

Physics at the grocery store

R. De Luca^{*1} 

¹Università degli Studi di Salerno, Dipartimento di Fisica “E. R. Caianiello”, Fisciano, Italy.

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The motion of a food can on an accelerated conveyor belt in a grocery store is described by means of Newtonian mechanics. By assuming that the food can may roll on the conveyor belt, when the cashier starts this device, one can prove that the rolling motion, observed in the belt reference system, is such that the can is seen to move away for the cashier. Experiments can be performed with common material: an empty and a full food can, and a sheet of paper as conveyor belt.

Keywords: Newtonian mechanics, rigid bodies dynamics, Galilean transformations.

1. Defining the Problem

Very often physics students are asked to solve problems involving axially symmetric bodies rolling on a rough surface [1, 2]. These problems can be solved using standard Newtonian mechanics. In general, point masses, strings, and pulleys are used to set in action the rolling motion of these bodies.

A simple physics experiment on this topic can be performed at the grocery store. In fact, you may place a cylindrical food can on the conveyor belt carrying goods to the cashier along the horizontal x -axis lying along the belt. If the longitudinal axis of the can is orthogonal to the x -axis, then the rolling motion of this cylindrical body will allow motion of the can’s center of mass along the x -direction. A schematic representation of the system is shown in Fig. 1, where all forces acting on the cylinder are shown. A similar problem, related to the crockery sliding on tablecloth, has been studied in Ref. [3]. In our case, however, only pure rotation in the conveyor belt reference system is assumed. Because of pure rotation, the component of the center of mass acceleration a'_{CM} along the x' -axis, parallel to the x -axis and attached to the belt, can be written as follows:

$$a'_{CM} = R \alpha, \quad (1)$$

Where α is the angular acceleration of the cylindrical food can. The forces acting on the cylinder are the following: its weight $M\vec{g}$, the normal reaction \vec{N} and the friction force \vec{F}_A . With the schematic representation in Fig. 1, we shall proceed to analyse the dynamical properties of the system.

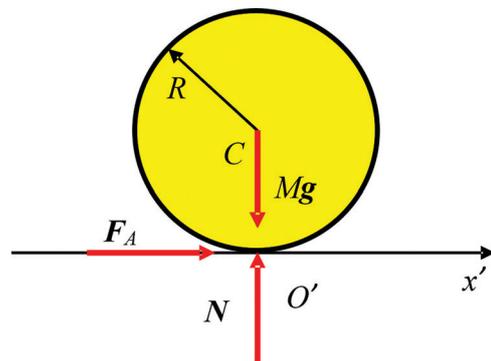


Figure 1: A cylinder of mass M and radius R can rotate about a longitudinal axis orthogonal the x' direction shown. In the reference system of the conveyor belt, with origin O' attached to the belt, the acceleration a'_{CM} of the center of mass of the cylinder can be expressed in terms of the angular acceleration α as follows: $a'_{CM} = R \alpha$. All forces acting on the cylinder are shown.

2. Solving the Problem

By applying Newton’s second law to the system in Fig. 1, we can write:

$$M\vec{g} + \vec{N} + \vec{F}_A = M\vec{a}_{CM}, \quad (2)$$

where $\vec{a}_{CM} = \vec{a}'_{CM} + \vec{a}_{O'}$, is the acceleration of the center of mass of the can in the inertial reference system of the grocery store, $\vec{a}_{O'}$ being the non-constant acceleration of the belt. In terms of the x - and y -components, we have, in the order:

$$F_A = M(a'_{CM} + a_{O'}), \quad (3a)$$

$$N - Mg = 0. \quad (3b)$$

* Correspondence email address: rdeluca@unisa.it

Furthermore, by writing the dynamical equation for the total torque $M_{ext} = -F_A R$, calculated with respect the centre of mass of the cylinder, we may write:

$$-F_A R = I_{CM} \alpha, \quad (4)$$

where $I_{CM} = kMR^2$ is the moment of inertia of the cylinder, k being a numerical factor. The constant k is equal to one, for example, if we consider a thin hollow cylinder. Therefore, by considering that $a'_{CM} = R\alpha$, Eq. (4) can be written as follows:

$$F_A = -kM a'_{CM}. \quad (5)$$

Substituting Eq. (5) into Eq. (3a) we obtain:

$$a'_{CM} = -\frac{1}{k+1} a_{O'}. \quad (6)$$

Therefore, the acceleration of the center of mass in the reference system of the belt is opposite to the acceleration of the belt. Because of this, the friction force is in the same direction indicated in Fig. 1, and

$$F_A = \frac{k}{k+1} M a_{O'}. \quad (7)$$

By imposing the condition for rolling without slipping, $F_A \leq \mu_S N$, where μ_S is the coefficient of static friction, by Eq. (3b) and Eq. (5) we have:

$$a_{O'} \leq \frac{k+1}{k} \mu_S g. \quad (8)$$

Therefore, for $a_{O'} > \frac{k+1}{k} \mu_S g$ slippery of the food can on the conveyor belt cannot be prevented.

3. In the Grocery Store

Consider now the way an observer at the grocery store looks at the can rolling on the conveyor belt. Let start by noticing that, by Eq. (6), the motion of the center of mass C (see Fig. 1) of the food can, observed in the reference system of the belt, can be obtained by reversing the sign of a'_{CM} and by rescaling this quantity by a factor $\frac{1}{k+1}$. Therefore, if the conveyor belt moves through a distance D , point C will move, with respect to point O' , of a distance $d' = \frac{D}{k+1}$ in the opposite direction of the velocity of the belt. As for the distance d travelled by the can toward the cashier, by Galilean transformations and by means of Eq. (6), we may write:

$$a_{CM} = a_{O'} + a'_{CM} = \frac{k}{k+1} a_{O'}. \quad (9)$$

In this way, to find the velocity v_{CM} of C , we integrate both sides of Eq. (9), so that:

$$v_{CM} = \frac{k}{k+1} v_{O'}, \quad (10)$$

where $v_{O'}$ is the velocity of the conveyor belt. In Eq. (10) we have made implicit use of the initial conditions: both

the conveyor belt and the can are at rest at $t = 0$ s, so that the integration constant is zero. By further integrating Eq. (10) to obtain the position x_{CM} of point C , we have:

$$x_{CM} = \frac{k}{k+1} x_{O'} + c, \quad (11)$$

where c is a constant and $x_{O'}$ is the position of an arbitrary point on the conveyor belt.

By Eq. (10), we may argue that the cashier will see the food can move toward him/her at a reduced speed with respect to the speed of the belt. And we hope that he/she does not get angry because physics works this way. Furthermore, from Eq. (11) we may argue that the distance Δx_{CM} travelled by the food can toward the cashier in a time interval Δt , as seen by a customer, is given by the following simple relation:

$$\Delta x_{CM} = \frac{k}{k+1} \Delta x_{O'}, \quad (12)$$

where $\Delta x_{O'}$ is the distance travelled by the sheet of paper in the same time interval Δt . In writing Eq. (12) we notice that the constant c in Eq. (11) does not play any role. The reduction factor, $\frac{k}{k+1}$, depends on the type of object we consider: for a thin hollow cylinder, for example, $k = 1$ and the reduction factor is equal to $\frac{1}{2}$. For a homogenous cylinder, on the other hand, $k = \frac{1}{2}$, so that $\Delta x_{CM} = \frac{1}{3} \Delta x_{O'}$. However, at the grocery store we rarely find either one of the two goods. What we find, in general, are aluminum cans filled with food. We assume that the food inside is rolling solidarily with the aluminum can.

We might now realize that it is a rather difficult task to calculate the moment of inertia with respect to the horizontal symmetry axis of all the rolling cans in the grocery store to obtain the constant k . Nevertheless, we now know a way to determine those moment of inertia, namely, by measuring the ratios $r = \frac{\Delta x_{CM}}{\Delta x_{O'}}$ for various time intervals Δt . By finding an average value \bar{r} for r and the standard deviation Δr , by Eq. (12) we solve for k and its absolute error Δk , obtaining:

$$\bar{k} = \frac{\bar{r}}{1 - \bar{r}}. \quad (13a)$$

$$\Delta k = \frac{1}{(1 - \bar{r})^2} \Delta r. \quad (13b)$$

Therefore, once we find $k = \bar{k} \pm \Delta k$, by recalling that $I_{CM} = kMR^2$, we may calculate, by weighing the whole object to get M , and by measuring its radius R , the moment of inertia of any rolling can in the grocery store.

4. Visualizing the Solution

Let us now consider a possible way of representing the non-constant acceleration of the conveyor belt in the grocery store. Imagine, therefore, that the clerk needs

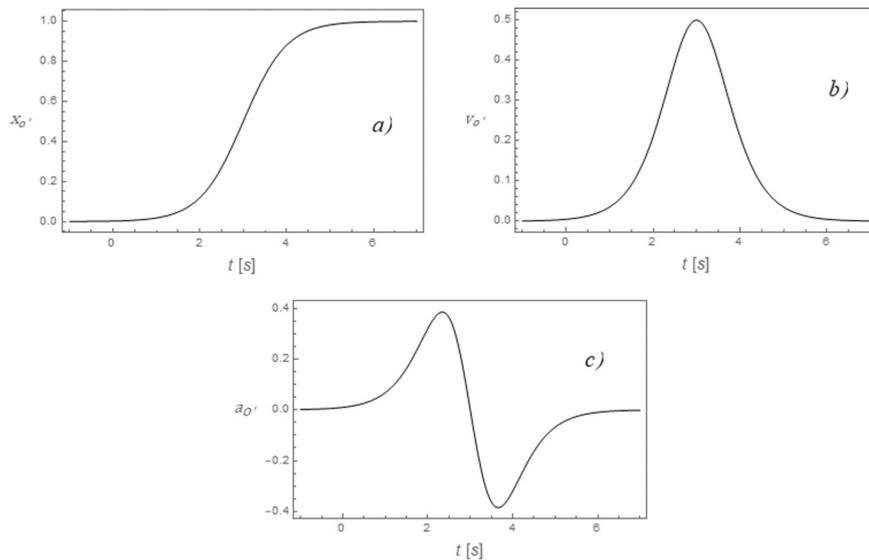


Figure 2: Graphical representation of the position $x_{O'}$ (panel a), the velocity $v_{O'}$ (panel b), and the acceleration $a_{O'}$ (panel c) as functions of time t . The following choice of parameters has been made: $A = 1.0\text{ m}$, $n = 3.0$, $m = 1.0\text{ Hz}$.

only to push a button to make the belt proceed forward of a distance A so that the point O' (see Fig. 1) is made to move according the following smooth function:

$$x_{O'}(t) = A \tanh(mt - n). \tag{14}$$

where n is a dimensionless quantity and m has dimensions of a frequency. The function in Eq. (14) is described in Fig. 2a for $A = 1.0\text{ m}$, $n = 3.0$, $m = 1.0\text{ Hz}$. In Fig. 2b and 2c, the velocity $v_{O'}(t)$ and the acceleration $a_{O'}(t)$ of point O' on the belt are shown, respectively, for the same choice of the parameters A , n , and m . Naturally, $v_{O'}(t)$ is obtained by differentiating $x_{O'}(t)$ with respect to the time, so that:

$$v_{O'}(t) = Am \frac{1}{\cosh^2(mt - n)}. \tag{15}$$

Furthermore, the acceleration $a_{O'}(t)$ is obtained by differentiating $v_{O'}(t)$ with respect to the time, so that:

$$a_{O'}(t) = -2Am^2 \frac{\tanh(mt - n)}{\cosh^2(mt - n)}. \tag{16}$$

By setting to zero the acceleration in Eq. (16), we may well realize that the maximum velocity of the belt is obtained at $t^* = n/m$, as it also appears from Fig. 2b, for which $t^* = 3.0\text{ s}$. For $t > t^*$ a deceleration takes place, to bring the belt at rest again after about $2t^*$.

Of course, the analytic description of the type of motion point O' undergoes on the belt wants only to reproduce, qualitatively, what happens when we stare at the can rolling.

By Eq. (6) we may see that the motion of the center of mass C (see Fig. 1) of the food can, observed in the reference system of the belt, can be obtained by reversing the sign of the kinematic quantities represented

in Fig. 2a–c and by rescaling them by a factor $\frac{1}{k+1}$. Therefore, point C will move, with respect to point O' , of a distance $\frac{A}{k+1}$ in the opposite direction of the velocity of the belt. As for the acceleration, the velocity, and the position of the center of mass of the cylindrical object in the grocery store reference system, we may refer to Eqs. (9), (10), and (11). Finally, the displacement of the center of mass of this rolling body is given by Eq. (12). Therefore, once the conveyor belt motion is defined, we may find the way the cylindrical body moves with respect to the belt, or with respect to the grocery store reference system.

5. Conclusions

A food can, capable of rolling on a conveyor belt in a grocery store, is seen to move slower toward the cashier than a point on the belt. This type of motion attracted the attention of the author, when shopping at the grocery store. A solution has been given relying only on Newtonian mechanics. In this way, the present analysis can be addressed to high-school or undergraduate physics students. Following the inquiry based science education approach [4], a video on this type of motion or a demonstration in class made with common material can be part of an “engage” phase of a physics lecture on Newton’s second law when non-inertial reference systems are considered. By using common material, in fact, a conveyor belt can be replaced by a sheet of paper which the instructor drags with his/her hands on a horizontal table. To simulate the start/stop motion of the conveyor belt, the instructor starts moving the sheet of paper from rest and puts it to stop after having pulled it for a certain distance D . Depending on the type of can used, students should be able to notice that, when the

phenomenon is observed in the reference system of the sheet of paper, the can rolls in such a way that its center of mass is seen to move in a direction opposite to the positive x -axis along which the sheet of paper slides. An observer in the laboratory, on the other hand, may see that the food can still moves, at a slower pace, along the positive x -direction. In fact, during the whole experiment, while the paper slides on the table for a distance D , the center of mass of the can moves along only by a distance $d = \frac{k}{k+1}D$. An analytic description of this phenomenon can also be given. Finally, by the present analysis, students may estimate the value of k , introduced in the expression for the moment of inertia I_{CM} , and may calculate I_{CM} of any food can in the grocery store.

Acknowledgments

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Supplementary Material

The following online material is available for this article:
Appendix

References

- [1] R.A. Serway and J.W. Jewett, *Physics for Scientists and Engineers with Modern Physics* (Cengage Learning, Boston, 2013).
- [2] D. Halliday, R. Resnick and J. Walker, *Fundamentals of Physics* (John Wiley and Sons, New York, 2011), 9 ed.
- [3] R. Cross, *The Physics Teacher* **54**, 76 (2016).
- [4] R.M. Gillies, *Inquiry-Based Science Education* (CRC Press, Boca Raton, 2020).