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Simulation of diets for dairy goats and growing doelings using nonlinear optimization procedures

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ABSTRACT - The objective of this study was to simulate total dry matter intake and cost of diets optimized by nonlinear programming to meet the nutritional requirements of dairy does and growing doelings. The mathematical model was programmed in a Microsoft Excel® spreadsheet. Increasing values of body mass and average daily weight gain for growing doelings and increasing body mass values and milk yield for dairy does were used as inputs for optimizations. Three objective functions were considered: minimization of the dietary cost, dry matter intake maximization, and maximization of the efficiency of use of the ingested crude protein. To solve the proposed problems we used the Excel® Solver® algorithm. The Excel® Solver® was able to balance diets containing different objective functions and provided different spaces of feasible solutions. The best solutions are obtained by least-cost formulations; the other two objective functions, namely maximize dry matter intake and maximize crude protein use, do not produce favorable diets in terms of costs.

Key Words: decision support system, diet formulation, diet optimization, goat nutrition

Introduction

Feeding is one of the most important components of the livestock activity. The productive animal must be fed properly to express its genetic potential, and feeding represents a high proportion of the total production costs. In two small dairy goat production systems in North-western Rio de Janeiro State, Brazil, 41 to 73% of the total effective operating costs consisted of concentrates (Vieira et al., 2009). Least-cost optimization procedures are used to find the most suited combination of foods that meets animal requirements (Agrawal and Heady, 1972). Linear programming tools generally able to solve the problem of diet formulation, such as the Simplex method (Agrawal and Heady, 1972; Tedeschi et al., 2000). Nevertheless, the complexity of the animal physiology and the interactions among the food eaten and digestive and metabolic processes that occur in the animal organism (Dijkstra et al., 2005) demand the use of nonlinear programming to obtain more accurate diet formulations (Jardim et al., 2013; 2015).

Nonlinear programming can be used to simulate scenarios from input data (De Los Campos et al., 2013). On the other hand, simulation studies can be used to predict specific virtual situations before making decisions or to improve our understanding of certain phenomena. From this perspective, the aim of this study was to simulate scenarios where three objective functions were optimized: least-cost of diets were minimized, dry matter intake of simulated diets were maximized, and ratios of metabolizable protein intake to crude protein intake were maximized. These problems were considered as general nonlinear programming problems, in which target performances and nutritional requirements of dairy does and growing doelings and usual dairy goat feeds were used as inputs.

Material and Methods

The Microsoft Excel® spreadsheet was used to program a mathematical model that combines the conceptual and mathematical structures of the CNCPS - Cornell Net Carbohydrate and Protein System (Fox et al., 2004) to estimate nutritive value of feeds, and the NRC (2007) equations to calculate nutrient requirement of growing doelings and lactating does. The steady-state pool size and

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digestibility of fiber in the ruminoreticulum were modeled according to Vieira et al. (2008a,b). Acronyms and symbols used in equations that describe the system are listed in Tables 1 and 2.

The diets for growing doelings and lactating does were formulated as a general nonlinear programming problem subjected to constraints of equalities and inequalities. Three different problems were optimized by considering three different objective functions separately:

Minimize: $Z = \sum_{i=1}^{m} c_i x_i$ (1)

Maximize: $W = \sum_{i=1}^{m} x_i$ (2)

Maximize: $K = \frac{MPI}{CPI}$ (3)

Subjected to:

$MEI \geq MEt$ (4)

$MPI \geq MPt$ (5)

$RFM_{max} = 8.5 \times BW$ (6)

$EFI = 200 + FI_j$ (7)

The objective function Z (Eq. 1) is represented by the linear combination of constant c_i, i.e., the unitary dry matter cost of the i-th ingredient; x_i represents the unknown dry matter intake of the i-th ingredient. The objective function W (Eq. 2) is the total dry matter intake, and the objective function K (Eq. 3) is the proportion of the crude protein ingested (CPI) transformed into metabolizable protein; MEI and MPI are the intakes of metabolizable energy (MJ/day) and metabolizable protein (g/day) intakes, respectively; MEt is the metabolizable energy required (MJ/day); and MPt is the metabolizable protein required (g/day; Table 3). The term RFM_{max} corresponds to the maximum fiber retention capacity of the rumen (g/day); EFI is the effective fiber concentration of the diet (g/kg of dry matter); and FI_j is the fiber increment added to the minimum fiber content set (200 g/kg of dry matter). FI_j values were increased successively by adding 50 g/kg of dry matter constant increments to the minimum concentration of effective fiber for dairy does, and 25 g/kg of dry matter constant increments for growing doelings until feasible solutions were no longer achieved.

Constraints to the use of urea were also added. It is recommended that the urea supply should not exceed 40 g per 100 kg of body weight (BW), and two hypothetical situations were considered to balance rumen ammonia nitrogen (RANB, g/d):

$RANB \geq 0$ (8)

or $RANB \geq -200$ (9)

The RANB is a relationship between ammonia and carbohydrates available to the rumen microorganisms (Russel et al., 1992; AFRC, 1993; Fox, 2004). In addition, a maximum limit of 50 g/kg of diet dry matter for crude fat concentration was set for all simulations (NRC, 2001).

Simulations for growing doelings were made by varying the mass of the animal from 17 to 35 kg of BW with 3 kg BW increments. The diets were optimized to meet maintenance requirements and nutrient demands generated by daily weight gains ranging from 0 to 150 g/day, with 25 g/day constant increments. The simulations for lactating does were made by varying the weight of the animal from 50 to 80 kg with 5 kg BW increments, and milk production ranging from 2 to 9 kg/day with 0.5 kg/day increments.

We solved the presented problems by using the Excel® Solver® spreadsheet. This tool uses a generalized reduced gradient algorithm to optimize nonlinear problems (Lasdon et al., 1978).

The prices of the feed ingredients used in the model (Table 4) were taken in December 2010, as current market prices in the northern and northwestern regions of Rio de Janeiro State. The nutritional composition of the feeds was obtained from tables contained in CNCPS (Sniffen et al., 1992), Nutrient Requirements of Beef Cattle (NRC, 1996), and Nutrient Requirements of Dairy Cattle (NRC, 2001).

Results

The Excel® Solver® was efficient to obtain feasible solutions to the proposed problems. Simulations with increments for daily gain and milk yield resulted in positive linear relationships between production levels and MEI, and production levels and MPI (Figures 1a, 1b, 1c and 1d). Sometimes, the space of feasible solutions differed remarkably. However, the increments in the fiber content of the diet caused an increased dry matter intake until a maximum point was achieved. Afterwards, a sharp decrease in the solution space was observed at higher fiber concentrations in the diet (Figures 1e and 1f). The number of feasible solutions was higher for fiber contents ≤500 g/kg of diet dry matter for lactating does (Figure 1e). However, for growing doelings, only the level of 725 g/kg of fiber in the diet reduced the space of feasible solutions considerably (Figure 1f).

The optimization for maximum dry matter intake, i.e., objective function W or Eq. 2, resulted in more expensive diets for growing doelings in comparison with the other objective functions (Figure 2a). The maximization of the crude protein utilization or objective function K (Eq. 3, Figure 2b) produced diets with intermediate costs and,

Table 1 - Definitions of symbols and acronyms used in the text[1]

Symbol	Definition (dimension)
[TDNa]	Content of total apparently digestible nutrients from the diet (dmls)
[DE]	Digestible energy content (MJ kg^{-1})
[ME]	Metabolizable energy content (MJ kg^{-1})
λ_r	Asymptotic age-dependent fractional rate for transference of particles from the raft to the escapable pool (h^{-1})
A	Protein fraction A for each feed (g kg^{-1} DM)
A'	Carbohydrate fraction A' (simple sugars) (g kg^{-1} DM)
ADE	Amount of digestible energy available to the animal provided by feed (MJ d^{-1})
ADG	Average daily gain (kg d^{-1})
AIM	Amount of indigestible dry matter (g d^{-1})
AME	Amount of metabolizable energy available to the animal provided by feed (MJ d^{-1})
AMNBACTFC	Ammonia nitrogen retained by microorganisms that use fibrous carbohydrates (g d^{-1})
AMNBACTNF	Ammonia nitrogen retained by microorganisms that use the non-fibrous carbohydrates (g d^{-1})
Apef	Amount of physically effective fiber from the feed (g d^{-1})
Ash	Amount of ash for each feed (g d^{-1})
B1	Protein B1 fraction for each feed (g kg^{-1} DM)
B1'	Carbohydrate B1' fraction (starch and soluble fibers) (g kg^{-1} DM)
B2	Protein B2 fraction for each feed (g kg^{-1} DM)
B2'	Carbohydrate B2' fraction (available NDF) (g kg^{-1} DM)
BACT	Cell biomass produced from total ruminal availability of carbohydrates supplied by feed (g d^{-1})
BactNBAL	Ruminal bacteria N balance
BACTred	Reduction in the amount of bacteria due to nitrogen deficiency in the rumen (g d^{-1})
BW	Animal body weight (kg)
C	Protein C fraction for each feed (g kg^{-1} DM)
C'	Carbohydrate C' fraction (indigestible) (g kg^{-1} DM)
CA'	Amount of A' for each feed (g d^{-1})
CB1'	Amount of B1' for each feed (g d^{-1})
CB2'	Amount of B2' for each feed (g d^{-1})
CC'	Amount of C' for each feed (g d^{-1})
Cd	Coefficient of intestinal digestibility of B1' (dmls)
CF	Content of crude fat for each feed (g kg^{-1} DM)
CPEPUP	Number of peptides degraded by bacteria that used more peptides that escaped the rumen (g d^{-1})
CPI	Crude protein intake (g)
DA'	Degradability of A' (g d^{-1})
DB2'	Potentially digestible NDF fraction available to rumen microbes (g d^{-1} DMI)
DB1	Degradability of B1 (g d^{-1})
DB1'	Degradability of B1' (g d^{-1})
DB2	Degradability of B2 (g d^{-1})
DisappTime	Time required for the ruminal disappearance of bacteria and peptides (h)
DMI	Dry matter intake (g d^{-1})
DRPEPh	Rate of release of peptides in the rumen (g h^{-1})
EBACTratio	Proportion of bacteria produced from the i-th feed in relation to total bacteria produced (dmls)
EFCBACT	Amount of FC bacteria when energy is limiting (g d^{-1})
ENFCBACT	Amount of NFC bacteria when energy is limiting (g d^{-1})
Fat	Amount of fat for each feed (g d^{-1})
FC	Amount of fibrous carbohydrates for each feed (g d^{-1})
FCBACT	Biomass of microbial cells produced from ruminal availability of fibrous carbohydrates supplied by the feed (g d^{-1})
FCBACTratio	Proportion of bacteria that use fibrous carbohydrates in relation to total bacteria (dmls)
FCBACTred	Reduction in the amount of bacteria that use fibrous carbohydrates due to nitrogen deficiency in the rumen (g d^{-1})
FCM	4% fat-corrected milk (g kg^{-1})
$FCNH_{3eq}$	Amount of NH_3 required by FC bacteria (g d^{-1})

Continues...

Table 1 (Continued)

Symbol	Definition (dimension)
$FCNH_{3avail}$	Amount of NH_3 available for FC bacteria usage
FCRed	Amount of fraction B2' not degraded by bacteria that use fibrous carbohydrates (g d^{-1})
FCRedratio	Ratio between the total FCRed and the total RAB2' (dmls)
Fiber	Amount of fiber for each feed (g d^{-1})
FLAsh	Fecal losses of ash derived from the feed (g d^{-1})
FLAshM	Fecal losses of ash of microbial origin (g d^{-1})
FLB1'	Fecal losses of B1' (g d^{-1})
FLB2	Fecal losses of B2 (g d^{-1})
FLB2'	Fecal losses of B2' (g d^{-1})
FLBACT	Fecal losses of microbial mass (g d^{-1})
FLC	Fecal losses of C (g d^{-1})
FLC'	Fecal losses of C' (g d^{-1})
FLEAsh	Endogenous fecal losses of ash (g d^{-1})
FLEFat	Endogenous losses of fat over feed intake (g d^{-1})
FLEP	Endogenous fecal losses of protein over feed intake (g d^{-1}).
FLMC	Fecal losses relative to total microbial carbohydrate (g d^{-1})
FLMCW	Fecal losses of microbial cell wall (g d^{-1})
FLMFat	Microbial fecal losses of fat (g d^{-1})
FLPA	Fecal losses of feed protein (g d^{-1})
FLTAsh	Fecal losses of ash from the diet (g d^{-1})
FLTCF	Fecal losses of carbohydrate from the feed (g d^{-1})
FLTCT	Fecal losses of carbohydrate from the diet (g d^{-1})
FLTFat	Fecal losses of fat from the diet (g d^{-1})
FLTMP	Total fecal losses of microbial protein (g d^{-1})
FLTPRO	Fecal losses of protein from the diet (g d^{-1})
F_{NDF}	NDF intake (kg d^{-1})
Growth Time	Time required for bacteria growth assuming liquid turnover time (h)
i	Subscript denoting diet ingredient
I	Subscript applied to variables denoting the order of time dependency that varies from 1 to N or Na (dmls)
IDB1	Intestinal digestibility of fraction B1 that escapes rumen degradation (g d^{-1})
IDB2	Intestinal digestibility of fraction B2 that escapes rumen degradation (g d^{-1})
IDCF	Intestinal digestibility of carbohydrates of feed origin (g d^{-1})
IDF	Intestinal digestibility of fat (g d^{-1})
IDFF	Intestinal digestibility of fat of dietary origin (g d^{-1})
IDMC	Intestinal digestibility of microbial carbohydrates (g d^{-1})
IDMF	Intestinal digestibility of microbial fat (g d^{-1})
IDMNA	Intestinal digestibility of microbial nucleic acids (g d^{-1})
IDP	Intestinal digestibility of protein (g d^{-1})
IDPF	Intestinal digestibility of total protein feed source (g d^{-1})
IDTC	Intestinal digestibility of total carbohydrates (g d^{-1})
IDTMP	Intestinal digestibility of microbial true protein (g d^{-1})
IMP	Percentage growth improvement due to the availability of peptides in relation to fractions A' and B1' (%)
k'_{d1}	Rate of digestion of fraction A' (h^{-1})
k_{d2}	Rate of digestion of fraction B2 (h^{-1})
k'_{d2}	Rate of digestion of fraction B1' (h^{-1})
k'_{d3}	Rate of digestion of fraction B2' (h^{-1})
k_e	Ruminal escape rate of fibrous particles to the remainder of the gastrointestinal tract (h^{-1})
k_{l-d}	Efficiency of use of ME_{l-d} (dmls)
k_m	Efficiency of ME utilization for maintenance (dmls)
k_{m1}	Ratio of maintenance of bacteria that use the fibrous carbohydrates (dmls)
k_{m2}	Ratio of maintenance of bacteria that use the non-fibrous carbohydrates (dmls)

Continues...

Table 1 (Continued)

Symbol	Definition (dimension)
k_{pl}	Passage rate of liquids (h^{-1})
k_r	Rate of passage of long fibrous particles newly ingested through portions of the ventral rumen (h^{-1})
k_{up}	Rate of utilization of peptides by bacteria that ferment non-fibrous carbohydrates (g peptide g^{-1} cells per h)
k_{upi}	Utilization rate of peptide-corrected supply of ionophores in the diet (g peptide g^{-1} cells per h)
ME_g	ME for whole body gain (MJ d^{-1})
ME_{l-d}	Dietary ME used for lactation (MJ d^{-1})
ME_m	ME requirement for maintenance (MJ d^{-1})
MFCP	Metabolic fecal crude protein (g d^{-1})
Milkyield	Milk production (kg d^{-1})
MM	Content of ash for each feed (g d^{-1})
MP	Amount of metabolizable protein from the diet (g d^{-1})
MP_g	MP for whole body gain (g d^{-1})
MP_{l-d}	Dietary MP used in milk production (g d^{-1})
MP_m	Metabolizable protein for maintenance (g d^{-1})
$MP_{required}$	Total metabolizable protein required (g d^{-1})
MRT	Retention time of fiber in the rumen (h^{-1})
N	Positive integer order of time dependency for transferring a particle from the raft to the escapable pool (dmls)
NBACTFC	Amount of nitrogen contained in the biomass produced in the rumen from the fibrous carbohydrates supplied by the feed (g d^{-1})
NBACTNF	Amount of nitrogen contained in the biomass produced in the rumen from the non-fibrous carbohydrates supplied by the feed (g d^{-1})
Nexc	N in excess of rumen bacterial N and tissue needs (g d^{-1})
NFC	Amount of non-fibrous carbohydrates for each feed (g d^{-1})
NFCBACT	Cell biomass produced from ruminal availability of non-fibrous carbohydrates supplied by the feed (g d^{-1})
NFCBACTaj	Biomass of bacteria that use non-fibrous carbohydrates adjusted for the ruminal escape of non-fibrous carbohydrates (g d^{-1})
NFCBACTmass	Biomass of bacteria that use non-fibrous carbohydrates (g d^{-1})
$NFCBACTNH_{3req}$	Amount of NH_3 required by NFC bacteria (g d^{-1})
NFCBACTpepup	Peptide uptake by bacteria that use non-fibrous carbohydrates (g d^{-1})
NH_{3bact}	Amount of NH_3 from peptide uptake not used by bacteria and produced as NH_3 (g d^{-1})
NH_{3diet}	Amount of NH_3 from the diet (g d^{-1})
$NH_{3recicled}$	Amount of NH_3 recycled (g d^{-1})
NH_{3req}	Amount of NH_3 required (g d^{-1})
NPEPUP	Nitrogen contained in peptides assimilated by microorganisms that use the non-fibrous carbohydrates (g d^{-1})
NPEPUPaj	Nitrogen contained in the peptides retained by microorganisms that use the non-fibrous carbohydrates adjusted for escape ruminal peptide (g d^{-1})
NperBACT	Microbial growth allowed by nitrogen availability in the rumen (g d^{-1})
PA	Amount of fraction A for each feed (g d^{-1})
PB	Amount of fraction crude protein for each feed (g d^{-1})
PB1	Amount of fraction B1 for each feed (g d^{-1})
PB2	Amount of fraction B2 for each feed (g d^{-1})
PC	Amount of fraction C for each feed (g d^{-1})
pef	Factor physical effectiveness of fiber in the feed ($0 \leq Fpe \leq 1$) (dmls)
PEPBAL	Peptides balance
PEPNEx	Amount of N required as peptide (g d^{-1})
PEPUP	Amount of peptides assimilated by microorganisms (g d^{-1})
PEPUPaj	Amount of peptides assimilated by microorganisms adjusted due to the escape of peptides in the rumen (g d^{-1})
PEPx	Potential amount of peptide uptake (g d^{-1})
PFC'	Fecal losses of C' (g d^{-1})
PRD	Total rumen degradable true protein (g d^{-1})
RAA	Ruminal availability of fraction A from the feed (g d^{-1})
RAA'	Ruminal availability of fraction A' from the feed (g d^{-1})
RAB1	Ruminal availability of fraction B1 from the feed (g d^{-1})
RAB1'	Ruminal availability of fraction B1' from the feed (g d^{-1})

Continues...

Table 1 (Continued)

Symbol	Definition (dimension)
RAB1aj	Ruminal availability of fraction B1 adjusted due to escaping ruminal peptide (g d^{-1})
RAB2	Ruminal availability of fraction B2 from the feed (g d^{-1})
RAB2'	Ruminal availability of fraction B2' from the feed (g d^{-1})
RAB2aj	Ruminal availability of fraction B2 adjusted due to escaping ruminal peptide (g d^{-1})
RAB2'aj	Ruminal availability of fraction B2' adjusted due to a deficiency of rumen nitrogen (g d^{-1})
RANB	Rumen ammonia nitrogen balance (g d^{-1})
RAPEP	Ruminal availability of peptides supplied by the feed (g d^{-1})
RATIO	Availability of peptides in relation to carbohydrate fractions A' and B1' (dmls)
REA'	Ruminally escaped carbohydrate A' (g d^{-1})
REAM'	Ruminally escaped microbial A' (g d^{-1})
REAsh	Ruminally escaped ash (g d^{-1})
REAshM	Ruminal escape of ash associated with the microbial biomass (g d^{-1})
REB1	Ruminally escaped B1 (g d^{-1})
REB1'	Ruminally escaped B1' (g d^{-1})
REB1aj	Adjusted ruminally escaped B1 (g d^{-1})
REB1M'	Ruminally escaped microbial B1' (g d^{-1})
REB2	Ruminally escaped B2 (g d^{-1})
REB2'	Ruminally escaped B2' (g d^{-1})
REB2aj	Adjusted ruminally escaped B2 (g d^{-1})
REB2'aj	Adjusted ruminally escaped B2' (g d^{-1})
REC	Ruminally escaped C (g d^{-1})
REC'	Ruminally escaped C' (g d^{-1})
RECF	Ruminally escaped fat (g d^{-1})
REMCP	Ruminally escaped microbial crude protein (g d^{-1})
REMTC	Ruminally escaped microbial carbohydrate (g d^{-1})
REMF	Ruminally escaped rumen microbial fat (g d^{-1})
REMNA	Ruminally escaped microbial nucleic acids (g d^{-1})
REPEP	Ruminally escaped peptides (g d^{-1})
REPMCW	Ruminally escaped protein linked to microbial cell wall (g d^{-1})
RFM_{max}	Retention capacity of fiber in the rumen (g kg^{-1}W)
RMF	Rumen mass of fiber (g kg^{-1}W)
RNPEPUP	Retention of nitrogen by microorganisms that use the non-fibrous carbohydrates (g d^{-1})
TC	Amount of total carbohydrates for each feed (g d^{-1})
TDNa	Total apparently digestible nutrients (g d^{-1})
TPD	True protein digestibility (g d^{-1})
VFA	Amount of volatile fatty acids produced from the feed (g d^{-1})
X	Content of crude protein from the diet (%)
x	Quantity of the i-th ingredient of the total diet optimized (g d^{-1})
Y	Recycling endogenous ammonia (g d^{-1})
Y1	Growth efficiency of microorganisms that use carbohydrates (fraction B2') supplied by the feed (g cells g^{-1} fibrous carbohydrate digested)
Y2	Growth efficiency of microorganisms that use non-fiber carbohydrates (fraction A') provided by food (g cells g^{-1} fibrous carbohydrate digested)
Y2'	Efficiency of microbial growth corrected for IMP (g cells g^{-1} NFC digested)
Y3	Growth efficiency of microorganisms that use non-fiber carbohydrates (fraction B1') provided by food (g cells g^{-1} fibrous carbohydrate digested)
Y3'	Efficiency of microbial growth corrected for IMP (g cells g^{-1} NFC digested)
YG1	Theoretical maximum yield of fiber carbohydrate bacteria, 0.4; g of bacteria per g of carbohydrate digested per h (g cells g^{-1} NFC digested)
YG2	Theoretical maximum yield of non-fibrous carbohydrates bacteria, 0.4; g of bacteria per g of carbohydrate digested per h (g cells g^{-1} NFC digested)

[1] All symbols and acronyms were based on definitions: Russel et al. (1992), Fox et al. (2004), NCR (2007), Vieira et al. (2008ab).
dmls - dimensionless.

Table 2 - Equations and variables used in the simulation to estimate the nutritive value of feeds. Acronyms are listed in Table 1.

No.	Equation	No.	Equation
1	$PA_i = x_i \times 0.001 \times A_i$	37	$REA'_i = CA'_i \times RAA'_i$
2	$PB1_i = x_i \times 0.001 \times B1_i$	38	$REB1'_i = CB1'_i \times RAB1'_i$
3	$PB2_i = x_i \times 0.001 \times B2_i$	39	$REB2'_i = CB2'_i \times RAB2'_i$
4	$PC_i = x_i \times 0.001 \times C_i$	40	$REB2'aj_i = FCRed + REB2'$
5	$PB_i = PA_i + PB1_i + PB2_i + PC_i$	41	$REC'_i = CC'_i$
6	$CA'_i = x_i \times 0.001 \times A'_i$	42	$YG1 = 0.4$
7	$CB1'_i = x_i \times 0.001 \times B1'_i$	43	$YG2 = 0.4$ if $\sum_{i=1}^{m} fiber_i / \sum_{i=1}^{m} x_i \geq 0.2$
8	$CB2'_i = x_i \times 0.001 \times B2'_i$	44	$YG2 = 0.4 \times \{1 - [(20 - 100 \times \sum_{i=1}^{m} Fiber_i / \sum_{i=1}^{m} x_i) \times 0.025]\}$, otherwise $\sum_{i=1}^{m} Fiber_i / \sum_{i=1}^{m} x_i < 0.2$
9	$CC'_i = x_i \times 0.001 \times C'_i$	45	$IMP_i = \exp^{0.404 \ln(100 \times RATIO_i) + 1.942}$, $\forall IMP \subset [0,18]$
10	$TC_i = CA'_i + CB1'_i + CB2'_i + CC'_i$	46	$FCBACT_i = Y1_i \times RAB2'_i$
11	$NFC_i = CA'_i + CB1'_i$	47	$Y2'_i = Y2_i \times (1 + IMP_i \times 0.01)$
12	$FC_i = CB2'_i + CC'_i$	48	$Y3'_i = Y3_i \times (1 + IMP_i \times 0.01)$
13	$TC_i = NFC + FC$	49	$NFCBACT_i = Y2'_i \times RAA'_i + Y3'_i \times RAB1'_i$
14	$Fat_i = x_i \times 0.001 \times CF_i$	50	$BACT_i = FCBACT_i + NFCBACT_i$
15	$Ash_i = x_i \times 0.001 \times MM_i$	51	$NBACTFC_i = 0.1 \times FCBACT_i$
16	$Fiber_i = x_i \times 0.001 \times F_i$	52	$NBACTNF_i = 0.1 \times NFCBACT_i$
17	$Apef_i = pef \times Fiber_i$	53	$PEPUP_i = \dfrac{k_{up} \times NBACTNF_i}{k_{up} \times NBACTNF_i + k_{pl} \times RAPEP}$
18	$RAA_i = PA_i$	54	$k_{up} = 0.07$
19	$RAB1_i = PB1_i.DB1_i$	55	$PEPUPaj = DRPEP_h \times 24$, if $NFCBACTpepup \times DisappTime > DRPEPh$
20	$RAB1aj_i = Maximum(0;RAB1 - REPEP \times RAB1/RAPEP)$	56	$PEPUPaj = NFCBACTpepup \times DisappTime \times 24$, otherwise $NFCBACTpepup \times DisappTime \leq DRPEPh$
21	$RAB2_i = PB2_i \times DB2_i$	57	$REPEP = \sum_{i=1}^{m} RAPEP_i - \sum_{i=1}^{m} PEPUPaj_i$
22	$DB2_i = kd2_i\{\sum_{i=1}^{N_j} \lambda_{r,j}^{i-1}/(\lambda_{r,j} + kd2_j)^i + \lambda_{r,j}^{N_j}/(\lambda_{r,j} + kd2_j)^{N_j} \times (ke_j + kd2_j)\}$	58	$CPEPUP = PEPUPaj_i + REPEP$
23	$RAB2aj_i = Maximum(0;RAB2 - REPEP \times RAB2/RAPEP)$	59	$NPEPUPaj = PEPUPaj/6.25$
24	$MRT = N/k_{ri} + 1/k_{ei}$	60	$RAB1aj_i = Maximum\{0, RAB1_i - [REPEP \times (RAB1_i / \sum_{i=1}^{m}(RAB1_i + RAB2_i))]\}$
25	$RAPEP_i = RAB1_i + RAB2_i$	61	$RAB2aj_i = Maximum\{0, RAB2_i - [REPEP \times (RAB2_i / \sum_{i=1}^{m}(RAB1_i + RAB2_i))]\}$
26	$REB1_i = PB1_i - RAB1_i$	62	$NPEPUP_i = PEPUP_i/6.25$
27	$REB1aj_i = REB1 + (REPEP \times RAB1/RAPEP)$	63	$RNPEPUP_i = NPEPUP$, if $NBACTNF_i \geq NPEPUP_i/0.66$
28	$REB2_i = PB2_i - RAB2_i$	64	$RNPEPUP_i = 0.66 \times NBACTNF_i$, if $NBACTNF_i < NPEPUP_i/0.66$
29	$REB2aj_i = REB2 + (REPEP \times RAB2/RAPEP)$	65	$AMNBACTFC_i = NBACTFC_i$
30	$REC_i = PC_i$	66	$AMNBACTNF_i = NBACTNF_i - RNPEPUP$
31	$RAA'_i = CA'_i \times DA'_i$	67	$X = 100 \times \sum_{i=1}^{m}(PA_i + PB1_i + PB2_i + PC_i)/\sum_{i=1}^{m} x_i$, $\forall X \subset [3,56]$
32	$DA'_i = kd1'_i / kd1'_i + kpl$	68	$Y = 12.7 - 12.01x + 0.325x^2$
33	$RAB1'_i = CB1'_i \times DB1'_i$	69	$PRD = \sum_{i=1}^{m}(RAB1aj_i + RAB2aj_i)$
34	$DB1'_i = kd2'_i\{\sum_{l=1}^{N_j} \lambda_{r,j}^{l-1}/(\lambda_{r,j} + kd1'_j)^i + \lambda_{r,j}^{N_j}/(\lambda_{r,j} + kd1'_j)^{N_j} \times (ke_j + kd1'_j)\}$	70	$RANB = Y/100 \times PB/6.25 + PRD/6.25 + \sum_{i=1}^{m} RNPEPUP_i/6.25 - \sum_{i=1}^{m} AMNBACTNF_i - \sum_{i=1}^{m} AMNBACTFC_i$
35	$RAB2'_i = CB2'_i \times DB2_i'$	71	$REB1aj_i = PB1_i - RAB1aj_i$
36	$DB2_i' = kd3'_i\{\sum_{l=1}^{N} \lambda_r^{l-1}/(\lambda_r + kd3'_i)^i + \lambda_r^N/(\lambda_r + kd3'_i)^N \times (k_e + kd3'_i)\}$	72	$REB2aj_i = PB2_i - RAB2aj_i$

Continues...

Table 2 (Continued)

No.	Equation	No.	Equation
73	$\mathrm{NperBACT}_i = (\mathrm{NPEPUP} + \mathrm{NH3}_{recicled} + \mathrm{NH3}_{diet})/0.1$	110	$\mathrm{IDMNA}_i = \mathrm{REMNA}_i$
74	$\mathrm{EBACTRatio}_i = \mathrm{FCBACT}_i + \frac{\mathrm{NFCBACT}_i}{\sum_{i=1}^{m}(\mathrm{FCBACT}_i+\mathrm{NFCBACT}_i)}$	111	$\mathrm{IDP}_i = \mathrm{IDPF}_i + \mathrm{IDTMP}_i + \mathrm{IDMNA}_i$
75	$\mathrm{NperBACT}_i = \mathrm{NperBACT} \times \mathrm{EBACTRatio}_i$	112	$\mathrm{VFA}_i = \mathrm{RAA'}_i + \mathrm{RAB1'}_i + \mathrm{RAB2'aj}_i$
76	$\mathrm{BACTred}_i = (\mathrm{FCBACT}_i + \mathrm{BACTCNF}_i) - \mathrm{NperBACT}_i$	113	$\mathrm{IDCF}_i = \mathrm{REA'}_i + \mathrm{Cd}_i \times \mathrm{REB1'}_i + 0.2 + \mathrm{REB2'aj}_i$
77	$\mathrm{FCBACTratio}_i = \mathrm{FCBACT}_i / \frac{\mathrm{FCBACT}_i +}{\mathrm{NFCBACT}_i}$	114	$\mathrm{IDMC}_i = 0.95 \times \mathrm{REMTC}_i$
78	$\mathrm{FCBACTred}_i = \mathrm{BACTred}_i \times \mathrm{FCBACTratio}_i$	115	$\mathrm{IDTC}_i = \mathrm{VFA}_i + \mathrm{IDCF}_i + \mathrm{IDMC}_i$
79	$\mathrm{NFCBACTaj} = \mathrm{NFCBACT} - \mathrm{BACTred} \times (1 - \mathrm{FCBACTratio})$	116	$\mathrm{RECF}_i = \mathrm{Fat}_i$
80	$\mathrm{FCred} = \mathrm{FCBACTred}_i / \mathrm{Y1}_i$	117	$\mathrm{IDFF}_i = \mathrm{RECF}_i$
81	$\mathrm{FCRedRatio} = 100 \times \sum \mathrm{FCRed} / \sum \mathrm{RAB2'}_i$	118	$\mathrm{IDMF}_i = 0.95 \times \mathrm{REMF}_i$
82	$\mathrm{RAB2'aj}_i = \mathrm{RAB2'}_i - \mathrm{FCRed}_i$	119	$\mathrm{IDF}_i = \mathrm{IDFF}_i + \mathrm{IDMF}_i$
83	$\mathrm{REB2'aj}_i = \mathrm{REB2'}_i - \mathrm{FCRed}_i$	120	$\mathrm{FLB2}_i = (1 - 0.8) \times \mathrm{REB2aj}_i$
84	$\mathrm{PEPx} = \mathrm{NFCBACTpepup} \times \mathrm{DisappTime}$	121	$\mathrm{FLC}_i = \mathrm{REC}_i$
85	$\mathrm{PEPNEx} = 0.66 \times \sum \mathrm{NFCBACT} \times 0.1$	122	$\mathrm{FLPA}_i = \mathrm{FLB2}_i + \mathrm{FLC}_i$
86	$\mathrm{PEPBAL} = \mathrm{NPEPUP} - \mathrm{PEPNEx}$	123	$\mathrm{FLB1'}_i = (1 - \mathrm{cd}_i) \times \mathrm{REB1'}_i$
87	$\mathrm{NFBACT(NH3req)} = 0.34 \times \sum \mathrm{NFCBACT} \times 0.625 / 6.25$	124	$\mathrm{FLB2'}_i = (1 - 0.2) \times \mathrm{REB2'aj}_i$
88	$\mathrm{NH3}_{bact} = \mathrm{Maximum}(0, \mathrm{BALPEP})$	125	$\mathrm{FLC'} = \mathrm{REC'}$
89	$\mathrm{NH3}_{diet} = \mathrm{RAA}_i / 6.25$	126	$\mathrm{FLTCF}_i = \mathrm{FLB1'}_i + \mathrm{FLB2'}_i + \mathrm{FLC'}_i$
90	$\mathrm{NH3\ Recicled} = 0.01 \times \mathrm{Y} \times (\sum_{i=1}^{m} \mathrm{PB}_i / 6.25)$	127	$\mathrm{FLAsh}_i = \mathrm{Ash}_i$
91	$\mathrm{Nexc} = \mathrm{NPEPUP} + \mathrm{NH3}_{diet} + \mathrm{NH3}_{recicled} - (\sum_{i=1}^{m}\mathrm{NBACTCF} + \sum_{i=1}^{m}\mathrm{NBACTCNF}) + (\sum_{i=1}^{m}\mathrm{MP} - \mathrm{MP}_{required})/6.25$	128	$\mathrm{FLMCW}_i = \mathrm{REPMCW}_i$
92	$\mathrm{FC}_{\mathrm{NH3}_{eq}}\mathrm{avail} = \mathrm{Maximum}(0, (\mathrm{NH3}_{bact} + \mathrm{NH3}_{diet} + \mathrm{NH3}_{recicled}) - \mathrm{NFCBACTNH3}_{req})$	129	$\mathrm{FLTMP}_i = \mathrm{FLMCW}_i$
93	$\mathrm{FC(NH3}_{eq}) = \mathrm{FCBACT} \times 0.625 / 6.25$	130	$\mathrm{FLAshM}_i = \mathrm{REAshM}_i$
94	$\mathrm{BactNBal} = (\mathrm{NPEPUP} + \mathrm{NH3}_{diet} + \mathrm{NH3}_{recicled}) - (\mathrm{PEPNEx} + \mathrm{NFCBACTNH3}_{req} + \mathrm{FCNH3}_{eq})$	131	$\mathrm{FLBACT}_i = \mathrm{FLTMP}_i + \mathrm{FLMC}_i + \mathrm{FLMFat}_i + \mathrm{FLAshM}_i$
95	$\mathrm{BactNBal} = (\mathrm{NPEPUP} + \mathrm{NH3}_{diet} + \mathrm{NH3}_{recicled}) - (\mathrm{PEPNEx} + \mathrm{NFCBACTNH3}_{eq} + \mathrm{FCNH3}_{eq})$	132	$\mathrm{FLMFat}_i = (1 - 0.95) \times \mathrm{REMF}_i$
96	$\mathrm{EFCBACT} = \mathrm{FCBACT}$	133	$\mathrm{FLEP}_i = 0.0387 \times \mathrm{CP}_i$
97	$\mathrm{ENFCBACT} = \mathrm{NFCBACT}$	134	$\mathrm{FLEFat}_i = 0.017 \times \mathrm{Fat}_i$
98	$\mathrm{REMCP}_i = 0.6 \times 0.625 \times \mathrm{Minimum}(\mathrm{BACT}_i, \mathrm{NperBACT}_i)$	135	$\mathrm{FLEAsh}_i = \mathrm{FLEFat}_i = 0.0119 \times \mathrm{Ash}_i$
99	$\mathrm{REPMCW}_i = 0.25 \times 0.625 \times \mathrm{Minimum}(\mathrm{BACT}_i, \mathrm{NperBACT}_i)$	136	$\mathrm{FLTPRO}_i = \mathrm{FLPA}_i + \mathrm{FLTMP}_i + \mathrm{FLEP}_i$
100	$\mathrm{REMNA}_i = 0.15 \times 0.625 \times \mathrm{Minimum}(\mathrm{BACT}_i, \mathrm{NperBACT}_i)$	137	$\mathrm{FLTC}_i = \mathrm{FLTCF}_i + \mathrm{FLMC}_i$
101	$\mathrm{REAM'} = 0.211 \times 0.8 \times \mathrm{Minimum}(\mathrm{NperBACT}; \mathrm{EFCBACT} + \mathrm{ENFCBACT})$	138	$\mathrm{FLTFat}_i = \mathrm{FLMC}_i + \mathrm{FLEFat}_i$
102	$\mathrm{REB1M'} = 0.211 \times 0.2 \times \mathrm{Minimum}(\mathrm{NperBACT}; \mathrm{EFCBACT} + \mathrm{ENFCBACT})$	139	$\mathrm{FLTAsh}_i = \mathrm{FLAsh}_i + \mathrm{FLAshM}_i + \mathrm{FLEAsh}_i$
103	$\mathrm{REMTC}_i = 0.21 \times \mathrm{Minimum}(\mathrm{BACT}_i, \mathrm{NperBACT}_i)$	140	$\mathrm{AIM}_i = \mathrm{FLTPRO}_i + \mathrm{FLTCT}_i + \mathrm{FLTAsh}_i + \mathrm{FLTFat}_i$
104	$\mathrm{REMF}_i = 0.12 \times \mathrm{Minimum}(\mathrm{BACT}_i, \mathrm{NperBACT}_i)$	141	$\mathrm{TDNa}_i = (\mathrm{CP}_i - \mathrm{FLTPRO}_i) + (\mathrm{TC}_i - \mathrm{FLTCT}_i) + 2.25 \times (\mathrm{Fat}_i - \mathrm{FLTFat}_i)$
105	$\mathrm{REAsh}_i = 0.044 \times \mathrm{Minimum}(\mathrm{BACT}_i, \mathrm{NperBACT}_i)$	142	$[\mathrm{TDNa}_i] = \mathrm{TDNa}_i / \sum_{i=1}^{m} \mathrm{x}_i$
106	$\mathrm{IDB1}_i = \mathrm{REB1aj}_i$	143	$\mathrm{Urea\ formation} = [(\mathrm{RANB} - \mathrm{NH3}_{recicled} + \mathrm{Nexc}) \times 0.0073] \times 4.184$
107	$\mathrm{IDB2}_i = 0.8 \times \mathrm{REB2aj}_i$	144	$\mathrm{ADE} = 0.001 \times 4.409 \times 4.184 \times \sum_{i=1}^{m} \mathrm{TDNa}_i$
108	$\mathrm{IDPF}_i = \mathrm{IDB1}_i + \mathrm{IDB2}_i$	145	$[\mathrm{DE}] = \mathrm{ADE} / \sum_{i=1}^{m} \mathrm{x}_i$
109	$\mathrm{IDTMP}_i = \mathrm{REMCP}_i$	146	$[\mathrm{ME}] = (1.01 \times [\mathrm{DE}]/4.184 - 0.45) \times 4.184$, if $\sum_{i=1}^{m} \mathrm{Fat}_i / \sum_{i=1}^{m} \mathrm{x}_i < 0.03$

Continues...

Table 2 (Continued)

No.	Equation	No.	Equation
147	$[ME] = [1.01 \times [DE]/4.184 - 0.45 + 0.0046 \times (\sum_{i=1}^{m} Fat_i / \sum_{i=1}^{m} x_i \times 100 - 3)] \times 4.184$	156	$RATIO_i = RAPEP_i / RAPEP_i + RAA'_i + RAB1'_i$
148	$AME = [ME] \times \sum_{i=1}^{m} x_i / 1000$	157	$FLMCW_i = (1 - 0.95) \times REMTC_i$
149	$MP_i = IDP_i - IDMNA_i$	158	$k_{upi} = k_{up} \times 0.66$
150	$RMF_i = \Sigma_i \{pef_i F_{NDFi} [B2'_i \{\Sigma_{i=1}^{N_j} \lambda_{r,i}^{i-1} / (\lambda_{r,j} + kd3'_i)^1 + \lambda_{r,i}^{N} / (\lambda_{r,j} + kd3'_i)^N + ((k_{e,j} + kd3'_j))\} + C'_j [N_j / \lambda_{r,j} + 1/k_{e,j}]]\}$	159	$NFCBACTmass = NFCBACT_i \times MRT_i / 24$
151	$Y1_i = kd3'_i \times YG1 / km1 \times YG1 \times kd3'_i$	160	$NFCBACTpepup = \sum_{i=1}^{m} NFCBACTmass_i \times k_{up}$
152	$Y2_i = kd1'_i \times YG2 / km2 \times YG2 \times kd1'_i$	161	$DRPEPh = \sum_{i=1}^{m} DRPEP_i / 24$
153	$Y3_i = kd2'_i \times YG2 / km2 \times YG2 \times kd2'_i$	162	$Growth\ time = 1/1 - kpl$
154	$km1 = 0.05$	163	$DisappTime = 1/(3600 \times \{(\log[(Growth\ time - 1)/(3600 \times DRPEPh)] / (BACTNFpepup/3600) + 1/\log[1 + (Growth\ time - 1)/3600])\}$
155	$km2 = 0.15$		

Table 3 - Equations used in simulations to compute nutritional requirements[1]

No.	Equation
164	$RFMmax = 8.5 \times BW$
165	$MP_g = 0.290 \times ADG$, for growing animals
166	$MFCP = 0.0267 \times DMI$
167	$TPD = 0.88 \times CPI$
168	$k_m = 0.35 \times [AME / \sum (x) \times 0.001 / 18.8]$
169	$MEm = (315 + 31.5 \times BW^{0.75}) / km$, for mature animals
170	$MP_{l-d} = 1.45 \times MP \times MPm \times 1000$, for mature animals
171	$ME_{l-d} = (1.4694 + 0.4025 \times FCM) \times Milkyield \times k_{l-d}$, for mature animals
172	$k_{l-d} = 0.624$
173	$MEm = 580 \times BW^{0.75}$, for growing animals
174	$MEg = 23.1 \times ADG$, for growing animals
175	$EUCP = 1.031 \times BW^{0.75}$
176	$MP_m = MFCP + EUCP + 0.2 \times BW^{0.6}$, for mature animals
177	$MEg = 28.5 \times ADG$, for mature animals

[1] All symbols and acronyms are based on definitions of AFRC (1993).

obviously, the minimum cost optimization (objective function Z, Eq. 1) was the most efficient objective function to minimize diet costs (Figure 2c). For all simulations, the increase in the production performance (milk yield or daily gain) increased diet costs (Figures 2 and 3). The RANB constraints (RANB ≥ 0 and RANB ≥ -200) did not influence the cost of the diets (Figures 3a, 3b, 3c and 3d). The space of feasible solutions was insensitive to the RANB constraint, and although the dietary cost increased with more challenging performance levels, the same solution space can be observed by comparing Figure 3a with 3b for milk yield, and Figure 3c with 3d for average daily gain.

Discussion

Linear optimization systems require an estimate of the dry matter intake as an input to solve the problem of least-cost diets (Tedeschi et al., 2000). However, the nonlinear nature of diet formulation is characterized by the interdependence between animal requirements and the food consumed (Jardim et al., 2013; 2015).Therefore, the solution or the optimized diet and its expected dry matter intake influences the values of the components of the constraints. In the model proposed in this study, intake is an output of the nonlinear optimization procedure.

The metabolizable protein and metabolizable energy intakes increase as animal production increases, because of higher demands for nutrients generated by growth, milk yield, and pregnancy (NRC, 2007). However, dry matter intake has a physical limit, imposed by the dietary fiber content, and the maximal capacity of fiber retention in the rumen (Mertens, 1994; Vieira et al., 2008b). The rumen size limits animal capacity due to fill, and because fiber generally passes from the reticulorumen more slowly, it has a great filling effect because of the distension it causes in rumen walls (Allen, 1996). The simulations with higher fiber content in the diet limits the space of feasible solutions (Figures 1e and 1f). According to Mertens (1987), higher milk productions constrain the fiber content in dairy cow diets, and this was observed here for dairy does diets (Gonçalves et al., 2001).

Speculations are made about the advantage of maximizing the dry matter intake of farm animals. Mertens (1987) developed simple mathematical models that can be used to predict maximum intake. However, simulations in

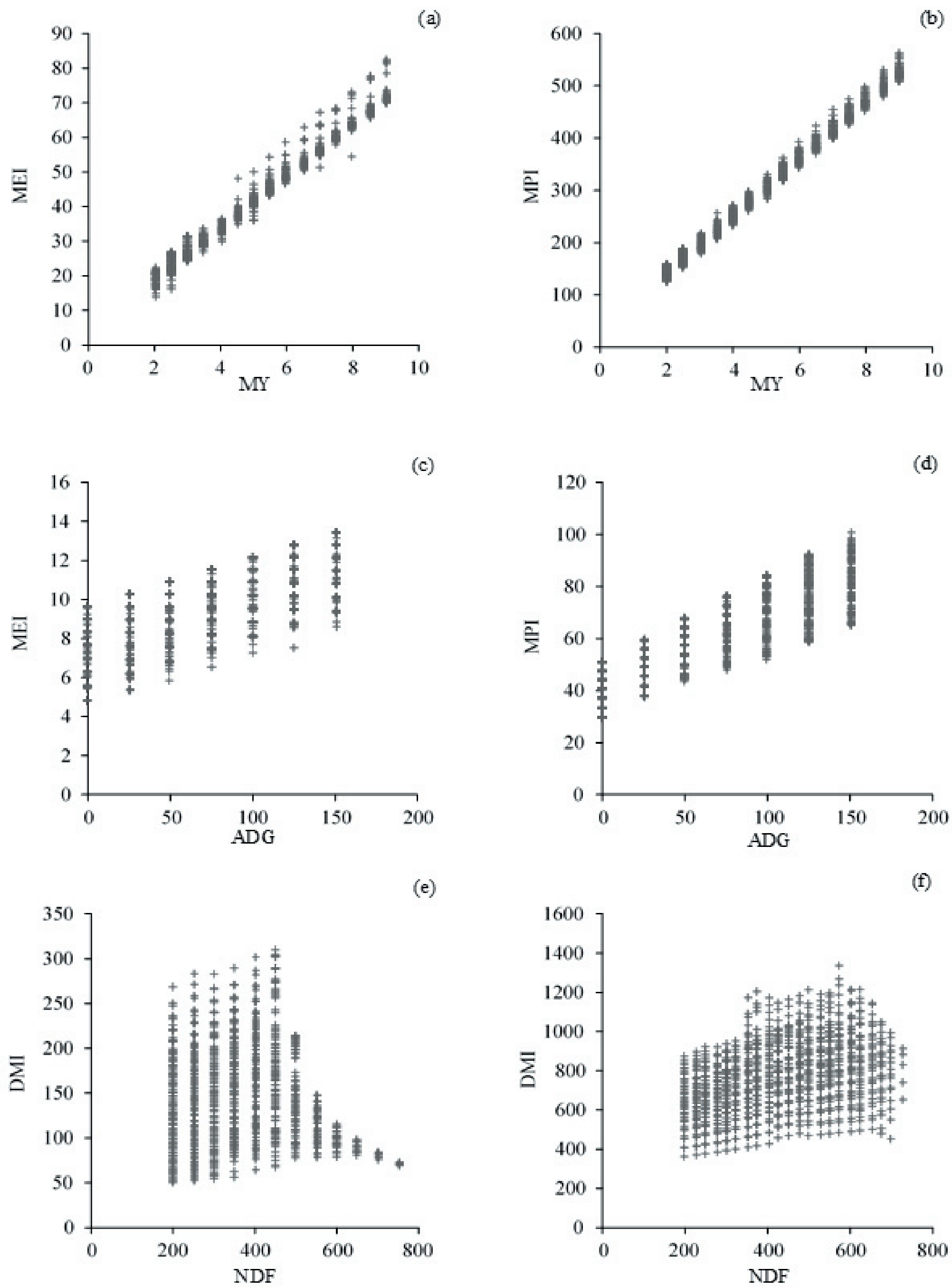

Figure 1 - Metabolizable energy intake (MEI, MJ/d) in relation to milk yield (MY, kg/d) (1a); metabolizable protein intake (MPI, g/d) in relation to MY(kg/d) (1b); MEI (MJ/d) in relation to average daily gain (ADG, g/d) (1c); MPI (g/d) in relation to ADG (g/d) (1d); dry matter intake (DMI, g/$kg^{0.75}$ per day) in relation to diet fiber content (NDF) for dairy goat (g/kg) (1e); DMI (g/$kg^{0.75}$ per day) in relation to NDF for growing goat (g/kg) (1f).

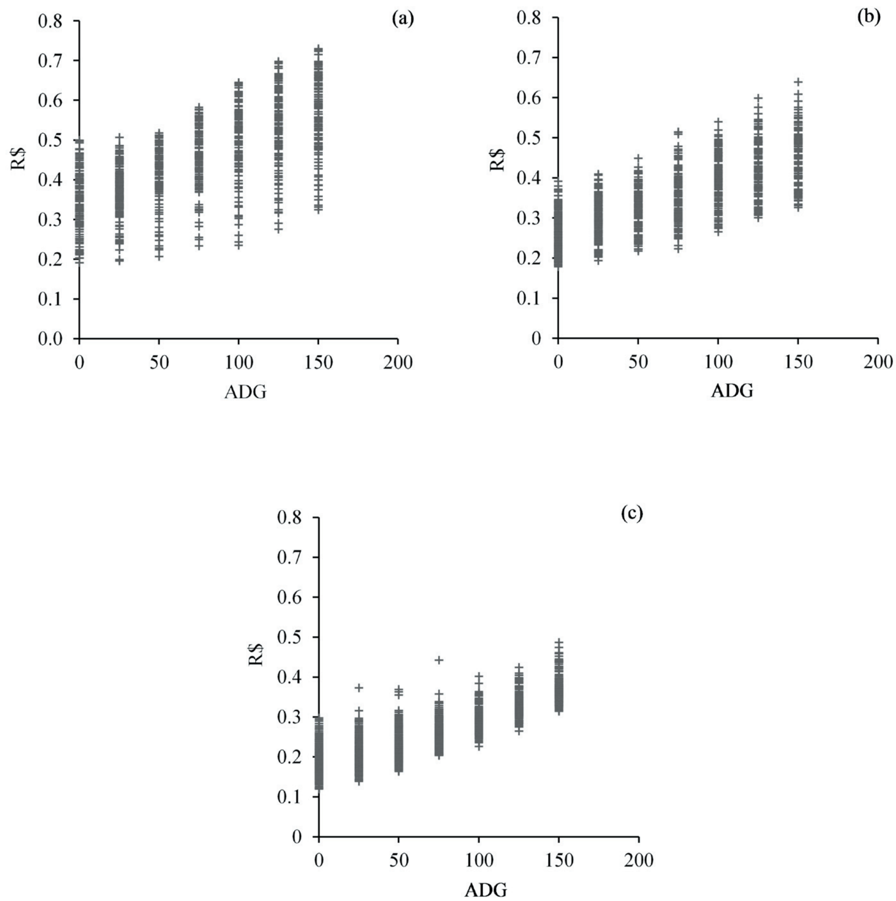

Figure 2 - Diet costs (R$) in relation to average daily gain (kg/d), using maximum intake as the objective function (2a); using maximum efficiency of utilization of crude protein as the objective function (2b); and using minimum cost as the objective function (2c).

which maximum dry matter intake was set as the objective function (Eq. 2) resulted in more expensive diets (Figure 2). The feedstuffs used as protein sources are, generally, more expensive than energy sources and, for this reason, simulations used to maximize protein utilization efficiency were made (Eq. 3). Nevertheless, the cost of protein-optimized diets using Eq. 3 as the objective function resulted in intermediary dietary costs (Figure 3). Least-cost diet formulation (Eq. 1) was the most effective procedure to reduce the cost of the diet under the same dietary constraints (Figure 3).

The rumen microorganisms can synthesize protein from non-protein nitrogen and ammonia is the main source of nitrogen for microbial protein synthesis (Russel et al., 1992). The amount of ruminal ammonia nitrogen can be estimated by the sum of endogenous nitrogen recycling with nitrogen originated from degradation of dietary protein and dietary non-protein nitrogen, and by discounting nitrogen retained by bacteria (Russel et al., 1992; Vieira et al., 2000a,c). The RANB indicates if rumen ammonia nitrogen is adequate to meet bacterial requirements. A positive RANB is essential to maximize ruminal degradation of the feed (Leng, 1990), and so, nitrogen deficiency decreases carbohydrate fermentation and the growth rate of fiber fermenting bacteria like *F. succinogenes* that become unable to ferment cellobiose if RANB < 0 (Maglione and Russel, 1997). The scenario in which RANB < 0 occurs in tropical pastures, specifically in the dry season, when the forage nutritive value and availability are reduced remarkably (Vieira et al. 2000b,c). The constraint expands the space for feasible solutions compared with RANB ≥ −200, which could result in cheaper diets, but the cost of the diets did

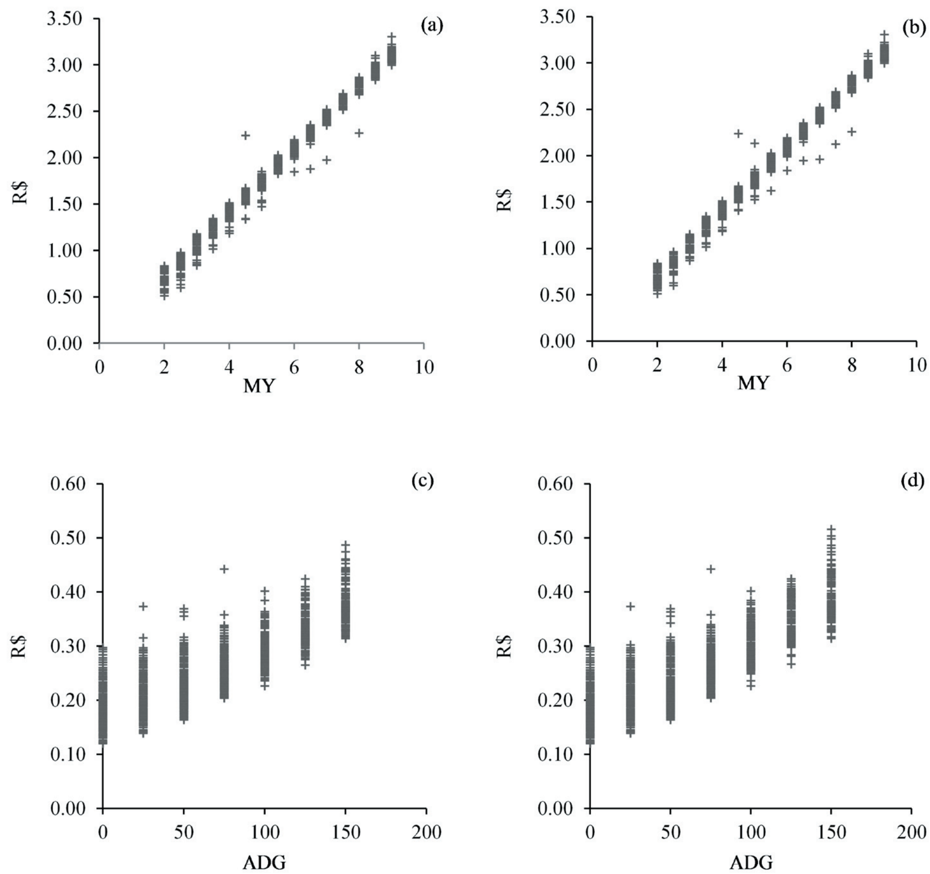

Figure 3 - Diet costs (R$) in relation to milk yield (kg/d) using RANB ≥ 0 (3a); in relation to milk yield (kg/d) using RANB ≥ –200 (3b); in relation to average daily gain (kg/d) using RANB ≥ 0 (3c); and in relation to average daily gain (kg/d) using RANB ≥ –200 (3d).

not differ between these two constraints (Figure 3). The CNCPS fractionation scheme is a useful tool to estimate the nutritive availability of protein and carbohydrate fractions in feeds and has been used to estimate the ruminal availability of protein and carbohydrates of tropical feeds (Cabral et al., 2000).

Dairy goat farming is an important activity that can generate income and wealth for farmers. This activity can produce enough wealth to the succession of the family business, which is an important tool for generating jobs and income (Vieira et al., 2009), mainly in the state of Rio de Janeiro, because of its unique goat milk production systems that favor the development of special products for specific markets (Santos Junior et al., 2008). Therefore, the control of production costs is mandatory. In that sense, nutrition models would assist in the optimization of small ruminant production scenarios (Tedeschi et al., 2010). Among all variables regarding nutrition of ruminants, passage rate estimates affect the utilization of fiber by small ruminants too; therefore, models based on the retention of fiber in the rumen are needed to properly formulate goat diets (Tedeschi et al., 2012; Regadas Filho et al., 2014a,b; Jardim et al., 2013; 2015). In this regard, the simulation of different scenarios could help in the decision-making process and to improve the understanding of the dynamics of goat nutrition and feeding.

Table 4 - Prices[1], chemical composition, digestion and passage parameters, and physical effectiveness of fiber of each feed ingredient[2,3]

Feed	Rkg_i$	A_i	$B1_i$	$B2_i$	C_i	A'_i	$B1'_i$	$B2'_i$	C'_i	CP_i	CF_i	Ash_i	NFC_i	$Fiber_i$	pef	Cd	N	kd_{1i}	kd_{2i}	kd'_{1i}	kd'_{2i}	kd'_{3i}	k_{ri}	k^4_{ei}	λ^4_r
Elephant grass	0.13	1.1	92.5	8.1	2.3	80.0	7.0	445.9	254.1	104.0	27.0	82.0	87.0	700.0	1.00	0.75	2	1.35	0.10	2.50	0.30	0.04	0.10	0.04	0.20
Cottonseed meal	0.90	76.8	318.5	3.4	21.3	19.0	171.0	244.2	63.8	420.0	19.0	63.0	190.0	308.0	0.36	0.75	1	0.71	0.06	3.00	0.10	0.03	10000	0.05	10000
Soybean meal	1.20	47.4	431.4	47.9	5.3	29.4	264.6	93.0	5.0	532.0	11.0	65.0	294.0	98.0	0.34	0.75	1	2.87	0.09	3.00	0.45	0.06	10000	0.05	10000
Wheat bran	0.66	58.0	96.0	21.0	5.0	35.8	322.2	314.6	52.4	180.0	45.0	50.0	358.0	367.0	0.02	0.75	1	2.45	0.04	3.00	0.70	0.12	10000	0.03	10000
Coast-cross hay	0.31	7.7	27.5	39.3	10.1	18.6	36.4	627.6	145.6	84.5	17.5	69.8	55.0	773.2	1.00	0.75	2	0.52	0.01	2.50	0.30	0.05	0.20	0.05	0.40
Alfalfa hay	0.50	54.7	106.8	9.5	19.0	171.0	19.0	300.0	200.0	190.0	20.0	100.0	190.0	500.0	0.92	0.15	2	1.50	0.09	2.50	0.30	0.05	0.18	0.06	0.36
Ground sorghum grain	0.48	3.2	76.9	15.5	3.3	100.5	630.5	59.8	52.2	98.8	41.0	17.0	731.0	112.0	0.34	0.75	1	0.25	0.06	1.50	0.12	0.05	10000	0.06	10000
Soybean oil	2.70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1000.0	0.0	0.0	0.0	0.00	0.80	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Maize silage	0.31	42.5	28.9	6.0	7.7	0.0	273.0	415.3	124.7	85.0	32.0	70.0	273.0	540.0	0.85	0.75	2	3.00	0.10	2.75	0.25	0.06	0.15	0.05	0.30
Ground corn	0.60	8.0	52.0	5.0	1.0	78.0	697.0	93.0	5.0	66.0	21.0	40.0	775.0	98.0	0.34	0.80	1	0.26	0.02	3.00	0.35	0.06	10000	0.06	10000
Soybean grain	1.00	26.9	248.2	44.1	22.8	117.0	149.0	0.1	134.9	342.0	179.0	78.0	266.0	135.0	1.00	0.75	1	0.25	0.03	3.00	0.45	0.06	10000	0.05	10000
Urea	1.45	2812.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2812.0	0.0	0.0	0.0	0.0	0.00	0.00	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

[1] Brazilian currency R$ 1.00 = US$ 0.64 in July, 2011; and R$ 1.00 = US$ 0.32 in July, 2015.
[2] The subscript i denotes the i-th feed ingredient.
[3] See Table 1 for definitions of symbols and acronyms.
[4] High values indicate that $\lambda_r \gg k_e$.

Conclusions

The Microsoft Excel® Solver® allows for the balance of diets for dairy goats and growing doelings using different objective functions. Least-cost formulations provide better solutions in terms of overall costs of the diets than maximization of dry matter intake or crude protein use do. There is no net improvement of maximizing both dry matter intake and efficiency of use of crude protein. The predictions obtained with this model are in accordance with ruminant nutrition theories, and the nonlinear programming problem of the diet can be modeled to simulate different scenarios for decision-making, which is useful for developing strategies for increasing profitability of dairy goat production systems.

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