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Work has nothing to do with energy expenditure

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We show that, contrary to common sense, the author of mechanical work does not necessarily provide the required energy. By the most simple example one can ever conceive – a standing person starting to move, this point is made clear: the floor does not spend energy and it does work on the person (in relation to its own reference frame). We analyse two additional examples in order to ilustrate the subtle aspect of Newtonian theory: despite something is said about where the work is going to, namely the Work – Kinetic Energy Theorem, absolutely nothing is said about where the work comes from (we explain in our conclusions, making reference to Aristotelian philosophy, why it happens in that way). In all these examples, explicit calculations of work are done in both inertial and non-inertial frames.

Keywords: Work and Energy, Newtonian Mechanics, Aristotelian Philosophy, Work – Kinetic Energy Theorem, Non-inertial systems.

1. Introduction

If you work hard all day long, you certainly will be very tired by the end of the day. This is so because you spent a great amount of energy, and thus work is commonly associated to energy. It is very usual to think in that way: since energy is necessary to do some work, the required energy for some work must be provided by the author of that work.

It seems to be a fair reasoning, but it is surprisingly not correct, and, perhaps, it is incredible that this question is not found in all major undergraduate textbooks. Of course, we do not argue the impossibility of situations in which work comes directly from the energy of its author, it happens indeed; but we would like to point out in this paper that this is not necessarily true.

In order to illustrate, in detail, the aspects of our reasoning, we analyse in following sections three pedagogical examples. The first of them (Section 2) is remarkable simple: a person just begins to walk. Perhaps it is the simplest example one could ever imagine, and still it already illustrates that the entity that does the work is not necessarily the source of energy.

In Section 3, we explore the second example, in which two persons, together, move away because one pushed other. In this case, we argue in a similar way, but, in addition, a non-trivial calculation of work in an accelerated frame is promptly done. In this case, the Work - Kinetic Energy Theorem has to be modified (see Ref. [1]), or some trick has to be used as we did actually.

Another interesting example is explored in Section 4, including not only human agencies, but also an engine coupled to a treadmill which can do work as well as spend energy. Next, we draw final comments and conclusions by analysing the underlying philosophy and in this way we explain why in Physics there is not any association between the entity that does work and where the necessary energy come from.

2. The simplest example: a person begins to walk

Let us consider a person with mass m standing still in an horizontal ground. Suddenly, this person starts to walk in such a way that he attains rapidly the constant velocity whose magnitude is v, relative to ground. We will calculate the work done by the person over the floor and the work done by the floor over the person, both in the reference frame attached to the ground, i.e., these quantities will be calculated in relation to the ground. Later, let us repeat these calculations in the reference frame attached to the person. Notice here that these quantities are absolutely precise and non-ambiguous if we mean "work done by body A over body B" simply by "work of the force F which is the force the body A exerts on body B":

$$W_{A-B} = \int \vec{F}_{A-B} \cdot d\vec{r}$$

where $\vec{F}_{A-B}$ is the force the body A exerts on body B and $d\vec{r}$ is the infinitesimal displacement of body B.

The force exerted by the floor on the person is a combination of the normal force (vertical) and the friction (horizontal). Of course, it is the friction the force which gives acceleration to the person, we mean, that force is precisely the resultant force on the person. Given that, we are able to apply the Work - Kinetic Energy Theorem

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to the person, which is

$$W = \Delta K$$

where W is the work of the resultant force acting on the person and ΔK is the variation of kinetic energy experienced by him. As the obvious result, we achieve $W = mv^2/2$ for the work done by the floor over the person in relation to the floor. Figure 1 illustrates the situation. One could say that this example is somewhat artificial because the floor is not actually at rest during the entire process in an inertial reference frame, because of the momentum conservation theorems (both angular and linear). But the mass of the Earth is so incomparable to the mass of a human being, that no Earth movement can be detected by all available means because a person just began to walk. Thus, the floor is effectively at rest in the apropriate inertial frame.

Now let us calculate the work done by the person on the floor in relation to the floor. This calculation is so straightforward that we bet most people would give the wrong answer. The floor is always at rest in relation to the floor itself, so, the work of any force on the floor is zero. Then, the work done by the person on the floor is zero. It is interesting to note here that in spite of the fact that the ground is doing non-trivial work and the person did not do any work on the floor, the floor did not spent energy at all, but the person did as everyone knows. The energy necessary to increase the person's kinetic energy comes from the muscular activity in the legs, so maybe it is easy to give a wrong answer precisely because of the following direct reasoning: "the person spent energy, so he is who actually did the work".

Thus we have shown that the reasoning according to which who spent energy also did work is absolutely false. For the sake of completeness, let us repeat those calculations in relation to the person. In this different reference frame, the application of Work - Kinetic Energy Theorem has to be treated with caution, because this reference system is not inertial [1]. Yet the calculation is very simple: the work done by the floor on the person is zero, because the person is always at rest in relation to himself, and the work done by the person on the floor is equal to $mv^2/2$, because the magnitude of the force exerted by the person on the floor is the same magnitude of the force exerted on the person by the floor, at each instant of time; and the infinitesimal displacements of the person as seen from the floor and of the floor as seen from the person are equal (and opposite).

So we have shown that the work done by the person in reference to himself is the amount of energy necessary to cause such a movement, $mv^2/2$. One can ask if the criterion of adopting the own referential of a body is the solution to relate work done to energy spent by the same body. The answer is no, because the work done by the floor in relation to the floor itself is non zero and it does not spend any energy at all.

3. Second example: two people move away from each other without friction

Now let us consider the example of two people (John and Mary) moving away from each other in a smooth and horizontal floor (without any friction). Initially, they were at rest, but Mary (with mass M) suddenly pushed John (with mass m) so their final velocity relative to the floor was V and v, respectively (see Figure 2). In relation to the ground, the work done by Mary on John is his variation of kinetic energy: $W_{MJ} = mv^2/2$. Similarly, the work done by John on Mary is $W_{JM} = MV^2/2$.

We knew in the previous example that the person is the body who actually spent energy, but what can be said about the case of John and Mary? If Mary pushed John, so it is clear that she has provided the energy for movement, but this information is absolutely necessary to answer such a question. Actually, both can spend energy in any proportion. Later on we will explain why this indeterminacy shows up.

What would be the work done by Mary in relation to herself? This calculation is a very interesting task and deserves attention. We are in a position to calculate the work of the force done by Mary on John in the Mary's reference frame, which is a non-inertial frame. First, let the total displacement of John while he is accelerated by Mary denoted by Δx (which can be positive or negative depending on the coordinate axis Ox) in relation to the ground. Figure 2 illustrates this example. Of course we already know that

$$F\,|\Delta x| = \frac{1}{2}mv^2 \qquad (1)$$

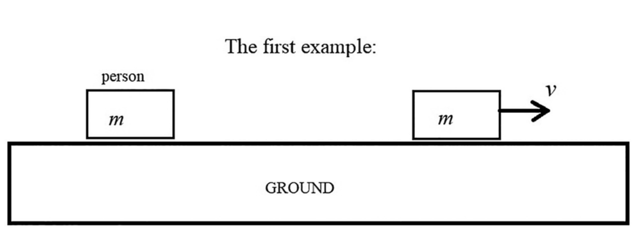

Figure 1: All entities are represented by blocks in this figure and in the next ones. Relative to the ground, the person, with mass m, acquires the velocity v. Thus, the work done on the person by the ground is $W = mv^2/2$.

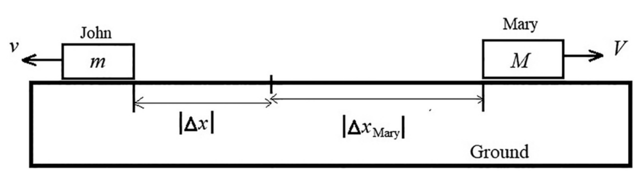

Figure 2: Mary pushes John and both get away. All quantities above are given in reference to the ground, but notice that the magnitude of relative displacement (or velocity) is equal to the sum of the magnitudes of each displacement (or velocity): $\left|\Delta x'\right| = |\Delta x| + |\Delta x_{Mary}|$ and $|V_{rel}| = V + v$.

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where F is the average of the magnitude of the force exerted by Mary on John.[1] The work done by Mary on John in respect to Mary's reference system is written then as

$$W = F\left|\Delta x'\right| \tag{2}$$

where $\Delta x'$ is the total displacement of John in relation to Mary while she is exerting force on John. Observe that the equation (1) can be seen also as the Torricelli's equation:

$$2\frac{F}{m}\left|\Delta x\right| = v^2$$

The same equation can be used for calculating $F\left|\Delta x'\right|$ in a very direct and simple way by noting that the net acceleration of John as seen by Mary, a_{rel} is the sum of each acceleration in relation to the ground: $a_{rel} = a_{John} + a_{Mary}$ We can write then

$$2a_{rel}\left|\Delta x'\right| = V_{rel}^2$$

where V_{rel} is the magnitude of the final relative velocity. As $V_{rel} = v + V$ and $a_{John} = F/m$ and $a_{Mary} = F/M$, we obtain

$$2\left(\frac{F}{m} + \frac{F}{M}\right)\left|\Delta x'\right| = (v + V)^2 \tag{3}$$

After using the relation $mv = MV$ (from momentum conservation) in the above equation, one can achieve the result

$$W = F\left|\Delta x'\right| = \frac{M + m}{M}\left(\frac{mv^2}{2}\right) \tag{4}$$

Thus we have calculated the work done by Mary on John in relation to herself. Again, this quantity has nothing to do with the energy spent by Mary, which is the total variation of kinetic energy in the ground's system of reference: $mv^2/2 + MV^2/2$. Notice that, in the Mary's reference, the variation of the total kinectic energy is greater:

$$\Delta K = \frac{1}{2}m\left(v + V\right)^2 = \left(\frac{M + m}{M}\right)^2\left(\frac{mv^2}{2}\right) \tag{5}$$

The above formula represents a naïve calculation of the work done by Mary, based on the Work - Kinetic Energy Theorem, but the error lies in the fact that the work of inertial force (or fictitious force) was neglected. Indeed, if one adds the work of fictitious force to the work in equation (4), the result obtained is exactly the right-hand side of equation (5).

4. Third example: a child begins to walk in a treadmill

Now consider a more complicated example, where a child (mass m) is moved by a long treadmill, like those in airports. Initially, the child is at rest in relation to the treadmill, which moves at a constant velocity u. Such velocity is kept unchanged by its engine. Thus, the child is moving with speed u in relation to the exterior ground. Suddenly, he starts to run at the same direction and reaches the constant speed v in relation to the treadmill, such that he has speed $u + v$ in relation to the ground (Figure 3).

When we look at this problem adopting the treadmill's reference system, it becomes clear that this is the same problem as the first example in Section 2 above. Then, the interesting tasks will be the calculation of the work done by the child and by the treadmill in relation to the ground. Here not only the child can spend energy, but also the treadmill's engine. The work done by the child on the treadmill deserves extra care because the force exerted by the child on the treadmill is not the total force and thus the Work - Kinetic Energy Theorem cannot be used. But the theorem can be used in calculating the work done by treadmill on the child:

$$W_{tread} = \frac{1}{2}m(u + v)^2 - \frac{1}{2}mu^2 = \frac{1}{2}mv^2 + muv$$

The work done by the child on the treadmill is the work done by the horizontal (friction) force exerted by the child on the treadmill during the time Δt in which child has acceleration. This work is given by

$$W_{child} = -Fd$$

where d is the travelled distance by treadmill, and this is negative because treadmill moves in the opposite direction from the force exerted on it. Thus, as $d = u\Delta t$, we get

$$W_{child} = -F\,\Delta t\,u$$

But $F\Delta t$ is nothing more than the magnitude of the impulse received by treadmill (which is equal to the magnitude of the impulse received by the child, mv). Thus,

$$W_{child} = -muv$$

Now it is interesting to stress that someone doing negative work does not necessarily mean positive work being done on the same person. If this was so, the work done by Mary in the previous example would be equal to the minus work done by John. Negative work means just negative work, nothing more.

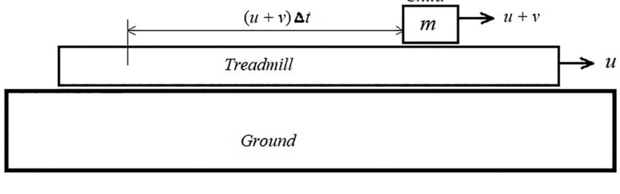

Figure 3: Diagram describing the third example. The child takes a time Δt to gain velocity $u + v$, by which the treadmill travels the distance d. Notice $d = u\Delta t$.

[1] Notice that, despite the actual force in each instant of time may have not magnitude F, this relation remains absolutely correct.

Because the first example corresponds to the current example described in relation to the treadmill's reference frame, we already know that the child is the agent who is promoting the action and who is spending the quantity of energy given by $mv^2/2$. Then, the remaining muv corresponds to the energy spent by the treadmill's engine. Notice that the work done by the child is numerically equal (up to the minus sign) to the energy spent by treadmill, but this is simply a coincidence.

5. Concluding remarks: the underlying philosophy

We have emphasize two main reasonings which seem to be correct but are actually wrong: (i) when something does work, it also spends the same amount of energy, and (ii) if somebody does, for example, $-200\,J$ of work, thus $200\,J$ of work were done on him. Observe, first, that the reason by which assertion (ii) appears to be correct is precisely the energy conservation.[2] But we say that this constraint does not exist, such that the conservation of energy will remain intact even if somebody does some amount of work and receives another different amount of work. Thus, the assertion (i) is the key error which underlies proposition (ii). Let us explain the philosophical bases behind the claim that (i) is wrong.

In the first place, let us note that Newtonian dynamics is clear and unambiguous: Work - Kinetic Energy Theorem establishes the exact equivalence between the total *work done on* a body and the (kinetic) energy variation of the same body. Rigorously nothing is said about the *work done by* a body, but just the work done on it. In other words, Physics tells us about where the work went to, not where it came from. There are some situations in which Physics can say about where the work done came from. Those cases are related to potential forces; we know indeed that the work done by any potential force is precisely the variation of the potential energy function defined for that force. In this case, one can say that the work done is equal to the loss of potential energy.

Observe that there is a very subtle difference between saying "The necessary energy for some work came from the potential energy" and "The necessary energy for some work came from John". *John* is different from *potential energy* not just because one is living and another is not, but because they dwell at different ontological levels. This kind of differentiation was pointed out by Wolfgang Smith in his book *O Enigma Quântico* [2]. John is referred to a concrete being, while "potential energy" is a physical concept based on kinematics and matter, measurable and translated into other concepts like velocity, acceleration and mass, expressed in the mathematical instance. Thus, the potential energy is an abstraction from corporeal world and, although it is something based on reality, it does not exist in the corporeal realm.

[2] As a matter of fact, assertion (ii) is correct when the First Law of Thermodynamics is applied to engines and refrigerators. But notice that it is not a coincidence that this law ensures the energy conservation. Besides, the work done by, say, an engine is independent on the reference system, so this case is different from the present approach.

From the above considerations, we would say that the case of potential forces is not even an exception for our claim that work is not energy loss, because we mean energy loss by someone or by something concrete in the corporeal world.

Concerning the four types of causes discovered by Aristotle[3], the very act of doing work seems to refer to the effective cause. John gained velocity because there were all necessary conditions for that (material and formal causes) and, most important, Mary wanted to push him (final cause). Eventually, then, Mary did the action: she was who transformed possibility into fact, or power into act. Before she pushes John, Mary was the missing part to actualize this power, so in this sense she was sufficient for that actualization. That is why the efficient cause is also called "sufficient cause".

At best, Newtonian mechanics deals with just material and efficient causes (as pointed out by Alexander Koyré in ref. [3]). In this way, the remaining final and formal causes went away in mechanical explanations. Indeed, mechanical notions such as mass and acceleration are abstractions from concrete reality – they are defined in the basis of quantity and place (two among the ten categories of Aristotle). The definition of mass[4], for example, is not about any substance, but is strictly a definition concerning the accidents of quantity and of place, both referred to truly unknown substances. The nature of a human being is obviously much more complex than the inorganic nature, but, even in the inorganic scenario, a mechanical body (homogeneous by definition, composed by a kind of materiality which is intelligible only by means of quantity and place) cannot produce motion. That is why the Newtonian mechanics cannot say anything about the causation of work but is restricted to describe its effects (variation of kinetic energy, heat production etc.).

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References

[1] R.A. Diaz, W.J. Herrera and D.A. Manjarrés, *American Journal of Physics* **77**, 270 (2009).

[2] W. Smith, *The Quantum Enigma: Finding the Hidden Key* (Sophia Perennis, Hillsday, 2005).

[3] A. Koyré *Newtonian Studies* (Harvard University Press, Cambridge, 1965).

[4] E. Mach*, History and Root of the Principle of the Conservation of Energy* (The Open Court Publishing, Chicago, 1911).

[3] The formal cause, the material cause (intrinsic causes), the efficient cause and the final cause (extrinsic causes).

[4] See, for example, the interesting point claimed about the definition of mass by Ernst Mach in Ref. [4].

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