# Application of $\mathrm{P} / \mathrm{F}$ Brass ratio method in the context of fast-paced adolescent fertility decline* 

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Adolescent fertility -fertility rates at ages 15-19- fell substantially (around 30 percent) between 2000 and 2010. It was the first time Brazil experienced such a decline in those ages since 1970, when the census included one question about children born in the past 12 months. This phenomenon has an important implication for the $P / F$ Brass ratio technique: it underestimates the cumulated current fertility up to age group 20-24 $\left(F_{2}\right)$, considering this cohort's previous fertility experience. Therefore, the $P_{2} / F_{2}$ value, used as an adjustment factor for the reported fertility level, is significantly overestimated. This paper discusses this issue and proposes an alternative to correct the reference period error in the 2010 Demographic Census in Brazil. The results of applying the proposed alternative in this specific context were very similar to those obtained using different techniques, thus supporting the strength of our alternative.

Keywords: P/F Brass. Fertility decline. Adolescent fertility. Demographic census.

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## Introduction

The availability of high-quality fertility data is undoubtedly, the ideal setting for any sound analysis. However, due to many contextual factors, this is not always the case. One of the techniques developed by Brass aimed at estimating age-specific fertility rates derived from poor data quality settings, such as the African one, and applying it to other contexts of equally poor data quality (BRASS; COALE, 1968).

The method, called the P/F Brass ratio technique, is a procedure for correcting the reference period error of reported period fertility in censuses or surveys. In order to apply the method, it is necessary to have the following information: female population age structure at reproductive ages; number of live births in the 12 months previous to the reference census date (period fertility); and the number of children ever born alive (retrospective fertility) (BRASS; COALE, 1968).

One of the conditions stated by Brass to apply the technique is constant fertility. However, this condition is unmet in the Brazilian case since the 1960s. A number of authors have drawn attention to the implications of unsatisfying this condition when estimating Age-Specific Fertility Rates (ASFR) and Total Fertility Rates (TFR) using the P/F Brass approach (CARVALHO, 1982; CASTANHEIRA; KOHLER, 2016; FEENEY, 1996; MOULTRIE; DORRINGTON, 2008; SCHMERTMANN et al., 2013; UNITED NATIONS, 1983). Nevertheless, in this paper we show that if the P/F ratio for age group 20-24 is employed as a good reference period error estimator, it is required that fertility at ages 15-19 (adolescent fertility) have remained constant in the 5 years before the census reference date.

In this case, to estimate the reference period error, the only information required is that pertaining to cumulated fertility of women aged 20-24 years (mean parity) and the period fertility of women aged 15-19 and 20-24 years. ${ }^{1}$ The mean parity at ages 20-24 is a cohort measure $\left(P_{2}\right)$. The reference period error is computed through the $P_{2} / F_{2}$ ratio, with $F_{2}$ representing the cumulated period fertility estimated from both $f_{1}^{*}-$ reported period $\mathrm{ASFR}_{15-19}$ - and from $f_{2}^{*}$ - reported period ASFR $_{20-24}$.

This paper has two goals: First, to discuss the application of the P/F Brass technique in a scenario of substantial adolescent fertility decline, demonstrating the limitation of its applicability and contributing to the ongoing debate on the topic. The fast adolescent fertility decline taking place in Brazil between 2000 and 2010 motivated this first topic (BERQUÓ; CAVENAGHI, 2014). Secondly, to demonstrate the empirical implication of such a dramatic decrease in the adolescent fertility schedule, as occurred in Brazil over the last decade, on the fertility estimates obtained by the Brass method.

[^1]Despite the initial motivation of this research in finding an alternative, within the $\mathrm{P} / \mathrm{F}$ Brass method framework, for achieving robust estimates of the level of fertility in Brazil, the issue of an overall discussion on the suitability of the method amidst changing fertility conditions, specifically at adolescent ages, became a more pressing matter. This discussion is of the utmost importance for different developing country contexts, both at the aggregate population level and for specific subgroups. Thus, in the following section, we present many views on this topic, some of which are often conflicting.

On one hand, Castanheira and Kohler (2016, p. 4) have a well-defined position against the application of the P/F Brass method in the current Brazilian scenario: "But as we illustrate in the case of Brazil, this adjustment in the context of contemporary low fertility and rapid fertility postponement may 'do more harm than good' [...]. We therefore suggest that the P/F method in official TFR statistics should be discontinued in this context". On the other hand, Cavenaghi and Alves (2016, p. 201) state that "Neither the correct TFR value in year 2010, nor the value on average two or three years before, are reason for dispute, since all available data and methods present some level of limitation." ${ }^{2}$

In this paper, we show that the P/F Brass method application, adjusted due to the fast decline of adolescent fertility, can still yield robust results. However, caution is advised, due to the fact that, in the Brazilian case, without proper adjustments, estimates are biased upwards, significantly affecting population projections, for instance.

It is important to note that, once the method is particularly applicable to census data, it allows for the study of population subgroups, whose identification are otherwise not possible from vital registration statistics. Hence, the method still proves relevant and useful, even when birth coverage is complete and universal.

Although this research focuses on the first decade of this Century (2000-2010), information from other periods and diverse data sources were employed in order to test for the alternative methodological approach presented. Altogether, we used the 1970, 1980, 1991, 2000 and 2010 Demographic Census, in addition to the 1990 and 2000 National Household Surveys (PNAD) as well as the 2009, 2010 and 2011 Vital Registration System (SINASC/Datasus).

Analyses for five of the major Brazilian regions requires data from the birth registration after proper assessment and quality evaluation; estimates obtained using the Gompertz Relational model (MOULTRIE, 2013a; ZABA, 1981) and the Gompertz Synthetic model (MOULTRIE, 2013b) were also used.

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## The P/F Brass method

Brass developed a technique to correct for the reference period error of reported period fertility in censuses or surveys (BRASS; COALE, 1968). Other types of error in reported fertility are usually considered negligible. However, if there is no selectivity according to childbearing age, these other errors are also corrected by the Brass technique.

In order to apply the method, two pieces of information on age-specific fertility are essential: the number of live births in the 12 months before the census or survey reference date; and the total number of children ever born (CEB) alive until the reference date. The average number (by woman) of live births in the 12 months before the census or survey reference date corresponds to the period fertility. The mean number (by woman) of CEB alive until the census reference date corresponds to the cumulated or retrospective fertility (or parity) of these women. In this sense, the latter is a stock variable for each cohort (BRASS; COALE, 1968).

The approach compares age-specific rates based on two pieces of information: retrospective fertility and cumulated period fertility. The mean number of CEB alive during a woman's reproductive life, for a given age or age group $i$, corresponds to the mean parity of this cohort $\left(P_{i}\right)$. The average number of live births during the previous 12 months for each woman of a given age or age group $i$, corresponds to the period age-specific fertility rate of these women ( $f_{1}$ or $A S F R_{i}$ ).

If the $A S F R_{i}$ are multiplied by five, considering women of reproductive age at five-year age groups, the resultant is the mean number of children that a woman would expect to bear during each age group. The cumulated value of these rates until the last age group represents the total average number of live births that a hypothetical cohort of women would bear throughout their reproductive lives, if it experiences throughout its reproductive life the current $A S F R_{i}$. The ratio parity to period cumulated fertility on each age is thus defined as $P_{i} / F_{i}$, where:
$P_{i}=$ mean parity, by woman, or the mean number of total CEB alive of women from the age group $i$, during their reproductive lives;
$F_{i}=$ period cumulated fertility, by woman, until age $i$.
In its original conception, the Brass technique involved some important assumptions and conditions (BRASS; COALE, 1968; CARVALHO, 1982). Such assumptions are: the reference period error is independent of mother's age and hence proportionally constant in each age; there is no omission in the report of the number of CEB or cumulated fertility among women aged 15-29; but it would increase by women age in the remaining groups (memory error). Among the conditions, the most important are constant fertility; population closed to migration or with no fertility differentials among migrants and non-migrants; and non-selective mortality towards fertility in a way that, for each age, the risk of childbearing among those women who died is the same as those who survived. ${ }^{3}$

[^3]The first assumption implies a proportional constant correction of the $A S F R_{i}$ levels, computed from reported period fertility, while the second assumption allows for estimating the reference period error through the comparison between parity $\left(P_{i}\right)$ and cumulated period fertility $\left(F_{i}\right)$ in the firsts age groups of the reproductive period. The three conditions and the first assumption guarantee that for any given cohort the past fertility is accurately represented by the distribution of the real age-specific fertility rates, which are the reported period fertility rates corrected for the reference period error.

If the ideal conditions are met, when the $\mathrm{P} / \mathrm{F}$ Brass technique is applied, the $\mathrm{P} / \mathrm{F}$ ratio declines throughout the age groups, as a consequence of the increasing memory error among older women (BRASS; COALE, 1968). As women's age increases, underestimation of $F_{i}$ will also increase, once $F_{i}$ represents the women's cumulated fertility experience at age $i$ since the beginning of their reproductive period. Considering a sustained fertility decline, underestimation in $F_{i}$, tends to offset the memory error in $P_{i}$.

It is extremely important to define which $P_{i} / F_{i}$ is to be used in order to correct the level of current fertility pattern (reported ASFR), i.e., to correct the reference period error. Information on the total number CEB alive of women aged above 25 or 30 years old, a stock variable, tends to incorporate an increasing memory error. ${ }^{4}$

In addition, the older the reproductive age group selected for estimating the reference period error, the higher the impact on the robustness of the estimator, of the non-fulfilment of the three ideal conditions for the technique application. Furthermore, the live births of women aged 15-19 years may suffer considerable fluctuations each year, not to mention other issues, such as higher sampling errors due to considerable small number of births, if the information source is not the census universe. In order to tackle these issues, Brass and Coale (1968) suggest using the ratio of parity to period cumulated fertility related to the 20-24 age group $\left(P_{2} / F_{2}\right)^{5}$.
$P_{2}$ refers to the mean parity of women aged 20-24 and is a cohort measure. $F_{2}$ is estimated as follows: (5) $f_{1}^{*}+\left(k_{2}\right) f_{2}^{*}$, with $f_{1}^{*}$ and $f^{*}$ being the reported ASFR at ages 15-19 and 20-24, respectively, and the $k_{2}$ term is a multiplier factor that allows to obtain fertility accumulated between ages 19.5 and 22.5 . The term $\left(k_{2}\right) f_{2}^{*}$ refers to the experience of this last cohort, which $P_{2}$ also corresponds to. ${ }^{6}$ In fact, the (5) $f_{1}^{*}$ term is the only one that does

[^4]not refer to the 20-24 cohort in the reference census or survey date, since it refers to the 15-19 cohort. $P_{2} / F_{2}$ will be erratic as an estimate of the reference period error, as long as the ASFR of women aged 15-19 in the current census date diverges from the ASFR of women aged 20-24 in the 5 years before the census date considered. Hence, the only reason why the current cumulated fertility $F_{2}$ is not the actual experience of the cohort aged 20-24, discounting the reference period error, is because the term (5) $f_{1}^{*}$ is borrowed from young women aged 15-19 (i.e., a younger cohort) in the census or survey date.

Thus, when using the $P_{2} / F_{2}$ ratio as an estimator for the reference period error, it would be necessary that only ASFR $_{15-19}$, in the reference census date be equal to the same age group 5 years before. ${ }^{7}$ In other words, only adolescent fertility should have remained constant, and, even so, for a short period.

Discussions regarding the suitability of the technique's application in a scenario of general fertility decline are not new (CARVALHO, 1982; CASTANHEIRA; KOHLER, 2016; CAVENAGHI; ALVES, 2016; FEENEY, 1996; SCHMERTMANN et al., 2013; UNITED NATIONS, 1983). However, we consider the focus on adolescent fertility should be a key part of the debate, and it has yet to be brought under the spotlight.

## The $\mathrm{P} / \mathrm{F}$ ratio in a scenario of adolescent fertility decline: the Brazilian case

Brass sustains that the P/F technique still yields good estimates, even when fertility is declining (BRASS, 1985). Also, according to Carvalho (1982), at the beginning of a transitional process of fertility decline, ASFR at ages 15-19 is relatively small when compared to the 20-24 age group, the technique may still yield good fertility estimates. In addition, the decline, when it happens, affects the beginning of the reproductive cycle to a lesser degree than that observed in older women.

In the Brazilian case, as was the case in several Latin American countries, the overall fertility decline happened hand in hand with an increase in adolescent fertility until year 2000, resulting in an error, by excess, in $F_{2}$ as an estimate of the cumulated fertility of the 20-24 age group. Assumptions that until 2000 women aged 20-24 years presented the same ASFR as women aged 15-19, five years earlier, yielded an error by excess in $F_{2}$ as an estimate of the cumulated fertility experienced by women aged 20-24 years at the census date, and, consequently, resulted in an underestimated $P_{2} / F_{2}$ correction factor. Nonetheless, this error would be small, since adolescent fertility had been slowly increasing. Brass's technique yields a TFR estimate with an error of between $2 \%$ and $3 \%$, assuming a linear variation in adolescent fertility during intercensal periods (1980 to 1991; 1991 to 2000) (MARQUES; CARVALHO, 2018).

Between 1980-1991 and 1991-2000, the ASFR of the first age group increased, respectively, 11,68\% and 9,38\%, for all regions in Brazil (Tables 1 and 2). When assuming

[^5]that $\operatorname{ASFR}_{15-19}$ declared in the 1991 and 2000 censuses were the same as $\operatorname{ASFR}_{20-24}$ in the previous 5 years, an error, by excess, of around 4.8 percent, in 1991, and 4.4 percent, in 2000, is produced in the term (5) $f^{*}{ }_{1}$ of $F_{2}$. Consequently, the quotient $P_{2} / F_{2}$, that should provide the correction factor for the reference period error, is underestimated by about 2.0 percent in both census years.

The 2010 Demographic Census was the first to show a $30 \%$ decline in the reported fertility among women aged 15-19 years at the country level, between 2000 and 2010. This remarkable negative change in adolescent fertility levels results in a significant overestimation of the $P_{2} / F_{2}$ as an adjustment factor of the reference period error when obtained by employing the traditional Brass approach, due to a substantial error in the denominator.

It is important to highlight that the applicability of the Brass's technique, in this specific context, requires a proper assessment of the adolescent fertility trajectory, mainly when it is declining, which is very often the case in the current developing settings. For example, if all fertility decline takes place at the beginning of a decade, the observed adolescent fertility rates in the reference census date will be the same as the one five years before. In this case, the application of the method is not compromised if the traditional approach of the $P_{2} / F_{2}$ ratio as an estimator of the reference period error is used. On the other hand, if all decline is concentrated towards the end of the decade, the adolescent fertility that provides a pertinent $F_{2}$ and yields a good $P_{2} / F_{2}$ reference period estimator, i.e., adolescent fertility from five years earlier will be the same as the one observed at the beginning of the decade.
$P_{2} / F_{2}$ estimates for Brazil and its main regions (Table 3 and Figure 1) present, in general, a reasonable stability in the three censuses between 1980 and 2000. This suggests that during the period there was no significant variation in the reference period error, at national and regional levels. However, the dramatic increase in the $P_{2} / F_{2}$ ratio during years 2000 and 2010 is due to a substantial decline in adolescent fertility and not to an increase in the reference period error.

Tables 1 and 3 show marked differences between the reported ASFRs and the $P_{2} / F_{2}$ ratios between 1970 and 1980. These are largely due to a change in the census question regarding the number of live births in the 12 months before the census reference date, implemented in $1980 .{ }^{8}$ Additionally, some fluctuations observed in the regional $P_{2} / F_{2}$ values for years 1980-2000 were also most probably due to sampling errors than to reference period errors. Nonetheless, the increase during the 2000-2010 period was systematic in all five major regions, as a clear consequence of the plummeting $\operatorname{ASFR}_{15-19}$, and thus leading to an increase in the $P_{2} / F_{2}$ ratio. The underestimated $F_{2}$ as representative of the cumulated fertility of women aged $20-24$ resulted in an overestimated $P_{2} / F_{2}$ ratio as a reference period error estimator.

[^6]TABLE 1
Reported age-specific fertility rates for women at the 15-19 age group
Brazil and Macro-regions - 1970-2010

| Regions | 1970 | 1980 | 1991 | 2000 | 2010 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Brazil | 0.0468 | 0.0582 | 0.0650 | 0.0711 | $\mathbf{0 . 0 4 9 7}$ |
| North | 0.0634 | 0.0941 | 0.0996 | 0.1060 | 0.0767 |
| Northeast | 0.0494 | 0.0630 | 0.0671 | 0.0792 | 0.0573 |
| Southeast | 0.0378 | 0.0500 | 0.0552 | 0.0586 | 0.0392 |
| South | 0.0517 | 0.0541 | 0.0641 | 0.0634 | 0.0420 |
| Center-West | 0.0664 | 0.0736 | 0.0790 | 0.0789 | 0.0534 |

Source: IBGE. Demographic Census 1970-2010.

TABLE 2
Variation in the reported age-specific fertility rates for women at the 15-19 age group Brazil and Macro-regions - 1980-2010

|  |  |  | In per |
| :---: | :---: | :---: | :---: |
| Regions | $\mathrm{f}^{\star} 1^{1991} / \mathrm{f}^{\star}{ }_{1} 1980$ | $\mathrm{f}^{\star}{ }_{1} 2000 / \mathrm{f}^{\star}{ }_{1}{ }^{1991}$ | $\mathrm{f}^{\star}{ }_{1} 2010 / \mathrm{f}^{\star}{ }_{1}{ }^{2000}$ |
| Brazil | 11.68 | 9.38 | -30.10 |
| North | 5.84 | 6.43 | -27.64 |
| Northeast | 6.51 | 18.03 | -27.65 |
| Southeast | 10.40 | 6.16 | -33.11 |
| South | 18.48 | -1.09 | -33.75 |
| Center-West | 7.34 | -0.13 | -32.32 |

Source: IBGE. Demographic Census 1980-2010.

TABLE 3
$P_{2} / F_{2}$ estimates according to Brass traditional method Brazil and Macro-regions - 1970-2010

| Regions | 1970 | 1980 | 1991 | 2000 | 2010 |
| :--- | ---: | ---: | ---: | ---: | :--- |
| Brazil | 1.308 | 1.115 | 1.117 | 1.105 | 1.193 |
| North | 1.362 | 1.139 | 1.180 | 1.151 | 1.236 |
| Northeast | 1.343 | 1.186 | 1.207 | 1.125 | 1.215 |
| Southeast | 1.298 | 1.069 | 1.069 | 1.084 | 1.179 |
| South | 1.270 | 1.110 | 1.042 | 1.091 | 1.167 |
| Center-West | 1.273 | 1.127 | 1.099 | 1.131 | 1.189 |

Source: Basic data from IBGE. Demographic Census 1970-2010.
Moultrie and Dorrington (2008) simulated the effects of violating the conditions of P/F Brass technique. Despite their discussions on varying mortality conditions, due to the strong prevalence of AIDS in the African continent, what interests us is their simulation on changing fertility schedules. Their results show that the greatest error in TFR estimation using the $\mathrm{P} / \mathrm{F}$ ratio results from changes in the fertility level. Nonetheless, even in the presence of those errors, the TFR estimated through Brass technique yielded more accurate estimates than other alternative approaches.

They found that when the fertility is declining, the magnitude of error is around $5 \%$ in the TFR. However, in an increasing fertility scenario, and after the increase reaches its peak, this error could amount to approximately 10\% (MOULTRIE; DORRINGTON, 2008). In
our case, we must consider that the age-specific fertility rate decline of Brazilian women aged $15-19$ between 2000 and 2010 was very strong (around $30 \%$ ), and thus the application of the traditional Brass method yields even greater error than that found by Moultrie and Dorrington (2008).

FIGURE 1
$P_{2} / F_{2}$ estimates according to Brass traditional method Brazil and Macro-regions - 1970-2010


Source: IBGE. Demographic Census 1970-2010. Own elaboration.

## Data and proposed alternative

In this paper, we use data from the 1970-2010 Demographic Censuses. The focus is on the 2010 census, since issues pertaining to the reference period estimate error are more acute for that year, due to the strong decline in $\mathrm{ASFR}_{15-19}$ during the first decade of the $21^{\text {st }}$ Century.

Additionally, data from the Brazilian National Household Survey (PNADs) of the 1990 and 2000 decades were also used. Information on the annual average number of live births from the National Information System on Live Births (SINASC/Datasus-Ministry of Health), period 2009-2011, was also used.

As mentioned previously in this work, the reference period error estimated in the traditional way, by means of the $P_{2} / F_{2}$ ratio using fertility data from the 2010 Demographic Census, is significantly overestimated due to the dramatic decline in adolescent fertility during the first decade of the $21^{\text {st }}$ Century. The assumption that period fertility information yields the true form of the fertility function (the $f_{i}^{*}$ distribution) is still valid though, as evidenced by a wide array of research conducted in different countries, Brazil included.

If we deem the $P_{2} / F_{2}$ ratio derived from the 2010 Brazilian census data inadequate for correcting the reference period error, then the question arises regarding what alternatives may exist in order to obtain a good fertility estimate derived from the reported period
fertility for the whole country including its regions and federal units. The issue narrows down to finding which approach provides the best estimate for the reference period error, in order to adjust the reported current fertility in the 2010 Brazilian Demographic Census.

A first approach to deal with this issue is to use the reported 2005 PNAD $f_{1}{ }^{*}$, or the average of all $f_{1}^{*}$ in years 2004, 2005 and 2006. This corresponds to the adolescent fertility experience, in the middle of the decade, of women aged 20-24 in 2010. However, due to reasons yet to be identified, the $f_{1}^{*}$ derived from the census data is systematically higher than the figure derived using PNADs (Figure 2). Hence, using the $f_{1}^{*}$ estimated in this way for computing the $P_{2} / F_{2}$ ratio leads to an overestimated correction factor of the reference period error.

FIGURE 2
Reported age-specific fertility rates for women aged 15-19 years
Brazil - 1980-2010


Years

- Census
- PNADs

Source: IBGE. Brazilian National Household Survey (PNAD) 1992-2009; Demographic Census, 1970-2010.
Another approach is to obtain the average $f_{1}^{*}$ from the 2000 and 2010 Demographic Censuses, in order to estimate the $2005 f^{*}$, as an approximate fertility experience of women aged 20-24 years in 2010, when they were part of the 15-19 age group in 2005. This results in a period cumulated fertility $\left(F_{2}\right)$ which should be closer to the experience of the cohort aged 20-24. However, the onset and pace of fertility decline in the various regions in Brazil were not the same (POTTER; SCHMERTMANN; CAVENAGHI, 2002). This was also the case for adolescent fertility. The series of reported age-specific fertility for women in the 15-19 age group (Figure 3) show the same results through the various PNADs, ${ }^{9}$ despite considerable regional fluctuations that are largely due to PNAD sampling errors.

[^7]FIGURE 3
Reported age-specific fertility rates for women aged 15-19 years
Brazil and Macro-regions - 1992-2009 (1)


Source: IBGE. Brazilian National Household Survey (PNAD) 1992-2009.
In the North and Northeast, the $f^{\star}{ }_{1}$ decline between 2000 and 2010 was apparently concentrated in the second half of the decade, while in the South and Center-West the onset of the decline took place at the beginning. Hence, it is not suitable to adopt the same criterion to select the $f_{1}^{*}$ to be cumulated in $F_{2}\left(f_{1}^{*}\right.$ average in the 2000 and 2010 Demographic Censuses) in all regions.

Another option is to compare births reported in the 2010 Demographic Census with the vital statistics from the Civil Registration System or SINASC. Given both databases have experienced considerable increase in recent years, mainly due to demands from social programs of conditional cash transfers (SZWARCWALD et al., 2011) this would be a reasonable approach. Nonetheless, it would still be important to account for delayed births registration, specifically when considering the Vital Registration System.

We would then use the ratio of total registered births to total reported births in the census $(R / C)$ to correct the level of current reported fertility, that is, to correct the reference period error. This procedure has two important limitations: the need to assume that registration coverage is complete in all regions and that the population coverage of the 2010 Demographic Census is also complete, or that the level of coverage are the same in all three data sources. The R/C would then be used to correct the level of reported fertility, that is, the traditional reference period error. The major issue is that those assumptions are not realistic; ${ }^{10}$ hence, this option was discarded.

An alternative approach, effectively adopted in this work, uses the same correction factor estimated using the 2000 Census data to correct the reference period error of 2010. Thus, the reference period error estimate derived from the 2000 Census is less affected by

[^8]variations in the $f_{1}^{*}$ than it would, using information on the 2010 Census, since there was less variation in the $f_{1}^{*}$ during the 1990s. In practice, the $P_{2} / F_{2}$ ratio used to correct the 2000 data is slightly underestimated, due to the fact that adolescent fertility was still increasing in the country as a whole and in the majority of regions (MARQUES; CARVALHO, 2018). See Table 2. We assumed that the reference period error in fertility for the whole country, its major regions and federal units in 2010 were the same as those estimated for year 2000.

The proposed alterative is robust for two reasons: First, the relative stability of the $P_{2} / F_{2}$ ratio in Brazil and its major regions between years 1980 and 2000, as previously shown. Second, despite the strong educational expansion in Brazil during this period (FAUSTINO, 2017), the reference period error is more related to cultural reasons (BRASS; COALE, 1968), and those are factors that take more than just one decade to change. This latter characteristic is indeed one of the factors explaining why in Africa the number of live births in the previous 12 months of the census date is overestimated, while in Latin America there is a clear trend in the opposite direction. It can be assumed that the significant expansion of the higher education system between 2000 and 2010 in Brazil did not considerably affect the current fertility reference period error between those years.

## Results

## Brazil and Macro-regions

Figure 4 presents the number of births, for Brazil and its Macro-regions, computed in two different ways: first, using the fertility function derived from the original $P / F$ ratio technique, exactly as proposed by Brass and Coale (1968); and secondly, using the alternative fertility function, as discussed in the aforementioned section in this paper. These figures are the product of the ASFR estimated using the number of women in reproductive age, as enumerated in the 2010 Demographic Census. We also present the average number of annual births in 2009, 2010 and 2011, as registered in the SINASC, which, as previously stated, has experienced a significant increase in both quality and coverage (SZWARCWALD et al, 2011). Despite the probable existence of birth underreporting in some regions in Brazil, particularly in the North and Northeast regions, they still provide a good benchmark for comparison. ${ }^{11}$

Related to this comparison, two important issues must be borne in mind: (1) the number of estimated births is certainly underestimated, due to census underreporting, an issue present on all surveys of this kind. The same is not true for the estimated ASFRs, if there is no selectivity regarding the women's fertility not enumerated in the census. (2) In a context of changing fertility, the estimated TFR does not correspond to the census year, but to some point in the 5-year period during 2005-2010. This is true because the fertility

[^9]level estimated is defined by $P_{2}$, which relates to the cumulated experience of women aged 20-24, until 2010. The level can then be allocated amidst the previous quinquennial.

The level of the estimated number of births obtained from the ASFRs derived using the traditional Brass technique is always higher than that obtained when using data from SINASC in 2010. This occurs in spite of the number of women in reproductive age (the census of female population which was underreported) used to generate the number of births is surely lower than the actual female population that generates the births in the SINASC. This is a clear indication that the 2010 Brazilian and Regional fertility function levels produced by the application of the traditional $P_{i} / F_{i}$ technique were overestimated. The number of births obtained through the proposed alternative present, a priori, consistent results, since it applies a correction factor, free from the effect caused by the strong adolescent fertility decline experienced in Brazil between 2000 and 2010.

FIGURE 4
Number of estimated births using P/F Brass technique, with and without the proposed adjustment factor, and number of registered births (1), by age of mother Brazil and Macro-regions - 2010


Source: IBGE. 2010 Demographic Census and Civil Registration Statistics; MS/SVS/Datasus. Information System on Live Births SINASC. (1) Annual average of registered births in 2009, 2010 and 2011.

The results for the whole country evidence surprisingly similar curves between our proposed alternative and that derived from SINASC, both when considering the level and
the pattern of the functions (Figure 4). Probably, the difference in level would be greater, due to the undercoverage of the 2010 Demographic Census (RIGOTTI, 2016). On the other hand, the level of the curve derived by the proposed alternative refers approximately to the average fertility in the 5-year period 2005-2010, and not precisely in the year 2010. Hence, it probably slightly overestimates the fertility level for 2010.

With regard to the 5 Macro-regions in the country, the shapes of birth distribution are very similar, slightly diverging only as regards their levels, with the exception of the South region. Besides the issue of allocating in time the number of births estimated by the P/F Brass technique, there are two hypotheses to explain the observed level differentials:

- the higher number of estimated births in the North, Northeast and Center-West regions can be, at least to a great extent, attributable to the higher degree of birth underreporting in the SINASC in these three regions (CAVENAGHI; ALVES, 2016), a factor that would offset the slightly overestimated figure derived from the traditional Brass method, since they do not precisely refer to 2010;
- the lower number of estimated births in the Southeast region can be due to the population census undercount in this region. There is evidence that, surprisingly, in 2010, serious issues in census coverage occurred in the Southeast regions of Brazil, especially in the states of São Paulo (RIGOTTI, 2016) and Rio de Janeiro.
Table 4 presents the TFRs estimated using original P/F Brass technique, as well as our proposed alternative and Gompertz relational model. Brass correction factors for year 2010 consistently yield overestimated TFRs. Consequently, the level of fertility in Brazil during the 2005-2010 period is lower than the one estimated by the original P/F Brass technique. The lower degree of TFR overestimation is observed in the Center-West (4.9\%) and South ( $6.5 \%$ ) regions.

TABLE 4
Estimated TFRs according to Brass, proposed alternative and Gompertz relational model Brazil and Macro-regions - 2005-2010

| Regions | A | B | B/A | Gompertz$(\approx 2010)$ | $\begin{gathered} \text { Synthetic } \\ \text { Gompertz } \\ (2000-2010) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Original Brass $(2005-2010)$ | Proposed alternative (2005-2010) | Variation (\%) |  |  |
| Brazil | 1.9051 | 1.7635 | -7.43\% | 1.7408 | 1.8975 |
| North | 2.4783 | 2.3073 | -6.90\% | 2.2363 | 2.4480 |
| Northeast | 2.0670 | 1.9135 | -7.43\% | 1.9320 | 2.1605 |
| Southeast | 1.7039 | 1.5671 | -8.03\% | 1.5736 | 1.6701 |
| South | 1.7823 | 1.6658 | -6.54\% | 1.6517 | 1.7419 |
| Center-West | 1.9287 | 1.8335 | -4.93\% | 1.7925 | 1.8981 |

Fonte: Basic data from IBGE. 2000 and 2010 Demographic Census.
The decline in adolescent fertility, in the Center-West and South regions, probably, occurred throughout the whole decade, as can be inferred from the set of $f_{1}^{*}$ for years from
the 1991 and 2000 census, as well as the trend in PNADs shown in Figure 3. This means that the cohort $f_{1}^{*}$ (women aged 20-24 in 2010) would be closer to the reported $f^{*}{ }_{1}$ in the 2010 Demographic Census, amounting to a smaller error when applying the original technique. It is important to note that the TFR for Brazil (1.76), hereby estimated by the proposed alternative and referring to approximately the average of the 5-year period 2005-2010, is equal to the one estimated using SINASC and also equal to that estimated by Castanheira and Kohler (2016), ${ }^{12}$ both referring to 2010. When using the traditional technique, the degree of overestimation is around $7.5 \%$ of fertility levels in the country, in the end of the decade.

We have also used the Gompertz relational model (MOULTRIE, 2013a; ZABA, 1981) and the Synthetic Gompertz Relational Model (MOULTRIE, 2013b). The first focuses on the 2010 Demographic Census date, while the second yields an average TFR for the 2000-2010 period. These techniques have different assumptions from those posed by Brass $\mathrm{P} / \mathrm{F}$; they both demand a certain level of fertility stability in years before the census reference date, but this does not imply constant fertility. They estimate fertility levels based on the parity of younger age groups, since parity at those ages is more stable and mortality selectivity as regards fertility is lower (MOULTRIE, 2013a, 2013b; ZABA, 1981).

Estimates generated using the $2000 P_{2} / F_{2}$ as correction factor for the 2000-2010 period were similar to those derived from the Gompertz Relational model (Table 4): 1.76 and 1.74. On the other hand, the TFRs resultant from the Synthetic Gompertz Model were higher (1.90) and consistent with their time reference, since in this technique the TFR corresponds to the average value in 2000-2010.

## Estimates for the six Brazilian Federal Units

As a means of further evaluating the alternative proposed in this work concerning the 2010 reference period error we adopted the same approach as the one applied for the whole country and its Macro-regions. The 2000 reference period correction factor was applied in each estate, in order to compute the TFR for 2005-2010. However, it is possible to adopt a second alternative, as the following sections will show.

The first step consists of computing for each region the $f^{*}$ estimate corresponding to adolescent fertility in 2005. When assuming that the reference period error was the same for the whole country and for the Macro-regions in 2000 and 2010, the value estimated by $P_{2} / F_{2}$ in year 2000 is the one considered real, not the 2010 figure. As a reminder, this is so because of the aforementioned problems in the $2010 F_{2}$, caused by the $f_{1}^{\star}$ of this year. It is therefore possible to estimate which should be, for each region, the $2010 f^{*}{ }_{1}$ value that would yield the same $2000 P_{2} / F_{2}$ value.

This $f_{1}^{*}$ would correspond to the fertility experience of women aged 20-24 in 2010, but 5 years earlier, in 2005, when they belonged to the 15-19 age group ( $f_{1}^{*}{ }_{1}^{2005}$ ).

[^10]Based on this, it was assumed that in each Brazilian state the $f_{1}^{*}{ }_{1}^{2005}$ is the same linear combination of the "observed" $f_{1}^{\star 2000}$ and the $f_{1}^{\star 2010}$ pertaining to each Macro-region.

$$
\begin{aligned}
& \frac{P_{2}^{2010}}{F_{2}^{* 2010}}=\frac{P_{2}^{2000}}{F_{2}^{2000}}=\lambda \\
& \lambda=\frac{P_{2}^{2010}}{\left(5 \cdot f_{1}^{* 2005}+k_{2} \cdot f_{2}^{* 2010}\right)} \\
& \frac{P_{2}^{2010}}{\lambda}=\left(5 \cdot f_{1}^{* 2005}+k_{2} \cdot f_{2}^{* 2010}\right) \\
& 5 \cdot f_{1}^{* 2005}=\frac{P_{2}^{2010}}{\lambda}-k_{2} \cdot f_{2}^{* 2010} \\
& f_{1}^{* 2005}=\frac{\left(\frac{P_{2}^{2010}}{\lambda}-k_{2} \cdot f_{2}^{* 2010}\right)}{5}
\end{aligned}
$$

Where:
$P_{2}^{2000}=$ average reported parity of women between 20-24 years of age, in 2000;
$P_{2}^{2010}=$ average reported parity of women between 20-24 years of age, in 2010;
$F_{2}^{2000}=$ current cumulated fertility of women between 20-24 years of age, in 2000, estimated according to the traditional Brass method;
$F_{2}^{* 2010}=$ current cumulated fertility of women between 20-24 years of age, in 2010, using $f^{*}$ as their estimated age-specific fertility rate, 5 years ago;
$\lambda=(1+$ reference period error) estimated in 2000;
$f_{1}^{* 2005}=$ period age-specific fertility rate of women between 20-24 years of age, five years ago;
$k_{2}=$ multiplier for estimating cumulated period fertility rate between ages 19.5 and 24.5 , obtained from the $P_{1} / P_{2}$ ratio;
$f_{2}^{* 2010}=$ period age-specific fertility rate of women between 20-24 years of age, in 2010.
The value of the estimated $f_{1}^{* 2005}$ in (1), is a linear combination of the known $f^{*}$ values, in 2000 and 2010. This $f_{1}^{* 2005}$ value, should we assume that the 2010 reference period error is the same as the one found for year 2000, is the estimate of the age-specific fertility rate of women currently aged 20-24, when they belonged to the 15-19 group in 2005. The distinct weights, for the whole country and for each region, are given by $\alpha$ and $\beta$ as presented from the system of equations shown below:

$$
\left\{\begin{array}{l}
f_{1}^{* 2005}=\alpha * f_{1}^{* 2000}+\beta * f_{1}^{* 2010}  \tag{2}\\
\alpha+\beta=1
\end{array}\right.
$$

To sum up, in order to estimate the reference period error for the Federation Units in Brazil in 2010, there are two possible alternatives. One is to adopt the same $P_{2} / F_{2}$ ratio
estimate for year 2000 in a given Federation Unit to adjust the reported ASFRs, i. e., to correct the reference period error. The other one, to employ the same P/F mechanism assumed by the traditional Brass approach, however substituting the $2010 f_{1}^{*}$ by the $f_{1}^{*}$ estimated in year 2005, according to the $\alpha$ and $\beta$ weights assigned to each Macro-region the Federal Units belong to.

In principle, the $2000 P_{2} / F_{2}$ estimate for the FUs (Federal Units) should be used also in 2010. However, in states where there is a small number of women in reproductive ages, and are thus more vulnerable to sampling errors, the $a$ and $\beta$ values of the region may be adopted. ${ }^{13}$

Table 5 shows for Brazil and its Macro-regions the $P_{2} / F_{2}$ adopted in 2010 (estimates based on the 2000 Demographic Census data), also its relative $\alpha$ and $\beta$ weights used to estimate the $2005 f^{*}$, derived from the reported 2000 and $2010 f^{*}$.

TABLE 5
Reestimated $P_{2} / F_{2}$ and the weights $a$ and $\beta$
Brazil and Macro-regions - 2010

| Regions | Reestimated $P_{2} / F_{2}(\mathbf{2 0 1 0})$ | Weight $a\left(f_{1}^{*}, 2000\right)$ | Weight $\beta\left(f_{1}^{*}, 2010\right)$ |
| :--- | :---: | :---: | :---: |
| Brazil | $\mathbf{1 . 1 0 4 6}$ | 0.3724 | 0.6276 |
| North | 1.1510 | 0.3719 | 0.6281 |
| Northeast | 1.1245 | 0.4116 | 0.5884 |
| Southeast | 1.0843 | 0.3624 | 0.6376 |
| South | 1.0910 | 0.2771 | 0.7229 |
| Center-West | 1.1307 | 0.2145 | 0.7855 |

Source: Basic data from IBGE. 2000 and 2010 Demographic Census.
Figure 5 shows the number of registered live births in SINASC (annual average for the three-year period 2009-2011), the number of births estimated from the conventional P/F Brass technique, the number of births computed from our proposed alternative and finally, the number of births estimated through $\alpha$ and $\beta$ pertaining to the Macro-region the FU refers to. Altogether, there are three states in the Southeast (SP, RJ and MG), one in the South (RS), one in the Center-West (GO) and one in the Northeast (BA). ${ }^{14}$ We show results for three states in the Southeast regions in Brazil because, oddly, there is robust evidence of problems regarding census coverage in the major states of São Paulo and Rio de Janeiro.

The levels of birth distribution (number of births) generated both by the proposed alternative and the $\alpha$ and $\beta$ of a given Macro-region, in each of the six Federal Units, are very similar. However, this does not indicate the degree of estimate precision, but only shows that in these states, the $f_{1}^{*}$ behavior in the last decade (not the level) was close to that of the respective region. Nonetheless, the differentials in behavior among the Macro-regions were quite large, as shown by the $\alpha$ and $\beta$ figures in Table 5.

[^11]In all states, with the exception of Goiás, the number of births directly generated by the Brass approach is higher than that provided by the proposed alternative in this paper. This differential reflects the dramatic decline in adolescent fertility $\left(f_{1}^{*}\right)$ during the decade, being the motivation of this work and the proposal of the alternative computation method discussed herein.

The case of Goiás is intriguing. The $f_{1}^{*}$ distribution in this state, as provided by the PNADs, in spite of fluctuations due to sample errors, is indicating a very small decline between years 2005-2010, contrary to what happened in the other states at different regions. Therefore, the increase in the $P_{2} / F_{2}$ ratio for Goiás, between 2000 and 2010 , when estimated using the conventional Brass method, yielded extremely small values (Table 6).

TABLE 6
P2/F2 estimated through traditional method
Selected federal units - 2000-2010

| Federal units | $P_{2} / F_{2}$ |  |  |
| :--- | :---: | :---: | :---: |
|  | $\mathbf{2 0 1 0}$ | $\mathbf{2 0 1 0 / 2 0 0 0 \Delta ( \% )}$ |  |
| Bahia | 1.10 | 1.25 | 13.26 |
| São Paulo | 1.09 | 1.16 | 6.79 |
| Rio de Janeiro | 1.07 | 1.19 | 10.87 |
| Minas Gerais | 1.08 | 1.21 | 11.38 |
| Goiás | 1.15 | 1.18 | 2.58 |
| Rio Grande do Sul | 1.05 | 1.19 | 13.45 |

Source: IBGE. 2000 and 2010 Demographic Census.
When comparing the levels of SINASC's figure of registered births with those from the other three distributions, it is important to take into account that SINASC data refers to, approximately, year 2010, while the other three refer to 12 months around July 2007 and June 2008.

In spite of this time divergence, and considering that this is a period of subtle decline in the fertility level, when comparing SINASC data with those estimated through the 2000 $P_{2} / F_{2}$ ratio for each state, the results for Bahia and Goiás suggest an under-registration of births in the SINASC information system, albeit not too significant.

We demonstrated that conventional P/F Brass estimates based on the 2010 Demographic Census overestimate the fertility level. Consequently, the number of births in the census population (not necessarily the true population, depending on the degree of census coverage) is also overestimated. The curves for São Paulo and Rio de Janeiro surprisingly indicate important issues related to census coverage in 2010. If the census coverage had been $100 \%$ or at least close to complete, the birth schedule derived from the conventional Brass method would be higher than that from SINASC, since Brass method yields overestimated fertility levels, as depicted by the cases of Rio Grande do Sul and Minas Gerais. It is important to note, however, that unfortunately, IBGE did not publish their evaluation of census coverage for year 2010, at the state level. Rigotti (2016), when
evaluating data on school enrollment from INEP, found solid evidence of census undercoverage for the year 2010 at the state level, particularly for the state of São Paulo.

FIGURE 5
Number of births estimated by P/F Brass, with and without the proposed adjustment, and registered births
Selected federal units - 2010


Source: IBGE. 2010 Demographic Census and Civil Registration Statistics; MS/SVS/Datasus. Information System on Live Births SINASC.
(1) Annual average of registered births in 2009, 2010 and 2011.

Table 7 presents the TFR estimated for five States using 2010 census data. We use the Brass traditional technique (A) and the two alternatives proposed in this paper: the ratio calculated using 2000 census data (B); and using relative weights ( $a$ and $\beta$ ) estimated for Large Regions in order to define $f^{\star}$ - in each State - for women aged 20-24 in 2010 five years earlier. i.e., when they were in the 15-19 age group. Both alternatives produce TFR lower than those obtained using Brass technique. In principle, those produced using the first alternative (B) would be closer to the actual fertility level since they do not assume the same teenager fertility path in each State defined for their correspondent Macro-region.

All TFR estimated using the $2000 P_{2} / F_{2}$, with the exception of Goiás, are significantly lower than those obtained through the traditional Brass technique.

TABLE 7
Total fertility rates estimates, Brass traditional method and proposed alternative Selected federal units - 2005-2010

| Federal units | Total fertility rates (TFRs) |  |  | Relative difference (\%) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Brass original (1) (2005-2010) (A) | $\begin{gathered} \text { Proposed } \\ \text { alternative } 1 \\ (2005-2010)(\mathrm{B}) \end{gathered}$ | $\begin{gathered} \text { Proposed } \\ \text { alternative } 2 \\ (2005-2010)(B) \end{gathered}$ | (A) and (B) | (A) and (C) |
| Bahia | 2.03 | 1.79 | 1.88 | 13.41 | 7.98 |
| Minas Gerais | 1.78 | 1.59 | 1.63 | 11.95 | 9.20 |
| Rio de Janeiro | 1.69 | 1.52 | 1.55 | 11.18 | 9.03 |
| São Paulo | 1.67 | 1.56 | 1.53 | 7.05 | 9.15 |
| Rio Grande do Sul | 1.75 | 1.55 | 1.62 | 12.90 | 8.02 |
| Goiás | 1.87 | 1.82 | 1.78 | 2.75 | 5.05 |

Source: IBGE. 2010 Demographic Census; IBGE (2013).
(1) Appling the $P_{2} / F_{2}$ ratio calculated with the 2000 Brazilian Demographic census to estimate the fertility level in 2005-2010. (2) Appling the a e $\beta$ weights census to estimate the fertility level in 2005-2010.

## Closing remarks

The aim of this work was to present the limitations imposed by the $\mathrm{P} / \mathrm{FB}$ Brass technique, a widely used approach for estimating fertility level in Brazil, in a scenario of rapid adolescent fertility change, highlighting how this affects the quality of fertility estimates derived from the method.

The results clearly show, assuming no selectivity of the reference period error and a scenario of changing fertility levels, that the only change actually affecting the quality of estimates derived using the $P_{2} / F_{2}$ correcting factor, is that pertaining to women aged 15-19 years in the five years before the census or survey date.

Based on this evidence, we proposed an alternative for applying the method on the 2010 Demographic Census, which showed a fast-paced decline in adolescent fertility. Despite the overall robustness of the P/F Brass technique, its standard application in a context of rapid adolescent fertility change in the five years before the census date yields strongly biased results. In the data analyzed herein, the effect is an overestimated fertility level when applying the technique on the 2010 Brazilian census data. On the other hand, when using the proposed alternative of adopting the $2000 P_{2} / F_{2}$ ratio to adjust for the 2010 reported ASFRs, results are similar to the vital statistics data from SINASC, which have consistently improved in quality throughout the last years in Brazil, as we have mentioned earlier in this work. Additionally, the proposed method yielded a TFR estimate similar to the ones resulting from other methods, such as the Gompertz Relational Model, as well as other studies. Hence, there is empirical support in favor of the alternative proposed in this research. Undoubtedly, the fertility schedule estimates from the 2010 Demographic

Census derived from the standard P/F Brass procedure are overestimated, posing serious consequences on Brazil`s demographic projections.

Probably, this adjustment will not be necessary for estimating fertility schedules in the 2020 Demographic Census, since it is likely that the pace of adolescent fertility decline will slow down between 2010 and 2020, as already suggested by the PNADs in this decade. However, for other population subgroups estimates, adjustments will continue to be necessary, always taking into account the behavior of adolescent fertility.

The P/F Brass technique can be applied even when the conditions originally stated by Brass are not met. However, it is necessary that the researcher grasps the rationale of this approach and adapts it according to the different possible scenarios that may emerge. The technique is still uniquely useful, especially with regard to its application on population subgroups defined by unavailable variables in the official birth registries, but present in censuses and surveys.

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## Resumo

Aplicação da técnica P/F de Brass em um contexto de rápida queda da fecundidade adolescente

O Brasil experimentou, entre 2000 e 2010, pela primeira vez desde 1970, quando se introduziu o quesito sobre filhos nascidos vivos nos 12 meses anteriores à data de referência do censo, queda significativa (em torno de 30\%) das taxas específicas de fecundidade declarada das mulheres entre 15 e $19 \operatorname{anos}\left(f_{1}^{*}\right)$. Esse fenômeno tem uma importante consequência para a aplicação da técnica P/F de Brass: gera um erro, por falta, na fecundidade corrente acumulada até o grupo etário de 20 a 24 anos $\left(F_{2}\right)$, se tomada como experiência pregressa dessa coorte, levando a um valor de $P_{2} / F_{2}$, usado para ajustar o nível da fecundidade declarada, significativamente sobrestimado. O presente trabalho discute detalhadamente este problema e, por fim, propõe uma alternativa para se corrigir o erro de período de referência da fecundidade corrente do Censo Demográfico de 2010 do Brasil. A alternativa proposta, neste contexto específico, gerou estimativas de taxa de fecundidade total muito próximas às produzidas por outras técnicas.

Palavras-chave: P/F de Brass. Queda da fecundidade. Fecundidade adolescente. Censo Demográfico.

## Resumen

Aplicación de la técnica P/F de Brass en un contexto de rápida caída de la fecundidad adolescente

Por la primera vez desde 1970 -cuando se introdujo la pregunta sobre nacidos vivos en los 12 meses anteriores a la fecha de referencia del censo- Brasil experimentó, entre 2000 y 2010,
una disminución significativa de aproximadamente $30 \%$ de las tasas específicas de fecundidad declarada de mujeres entre 15 y 19 años $\left(f_{1}^{*}\right)$. Este fenómeno trae una consecuencia importante para la aplicación de la técnica P/F de Brass: genera un error por falta en la fecundidad actual acumulada para el grupo de edad de 20 a 24 años $\left(F_{2}\right)$, lo que concomitantemente provoca una significativa sobrestimación en el valor de $P_{2} / F_{2}$-utilizado para corregir el nivel de la fecundidad declarada-. Este trabajo discute este problema y propone finalmente una adaptación de la técnica original de Brass para aplicarla a los datos del censo de 2010. La alternativa propuesta generó, en este contexto específico, estimaciones de la tasa global de fecundidad similares a las producidas por otras técnicas.

Palabras clave: P/F de Brass. Disminución de la fecundidad. Fecundidad adolescente. Censo demográfico.

Received for publication in 03/14/2018
Approved for publication in 04/12/2018

## APPENDIX

Multipliers to estimate period cumulated fertility in the first three years of the 5 -year age groups with a 0.5 -year shift ( 14.5 to $19.5,19.5$ to 24.5 , etc.).

| 15 to 19 years | 1,120 | 1,310 | 1,615 | 1,950 | 2,305 | 2,640 | 2,925 | 3,170 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 20 to 24 years | 2.555 | 2.690 | 2.780 | 2.840 | 2.890 | 2.925 | 2.960 | 2.985 |
| 25 to 29 years | 2.925 | 2.960 | 2.985 | 3.010 | 3.035 | 3.055 | 3.075 | 3.095 |
| 30 to 34 years | 3.055 | 3.075 | 3.095 | 3.120 | 3.140 | 3.165 | 3.190 | 3.215 |
| 35 to 39 years | 3.165 | 3.190 | 3.215 | 3.245 | 3.285 | 3.325 | 3.375 | 3.435 |
| 40 to 44 years | 3.325 | 3.375 | 3.435 | 3.510 | 3.610 | 3.740 | 3.915 | 4.150 |
| 45 to 49 years | 3.640 | 3.895 | 4.150 | 4.395 | 4.630 | 4.840 | 4.985 | 5.000 |
| $\mathrm{f}_{1} / \mathrm{f}_{2}$ | 0.036 | 0.113 | 0.213 | 0.330 | 0.460 | 0.605 | 0.764 | 0.939 |
| $\overline{\mathrm{~m}}$ | 31.7 | 30.7 | 29.7 | 28.7 | 27.7 | 26.7 | 25.7 | 24.7 |
| $\mathrm{P}_{1} / \mathrm{P}_{2}$ | 0.014 | 0.045 | 0.090 | 0.143 | 0.205 | 0.268 | 0.330 | 0.387 |

[^12]
[^0]:    * The authors thank comments and suggestions provided by Bernardo Lanza Queiroz, Cássio Maldonado Turra and Laura Lídia Rodríguez Wong. Acknowledgements are due, also to Helena Castanheira, the first one to draw the authors' attention to the problems in applying the traditional Brass' P/F technique due to the significant adolescent fertility decline between 2000 and 2010.
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[^1]:    1 In this paper, we always use information of the 20-24 female age group to estimate the reference period error. Brass also considers retrieving information from the 25-29 age group for the estimates. Since our main goal is to analyze the effects of changing fertility schedules at adolescent ages on the reference period error estimates, we disregard information from the 25-29 age group. The reason for this is that the negative impacts of varying fertility schedules at adolescent ages and at the 20-24 age group on estimating the reference period error are significantly higher for the 25-29 age group.

[^2]:    $\overline{2}$ Translated passage from the original in Portuguese: "O valor correto da TFT em 2010, ou na média dos dois ou três anos anteriores, não pode ser motivo de disputa, visto que todos os dados e métodos disponíveis no momento apresentam algum tipo de problema".

[^3]:    ${ }^{3}$ If the period reference error for both migrant and non-migrant women is the same and considering the $P_{2} / F_{2}$ estimate in an open population, then the effect of migration on the estimator will depend on the fertility function differentials and the weight of female migration only between the beginning of the reproductive period and age 25 . The same case occurs with regard to the selectivity of female mortality on fertility.

[^4]:    ${ }^{4}$ Despite being called a memory error, it is probable that part of the omission derives from the fact that the older the mother, the higher the probability of this information to be provided by other parties. Usually, those parties are less aware of the total number of CEB by the woman, especially if some of them are dead or away from the mum's home.
    5 "As a rule, therefore, the P/F ratio for the age group 20-24 years will be used to adjust the level of reported period fertility rates unless there is evidence that the ratio at 20-25 is distorted or inconsistent with the trend of the values at later ages" (BRASS; COALE, 1968, p. 96). "Under certain assumptions the ratio $P_{2} / F_{2}$ for the age group 20-24 years or $P_{3} / F_{3}$ for 25-29 years is a factor which can be used to multiply the age specific fertility rates as an adjustment for the under-reporting or over-reporting of the current births" (BRASS, 1985, p. 69).
    ${ }^{6} k$ is the multiplier to allow the exact comparison of $P_{i}$ and $F_{i}$. It is estimated using a parameter of the age fertility pattern. The $P_{1} / P_{2}$ is more appropriate to do so for the first three reproductive age groups, instead of adopting the $f_{1} / f_{2}$ ratio. Thus, random fluctuations and sampling errors are mitigated, especially at $f_{1}$. Professor William Brass, who actually adopts the same approach for estimating childhood mortality, approved this alternative, proposed by one of the authors. See the multipliers in the Appendix.

[^5]:    ${ }^{7}$ If the $P_{3} / F_{3}$ ratio is taken as an estimator for the reference period error, adolescent fertility would have to be constant in the last 10 years, and the 20-24 age group, in the last 5 years.

[^6]:    ${ }^{8}$ Women of reproductive age were asked about the number of live births in the 12 months before the census reference date. Since 1980, the enquiry changed to the month and year of birth of the last live birth.

[^7]:    ${ }^{9}$ The $f^{\star}{ }_{1}$ of census years are not shown because due to reasons not yet identified, the rates derived from the censuses are systematically higher than those from PNADs.

[^8]:    ${ }^{10}$ Figure 5 shows the average number of annual registered births in the period and the reported births in the 2010 Census. The figures clearly indicate the coverage differentials in the federal units.

[^9]:    ${ }^{11}$ This analysis is based on the number of births instead of the ASFRs to avoid the use of the number of women (in the denominator) from the 2010 Census that can introduce another bias due to under-coverage of this population segment.

[^10]:    $\overline{12}$ TFR was estimated with an adjustment of the birth counts in SINASC (CASTANHEIRA; KOHLER, 2016).

[^11]:    ${ }^{13}$ For other population subgroups, it is important to carefully analyze the $f^{*}{ }_{1}$ evolution between years 2000 and 2010, in order to adopt a more appropriate procedure.
    ${ }^{14}$ Estimates for other states are found in: Carvalho, Gonçalves and Silva (2017) <http://www.cedeplar.ufmg.br/pesquisas/ td/TD\%20564.pdf〉.

[^12]:    Source: Brass and Coale (1968) for $\bar{m}$ and $\mathrm{f}_{1} / \mathrm{f}_{2}$ values. Professor William Brass kindly made the introduction of the $P_{1} / P_{2}$ parameter available in 1973, due to the request of one of the authors of this paper.

