition, we must then analyse what are the determinants of this elasticity. Given that we adopt the Lucas supply curve,

the underlying framework of our model is based on the assumption of information rigidity. Therefore, the more rigid the information, the slower the adjust process in the economy, and the more inelastic the output with respect to inflation surprise (the higher the supply curve's slope). Therefore, the case for the naive central banker requires that the economy presents enough information rigidity. In addition, we conjecture that this result does not change when one adopts assumption of price or wage stickiness.

Because rigidity is a relatively common feature of many modern economies, a natural conclusion would be that many central banks should be headed by a naive individual. Yet, there is no empirical evidence of this fact. In fact, it is only possible to find few researchers and politicians which recommend such an option. Our conjecture for this absence is that a naive central banker will in general be more populist than the stabilizer one. The reason is on the nature of the naive behavior, described in section 3, which is more subject to political pressures. Then, the more populist (the less conservative) the naive relative to the stabilizer, the more rigid the economy must be for the naive central banker delivers inflation lower than the stabilizer one. For example, as we have seen, if $\phi/\lambda = 1$, then the condition is $a < 1$, but if $\phi/\lambda = 4$, then the output should be four times more inelastic than the previous case. In short, the more populist the naive central banker, the "weaker" the case for him.

As the remainder literature on inflationary bias, our model can be extended by considering that the central banker chooses inflation indirectly through his instruments (money supply, for instance). This extension would improve the discussion about rules and discretion in monetary policy, virtually absent in our analysis. A richer modification in our original framework would allow us to build our results on a New Keynesian approach. By adopting such an alternative, we would be able to study the behavior of interest rate, inflation and output simultaneously. In addition, if the chosen model were a DSGE, one would be possible to consider long run effects as well as impacts of other shocks on the naive behavior. However, we believe that the main contribution of adopting such models is to overcome the limitation imposed by the lack of dynamics. By using the New Keynesian framework, it is possible to study reputation issues and how penalizations can affect the behavior of the naive central banker over time. We conjecture that this modification would make the results of both types of central bankers (naive and stabilizer) even more similar, because the effects of reputation incentives. Yet, this extension may not be so direct, given the non-smoothness of the loss function, but it would add realism on the monetary policy analysis and would make the results closer to the state of the art in monetary theory. Finally, our results would be enriched if we added issues concerning to labor market, for example, like those studies of Guzzo & Velasco (1999); Jerger (2002); Lippi (2002).

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APPENDIX A. THE STABILIZER CENTRAL BANKER’S OPTIMAL MONETARY POLICY

Discretion

The loss function of the stabilizer central banker as a function only of inflation is given by

$$L_S(\pi) = \frac{1}{2} \left[(\pi - \pi^*)^2 + \lambda(\bar{y} + a(\pi - \pi^e) + \epsilon - y^*)^2 \right],$$

which yields the following first order condition (FOC):

$$(\pi - \pi^*) + \lambda a(\bar{y} + a(\pi - \pi^e) + \epsilon - y^*) = 0. \tag{A-1}$$

By rearranging the terms of (A-1) and using the fact of $\pi^e = \pi$ we have

$$\begin{aligned} \pi_S^D &= \pi^* + \lambda a(y^* - \bar{y} - \epsilon) \\ \mathbb{E}[\pi_S^D] &= \pi^* + \lambda a(y^* - \bar{y}) \\ \mathbb{V}[\pi_S^D] &= \lambda^2 a^* \sigma_\epsilon^2. \end{aligned} \tag{A-2}$$

By substituting (A-2) into (1) we have the economy's output:

$$\begin{aligned} y_S^D &= \bar{y} + (1 - \lambda a^2)\epsilon \\ \mathbb{E}[y_S^D] &= \bar{y} \\ \mathbb{V}[y_S^D] &= (1 - \lambda a^2)^2 \sigma_\epsilon^2. \end{aligned}$$

Commitment

We have already seen that when the commitment is binding $\pi = \mathbb{E}[\pi] = \pi^e = \pi^*$, which yields $y = \bar{y} + \epsilon$, $\mathbb{E}[y] = \bar{y}$ and $\mathbb{V}[y] = \sigma_\epsilon^2$. On the other hand, when the commitment is not binding, the central banker's optimization problem yields (A-1) as its FOC, but now $\pi^e = \pi^*$. Then, we have

$$\begin{aligned} \pi_S^C &= \pi^* + \frac{\lambda a}{1 + \lambda a^2}(y^* - \bar{y} - \epsilon) \tag{A-3} \\ \mathbb{E}[\pi_S^C] &= \pi^* + \frac{\lambda a}{1 + \lambda a^2}(y^* - \bar{y}) \\ \mathbb{V}[\pi_S^C] &= \frac{\lambda^2 a^2 \sigma_\epsilon^2}{(1 + \lambda a^2)^2}. \end{aligned}$$

By substituting (A-3) into the supply curve (1),

$$\begin{aligned} y_S^C &= \frac{\bar{y} + \lambda a^2 y^* + \epsilon}{1 + \lambda a^2} \\ \mathbb{E}[y_S^C] &= \frac{\bar{y} + \lambda a^2 y^*}{1 + \lambda a^2} \\ \mathbb{V}[y_S^C] &= \frac{\sigma_\epsilon^2}{(1 + \lambda a^2)^2}. \end{aligned}$$

The gain of the deviation under commitment

Let us calculate the elasticity η_S :

$$\begin{aligned} \eta_S &= \frac{(\pi_S^C - \pi^*) + \lambda a(\bar{y} + a(\pi_S^C - \pi^*) + \epsilon - y^*)\pi_S^C}{\frac{1}{2}[(\pi_S^C - \pi^*)^2 + \lambda(\bar{y} + a(\pi_S^C - \pi^*) + \epsilon - y^*)^2]} \Bigg|_{\pi_S^C = \pi^*} \\ &= \frac{\lambda a(\bar{y} + \epsilon - y^*)\pi^*}{\frac{1}{2}\lambda(\bar{y} + \epsilon - y^*)^2} \\ &= \frac{-2a\pi^*}{y^* - \bar{y} - \epsilon}. \end{aligned}$$

APPENDIX B. THE NAIVE CENTRAL BANKER'S OPTIMAL MONETARY POLICY

The steps of the results of commitment and discretion are shown in the text.



The gain of the deviation under commitment

By using Proposition 2, one can see that π^* is lower than π_N^C , such that it is in decreasing part of L_N (see (10)). Thus, $\partial L_N / \partial \pi = -a\phi$ and $L_N(\pi) = \phi \left| \bar{y} + a(\pi_N^C - \pi^*) + \epsilon - y^* \right|$, which yields

$$\begin{aligned} \eta_N &= \frac{-a\phi\pi_N^C}{\phi \left| \bar{y} + a(\pi_N^C - \pi^*) + \epsilon - y^* \right|} \Bigg|_{\pi_N^C = \pi^*} \\ &= \frac{-a\phi\pi^*}{\phi(y^* - \bar{y} - \epsilon)} \\ &= \frac{-a\pi^*}{(y^* - \bar{y} - \epsilon)}, \end{aligned}$$

where we use the fact of $y^* > \bar{y}$ and assume that ϵ is small, since it is a shock.

APPENDIX C. OMITTED PROOFS

Proof. **Proposition 1**

(i) $\mathbb{E} \left[\pi_N^C \right] \geq \mathbb{E} \left[\pi_S^C \right]$

$$\begin{aligned} \pi^* + \frac{\phi(y^* - \bar{y})}{1 + a\phi} &\geq \pi^* + \frac{\lambda a(y^* - \bar{y})}{1 + \lambda a^2} \\ \frac{\phi(y^* - \bar{y})}{1 + a\phi} &\geq \frac{\lambda a(y^* - \bar{y})}{1 + \lambda a^2} \end{aligned} \tag{C-4}$$

$$\begin{aligned} \phi + \phi \lambda a^2 &\geq \lambda a + \phi \lambda a^2 \\ \phi - \lambda a &\geq 0 \Leftrightarrow a \leq \frac{\phi}{\lambda}. \end{aligned} \tag{C-5}$$

(ii) $\mathbb{V} \left[\pi_N^C \right] \geq \mathbb{V} \left[\pi_S^C \right]$

$$\begin{aligned} \frac{\phi^2 \sigma_\epsilon^2}{(1 + a\phi)^2} &\geq \frac{\lambda^2 a^2 \sigma_\epsilon^2}{(1 + \lambda a^2)^2} \\ \frac{\phi^2}{(1 + a\phi)^2} &\geq \frac{\lambda^2 a^2}{(1 + \lambda a^2)^2}, \end{aligned}$$

which implies (C-4) and then (C-5).

(iii) $\mathbb{E} \left[y_N^C \right] \geq \mathbb{E} \left[y_S^C \right]$

$$\bar{y} + \frac{a\phi y^*}{1 + a\phi} \geq \frac{\bar{y} + \lambda a^2 y^*}{1 + \lambda a^2}$$

$$\frac{a}{(1 + a\phi)(1 + \lambda a^2)} [a\bar{y}(\lambda + a\phi) + y^*(\phi - \lambda a)] \geq 0,$$

such that a sufficient condition is $\phi - \lambda a \geq 0$, that is, $a \leq \phi / \lambda$.

(iv) By applying the L'Hôpital rule in (4), (5), (6), (8), (9), and (10) we can obtain

$$\begin{aligned} \lim_{a \rightarrow +\infty} \mathbb{E} \left[\pi_N^C \right] &= \lim_{a \rightarrow +\infty} \mathbb{E} \left[\pi_S^C \right] = \pi^* \\ \lim_{a \rightarrow +\infty} \mathbb{V} \left[\pi_N^C \right] &= \lim_{a \rightarrow +\infty} \mathbb{V} \left[\pi_S^C \right] = 0 \\ \lim_{a \rightarrow +\infty} \mathbb{E} \left[y_N^C \right] &= \bar{y} + y^* > y^* = \lim_{a \rightarrow +\infty} \mathbb{E} \left[y_S^C \right]. \end{aligned}$$

□

Proof. Proposition 2

Let π_1 and π_2 be the solutions of $\pi - \pi^* = \phi[\bar{y} - y^* + a(\pi - \pi^e) + \epsilon]$ and $\pi - \pi^* = -\phi[\bar{y} - y^* + a(\pi - \pi^e) + \epsilon]$, respectively. For given π^* , y^* , a , ϕ and π^e , it suffices to show that $|\pi_1 - \pi^*| > |\pi_2 - \pi^*|$, because $|\pi_i - \pi^*| = \phi|y_i - y^*|$ for $i = 1, 2$. We already computed π_2 when we solved the model with discretion, such that $\pi_2 = \pi^D$, given by (11). For π_1 , we have

$$\pi_1 - \pi^* = \phi[\bar{y} - y^* + a(\pi_1 - \pi^e) + \epsilon].$$

When one solves for π_1 , one obtains

$$\pi_1 = \frac{\pi^* - \phi[(y^* - \bar{y}) + a\pi^e + \epsilon]}{1 - a\phi}.$$

Then, by subtracting π^* of both sides and taking the absolute value, it results in

$$|\pi_1 - \pi^*| = \frac{|-\phi[(y^* - \bar{y}) + a(\pi^e - \pi^*) + \epsilon]|}{|1 - a\phi|}. \quad (\text{C-6})$$

For π_2 , observe that we can once again subtract π^* in both sides of (11) and then take the absolute value, such that

$$|\pi_2 - \pi^*| = \frac{|\phi[(y^* - \bar{y}) + a(\pi^e - \pi^*) + \epsilon]|}{|1 + a\phi|}. \quad (\text{C-7})$$

Now, given that $|x| = |-x|$ for all x , the numerators of (C-6) and (C-7) are equal. Still, recall that $\phi a > 0$, such that $|1 - \phi a| < |1 + \phi a|$. Therefore, $|\pi_2 - \pi^*| < |\pi_1 - \pi^*|$, which implies that $L(\pi_2, y_2) < L(\pi_1, y_1)$. □

Proof. Proposition 3

$$(i) \quad \mathbb{E}[\pi_N^D] \geq \mathbb{E}[\pi_S^D]$$

$$\pi^* + \phi(y^* - \bar{y}) \geq \pi^* + \lambda a(y^* - \bar{y})$$

$$\phi \geq \lambda a \Leftrightarrow \frac{\phi}{\lambda} \geq a.$$

$$(ii) \quad \mathbb{V}[\pi_N^D] \geq \mathbb{V}[\pi_S^D]$$

$$\phi^2 \sigma_\epsilon^2 \geq a^* \lambda^2 \sigma_\epsilon^2$$

$$\frac{\phi}{\lambda} \geq a.$$

□



APPENDIX D. GRAPHICS

Figure D-1. Gráficos.

