

The Inertia of Mass and the Newtonian Force

Fabiana Botelho Kneubil*¹

¹Instituto Educacional Futuro da Ciência, São Paulo, SP, Brasil.

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In this paper a relation between inertia of mass and force is presented. In educational contexts, the inertia is generally associated with momentum and energy. However, a direct connection between inertia and force can be also verified in a simple instance of collision. The description of this situation, from the action-reaction pair forces perspective, enlarges the meaning of inertia and highlights the coherence of Newtonian mechanics and the interrelation between concepts and their three laws.

Keywords: Inertia of mass, Newtonian mechanics, Principle of inertia and collisions.

1. Introduction

The mass is one of the most fundamental concepts usually taught at the beginning of high school physics courses. Together with other founding ones, such as force, acceleration, momentum, impulse, space and time, the mechanics theory is organized by means of the Newton's dynamic laws [1].

In general, mass is introduced as a measure of the amount of inertia and, this latter is a fundamental property of bodies related with the natural tendency to resist changes in their state of motion. Even with this definition, the meaning of inertia is not easy to be apprehended by words. In physics, the meaning of a concept is not given by itself but, rather, is developed through its relationship with both actions and other concepts. Bachelard already drew attention that a concept is defined by a *body of notions* and not just by a single element [2].

In this sense, the knowledge can be represented by a network, with concepts, quantities, laws and fundamental principles interrelating with each other [3]. This feature of physical knowledge hampers to define one concept without mentioning others. This way, in didactic approaches, inertia is usually linked with two others, namely momentum and energy. Inertia is also described by Newton's first law and albeit it has a conceptual nature and does not allow to perform any calculation directly, this principle appears in practical situations, such as collisions.

One argues in this work that inertia of mass is also deeply connected with force, but not through the second law $\vec{F} = m\vec{a}$, but rather by means of a kind of reaction to the proper movement. This feature is presented in a simple instance of collision. The main goal is to evidence the relationship of inertia with the force, in order to

enlarge its meaning and show the connection between the concepts of mechanics and the Newton's three laws.

2. The Inertia of Mass

The motion state of a body is associated with both kinetic energy K and momentum \vec{p} . The former is a scalar entity whereas, the latter, vectorial and, these two depend on the velocity and the mass, written as

$$K = \frac{1}{2}mv^2$$
$$\vec{p} = m\vec{v}.$$

These two expressions allow students to construct an intuition about the concept of inertia and to relate K and \vec{p} with the inertia of a body. As momentum and energy are directly proportional to the mass, if the mass increases, the momentum and the energy also increase. Considering the mass a 'measure of inertia' of a body, one may connect the increase of momentum directly with the increase of inertia. It's like mass and inertia are synonymous.

The relationship between \vec{p} and K with the inertia is clearer in collision situations. After two bodies collide with each other, \vec{p} and K of individual bodies may be changed, however, the total momentum is conserved and, for elastic collision, there is also conservation of total kinetic energy.

Considering a simple and well known example of collision, its description can be made from another point of view. From the force perspective, one uses the *Impulse theorem* to understand the cause of the change in momentum. One takes three situations, where two balls of masses m and m' collide. In the three cases, the mass m and the velocity \vec{v}_1 of the first ball are fixed and, the second ball is at rest. The mass of second ball m' is the only thing that changes from one case to another, as presented in the sequence.

* Correspondence email address: fkneubil@gmail.com

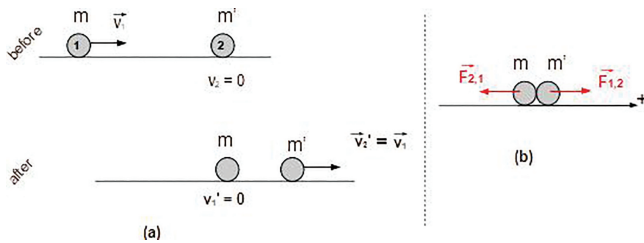


Figure 1: Collision between two masses, where $m = m'$.

• **Situation 1:** $m = m'$

Supposing an elastic collision and taking the initial conditions, namely the masses are equal, and the second ball is at rest ($v_2 = 0$), as shown in Fig. 1(a), one obtains $v_1' = 0$ and $v_2' = v_1$.

The collision may be analyzed from the interaction between two balls, i.e., considering the forces exchanged between them. These forces constitute a *action-reaction pair* and, hence, the force exerted by the ball 1 ($\vec{F}_{1,2}$) on ball 2 is equal to the force exerted by the ball 2 ($\vec{F}_{2,1}$) on ball 1. The Fig. 1(b) shows the forces $\vec{F}_{1,2}$ and $\vec{F}_{2,1}$, which only act during a tiny time interval of contact between two balls. As the intensity of two forces are equal, they provoke the same impulse on the balls. Using the *Impulse theorem* and taking the positive frame to the right, one writes

$$\begin{aligned} -I_1 &= \Delta p = p_F - p_i & I_2 &= \Delta p = p_F - p_i \\ -I_1 &= 0 - m v_1 & I_2 &= m v_2' - 0 \\ I_1 &= m v_1 & I_2 &= m v_2' \\ F_{2,1} \Delta t &= m v_1 & F_{1,2} \Delta t &= m v_2' \end{aligned}$$

• **Situation 2:** $m < m'$

Increasing the mass of ball 2, the situation and the result change. The Fig. 2(a) shows the initial conditions and, Fig. 2(b) represents again the forces exchanged between two balls during the collision. As $m < m'$, $v_2' < v_1$ and the velocity of ball 1 after collision is not zero. It means that the velocity of ball 2 is smaller when comparing with situation 1 and the ball 1 rebounds with v_1' , so that the momentum conservation equation is

$$m v_1 = -m v_1' + m' v_2'. \tag{1}$$

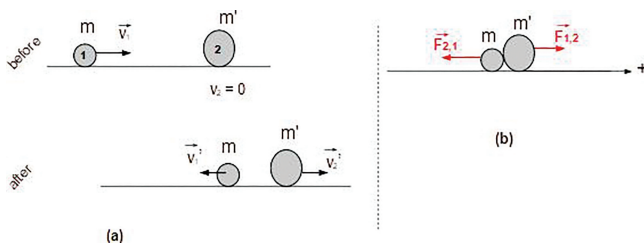


Figure 2: Collision between two masses, where $m < m'$.

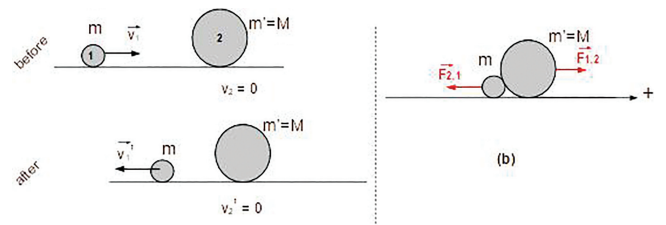


Figure 3: Collision between two masses, where $m \ll m' (= M)$.

The cause of momentum variation is the force and the impulse due to the force in each ball yields

$$\begin{aligned} -I_1 &= -m v_1' - m v_1 & I_2 &= m' v_2' - 0 \\ I_1 &= m (v_1' + v_1) & I_2 &= m' v_2' \\ F_{2,1} \Delta t &= m (v_1' + v_1) & F_{1,2} \Delta t &= m' v_2' \end{aligned}$$

Considering $|\vec{F}_{1,2}| = |\vec{F}_{2,1}|$, the impulses are equal: $I_1 = I_2$ and, therefore, one writes $m (v_1' + v_1) = m' v_2'$, which agrees with eq. (1). Furthermore, as $m < m'$, one concludes that $(v_1' + v_1) > v_2'$ and $v_1 > v_1'$. These results are placed in Table 1.

• **Situation 3:** $m \ll m' (= M)$

The collision in situation 3 is such that the mass of ball 2 is much bigger than mass of ball 1, which allows one to suppose that ball 2 does not vary its momentum and, consequently, the ball 1 inverts the initial velocity. The Fig. 3(a) illustrates this situation and Fig. 3(b) shows the forces exchanged between the balls.

One considers, thus, $v_1' = -v_1$ and $v_2' \cong 0$. The impulse of each ball is written as

$$\begin{aligned} -I_1 &= -m v_1' - m v_1 & I_2 &= M v_2' \\ I_1 &= 2 m v_1 & I_2 &\cong 0 \end{aligned}$$

The results of these three situations are summarized in Table 1, where one just emphasizes the momentum change of ball 1, whose mass is fixed.

Table 1: Momentum change of ball 1 with the increase in m' .

Situation	Relation between masses	$ \Delta \vec{p}_1 $
1	$m = m'$	$m v_1$
2	$m < m'$	$m (v_1' + v_1)$
3	$m \ll m' (= M)$	$2 m v_1$

3. The Inertia And the Force

Observing $|\Delta \vec{p}_1|$ of ball 1 in each situation, as the mass of ball 2 increases, the momentum change of ball 1 also increases. From the point of view of ball 1, it is interesting to note this increase, since the ball 1 has both the same mass and initial velocity in the three situations. The variation of momentum $|\Delta \vec{p}_1|$ is caused by the force

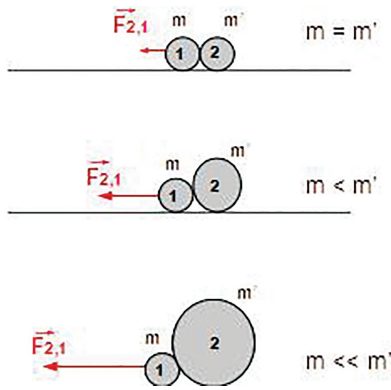


Figure 4: Force $\vec{F}_{2,1}$ with the increase in m' .

applied by ball 2 on ball 1 ($\vec{F}_{2,1}$). It means that this force $\vec{F}_{2,1}$ increases accordingly with the increase of the mass of ball 2. This idea is illustrated in Fig. 4.

This fact can be interpreted from the perspective of the inertia of ball 2. When its mass increases, its inertia also increases and, consequently, the ball 2 resists to the change of its state of motion. In the extreme situation, remains at rest after collision. This resistance is a kind of reaction, manifested by the force applied on ball 1. In other words, the resistance to the movement of ball 2 is associated with the force on ball 1 ($\vec{F}_{2,1}$), as shown in Fig. 4. The mass of ball 2 seems to command the intensity of the force.

The mass of ball 2 generates a kind of rejection to the movement and, the appearance of force $\vec{F}_{2,1}$ is the expression of inertia, resulting in a reaction to the information that it (ball 2) must get moving. In so far as its mass increases, the reaction force exerted on ball 1 increases accordingly and, therefore, the force $\vec{F}_{2,1}$ is deeply connected with the inertia of ball 2. This idea is sketched below.

mass \rightarrow inertia \rightarrow force \rightarrow reaction to movement

This force has a different nature than the force contained in Newton's second law $\vec{F} = m\vec{a}$. This latter plays an active role in the description of movement and, this force related with inertia is more passive. $\vec{F}_{2,1}$ appears when requested and, from epistemological point of view, it distinguishes itself from \vec{F} for being part of Newton's third law.

4. Final Comments

The Newtonian mechanics is a consistent and self contained theory. The organization of concepts and laws constitutes a rigid structure and highly coherent [3]. In Physics, concepts acquire meaning through links with other concepts [4].

A fundamental property such as inertia of mass, in general, is associated with momentum and energy of a body. Moreover, the action of force in mechanics appears

frequently by means of second law [5]. However, in a simple example of collision, it was possible to show an unusual relationship between force and mass. This relation enlarges the meaning of inertia of mass. The presentation of this force as a kind of reaction of mass to the movement shows a tacit link between the first and third Newton's laws, highlighting the coherence of mechanics theory as a whole.

References

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