Revista Ciência Agronômica, v. 54, e20207523, 2023
Centro de Ciências Agrárias - Universidade Federal do Ceará, Fortaleza, CE
www.ccarevista.ufc.br

Scientific Article
ISSN 1806-6690

Evaluation of irrigation requirement for the design of an irrigation system using a probabilistic approach for the estimation of evapotranspiration and rainfall[1]

Verônica Gaspar Martins Leite Melo[2], José Antônio Frizzone[2], Antonio Pires de Camargo[2]*, Leonardo Leite de Melo[2]

ABSTRACT - Reference evapotranspiration (ET_0) and rainfall are basic variables for estimating the net irrigation depth (NID). The objective of this study was to estimate the NID for designing irrigation systems in Piracicaba, SP, Brazil, using ET_0 and rainfall probability distributions. A 30-year ET_0 and rainfall dataset (1990–2019) was obtained from the ESALQ/USP weather station. The water balance between ET_0 and rainfall indicated July, August, and September as months of higher water deficit. Based on the first-order Markov chain, August presented the highest water deficit. Rainfall and ET_0 were estimated on 19 probability levels, and four probability distributions such as normal, log-normal, beta, and mixed gamma were evaluated. The analysis of historical August series using accumulated values in periods of five, ten, or 15 days is recommended for sizing irrigation designs in Piracicaba, SP, Brazil. The log-normal and mixed gamma probability distributions presented the best fit for ET_0 and rainfall data, respectively. To reach a crop coefficient K_c = 1 in Piracicaba, SP, Brazil in August, the irrigation system should be designed for an NID of 4.1 mm day^{-1}. The use of mean monthly rainfall and ET_0 values for designing irrigation systems underestimates the NID by a mean of 26.6% compared to estimates made at a probability of 75% at five-, ten-, and 15-day intervals because the mean rainfall values occurred with exceedance probabilities of < 36%, and mean ET_0 values occurred with non-exceedance probabilities of < 56%.

Key words: Net irrigation depth. Supplementary irrigation. Probable rainfall. Probable evapotranspiration.

DOI: 10.5935/1806-6690.20230048
Editor-in-Article: Prof. Carlos Alexandre Gomes Costa - costacag@gmail.com
*Author for correspondence
Received for publication on 01/08/2020; approved on 10/04/2023
[1]Biosystems Engineering Department, College of Agriculture “Luiz de Queiroz”, University of São Paulo, CEP 13418-900, Piracicaba-SP, Brazil
[2]Agricultural Engineering College, University of Campinas-SP, Brazil, veronica.martins@usp.br (ORCID ID 0000-0003-0665-3726), frizzone@usp.br (ORCID ID 0000-0002-4251-1496), apcpires@unicamp.br (ORCID ID 0000-0001-5164-2634), leoleitemelo@usp.br (ORCID ID 0000-0002-1866-883X)

INTRODUCTION

The quantity of water required for irrigation is one of the main parameters in designing and managing irrigation systems and in assessing water availability. An overestimated net irrigation depth (NID) results in oversized irrigation systems and increased costs per unit area. However, an underestimated NID results in undersized irrigation systems and its consequent incapacity to service the entire design area.

Crop water requirement is the quantity of water required in a given period without limiting yield under local climatic conditions. The NID, on the other hand, represents the quantity of water to be supplied by irrigation systems to complement rainfall to satisfy the quantity required by the crop (WALLER; YTAYEW, 2016).

The recommended periods for analyses of rainfall and reference evapotranspiration (ET_0) are five, ten, and 15 days, or monthly for irrigation designs under wet climate conditions (FERNANDES *et al*., 2019). Ideally, the analysis period should synchronize with the irrigation shift (SAAD *et al*., 2002). The NID can be estimated using the simplified water balance equation, considering the difference between potential crop evapotranspiration and rainfall (BERNARDO *et al*., 2019).

ET_0 and rainfall are variables with random components; however, these are fundamental for estimating crop irrigation requirements. They have great variability, which results in considerable dispersion of the calculated NID and requires an analysis of their probability distribution values. Rainfall presents the most dispersion in estimation models of crop irrigation (SOUZA *et al*., 2019). Agricultural designs involving hydrological variables require a study of probability distribution values accumulated over a specific time interval (MESQUITA; GRIEBELER; CORRECHEL, 2013).

The objective of this study was to estimate the NID for designing irrigation systems in Piracicaba, SP, Brazil, using analyses of ET_0 and rainfall probability distribution.

MATERIAL AND METHODS

This study was developed using data from the ESALQ/USP Conventional Weather Station, in Piracicaba, SP, Brazil. The station is located at the geographic coordinates 22° 42′ 30″ S and 47° 38′ 00″ W, with Köppen–Geiger *Cwa* mesothermal climate, dry winters, at 546 m altitude, and a mean annual rainfall of 1,300 mm, most of it in the summer, with 45% of this total occurring from January to February (LEB, 2020).

A 30-year dataset (1990–2019) was used to calculate ET_0 and rainfall frequency distribution. At the beginning of this study, ET_0 and rainfall were analyzed using accumulated decennial values, which showed that July, August, and September present the greatest water deficiency. ET_0 (mm day^{-1}) was calculated using the Penman–Monteith model standardized by the American Society of Civil Engineers (ALLEN *et al*., 2005).

ET_0 and rainfall data accumulated over monthly or five-, ten-, and 15-day intervals were analyzed. The irrigation requirement for the analyzed periods is defined by equation 1:

$$NID = K_c ET_0 - \text{rainfall} \tag{1}$$

where NID is the net irrigation depth in mm, ET_0 is the reference evapotranspiration in mm, K_c is the crop coefficient; and rainfall in mm.

The value of rainfall used in equation (1) is the probable rainfall, which represents the minimum quantity of rainfall expected at the specified probability level. The ET_0 is the maximum expected value at the specified probability level. Thus, the minimum expected rainfall and maximum ET_0 values at specified probability levels are obtained and used in equation (1) to calculate the NID for the design.

The first-order Markov chain was used to analyze the probabilities of dry days [P(D)], i.e., the probabilities of ET_0 exceeding the rainfall (BONAMENTE, 2017; MINUZZI, 2016) in ten-day period. The dry period and the sequences of consecutive dry days were characterized based on this theory. The equations used are as follows:

$$P(D) = \frac{F(D)}{F(D) + F(W)} = \frac{F(D)}{N} \tag{2}$$

$$P(W) = \frac{F(W)}{F(D) + F(W)} = 1 - P(D) \tag{3}$$

where F(D) is the frequency of dry days in a given period, F(W) is the frequency of wet days in a given period, and P(D) is the probability of a dry day in a given period, P(W) is the probability of a wet day in a given period, and N is the size of the historical series.

Conditional or transition probabilities were defined as:

$$P(D/D) = \frac{F(D/D)}{F(D/D) + F(W/D)} = \frac{F(D/D)}{F(D)} \tag{4}$$

$$P(U/S) = \frac{F(W/D)}{F(D/D) + F(W/D)} = \frac{F(W/D)}{F(D)} = 1 - P(D/D) \tag{5}$$

$$P(W/W) = \frac{F(W/W)}{F(W/W) + F(D/W)} = \frac{F(W/W)}{F(W)} \tag{6}$$

$$P(D/W) = \frac{F(D/W)}{F(W/W) + F(D/W)} = \frac{F(D/W)}{F(W)} = 1 - P(W/W) \tag{7}$$

where F(D/D) is the frequency of dry days in a period, considering that the previous day was dry, F(W/D) is the frequency of wet days in a period, considering that the previous day was dry, F(W/W) is frequency of wet days in a period, considering that the previous day was wet, and F(D/W) is the frequency of dry days in a period, considering that the previous day was wet, and P(D/D), P(W/D), P(W/W), P(D/W) are the corresponding probabilities, respectively.

Rainfall and ET_0 were estimated at 19 probability levels from 0.05 to 0.95. The rainfall values recommended for irrigation design generally correspond to probability levels of 0.75 or 0.80 (BERNARDO *et al*., 2019). The level of probability used depends on the water availability and crop value. The analysis of rainfall values in descending order shows that a 0.75 probability provides a rainfall value with a probability of 75% being equaled or exceeded, i.e., the mean rainfall value that should be equaled or exceeded at least once every 1.33 years. In this condition, there is a probability of 25% that the rainfall event will not be equaled (lower than the estimate) at least once a year and a probability of < 0.1% that the event will not be equaled at least once every five years. As for ET_0, Saad *et al*. (2002) and Souza *et al*. (2019) recommend that the probable value for irrigation design is obtained at a probability of 25%, i.e., a probability of 75% of not exceeding. Under this condition, on average, ET_0 is expected to be equaled or exceeded once every four years.

Four probability distribution models (normal, log-normal, beta, and mixed gamma) that are typically used for climatological data were analyzed for ET_0 and rainfall (ASSIS; ARRUDA; PEREIRA, 1996). The Kolmogorov–Smirnov (K–S) test was used to assess whether the rainfall and ET_0 samples are from a population with a specific distribution. Thus, the K–S statistic was defined as the highest absolute difference between the empirical and estimated cumulative frequency curves (D_{sup}) for each distribution function.

D_{sup} was compared with the quantile $D_{(1-\alpha)}$ or $D_{critical}$ given in the quantile table for the K–S statistic test, at a significance level of $\alpha = 0.05$, (BONAMENTE, 2017; BRADLEY, 2013 **)**. The distribution with the lowest D_{sup} was accepted as best fit. The lower this deviation, the better the quality of fit.

Regardless of the probability distribution, the probability of a continuous random variable x within the interval [a,b] is given by equation 8 (probability distribution function or cumulative probability function), where f(x) represents the probability density function of the distribution of interest.

$$P(a \le X \le b) = \int_a^b f(X)dx \tag{8}$$

For a probability distribution of continuous random variables, the mean or expected value E(x) describes the center of gravity of the probability distribution, while the variance V(x) is a measure of the dispersion of the possible x values within the distribution. Table 1 presents the probability density function, expected value, and variance of the assessed probability distributions (MONTGOMERY; RUNGER, 2014).

Rainfall data and ET_0 series were sorted in descending and ascending orders, respectively. Probability density function and cumulative probability function values for normal, log-normal, beta, and gamma distributions were obtained using functions available in the Microsoft Excel software.

For normal distribution, the population mean (μ) and population standard deviation (σ) parameters were approximated using the arithmetic mean ($\overline{X}$) and sample standard deviation (S_x) of the dataset.

For log-normal distribution, data were transformed using ln(x). Transformed data were used to calculate the arithmetic mean (θ) and sample standard deviation (ω) characteristic of the log-normal probability distribution. Beta distribution parameters were estimated using the method of moments, as proposed by Assis, Arruda, and Pereira (1996), and Denski and Back (2015) (equations 9 and 10). These equations use the arithmetic mean ($\overline{X}$) and the sample standard deviation (S_x) of normalized data in the range 0–1, according to equation (11).

Table 1 - Probability density function, mean, and variance of the assessed probability distributions

Distribution	Probability density function	Expected value	Variance
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\alpha}\right)^2} \quad -\infty < \chi < \infty; -\infty < \mu < \infty; \sigma > 0$	$E(\chi) = \mu$	$V(\chi) = \sigma^2$
Log-normal	$f(\chi) = \frac{1}{\chi\omega\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{\ln(\chi)-\theta}{\omega}\right]^2} \quad 0 < \chi < \infty$	$E(\chi) = e^{\theta+\frac{\omega^2}{2}}$	$V(\chi) = e^{2\theta+\omega^2}\left(e^{\omega^2} - 1\right)$
Beta	$f(\chi) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \chi^{\alpha-1}(1-\chi)^{\beta-1} 0 \le \chi \le 1; \alpha > 0; \beta > 0$	$E(\chi) = \frac{\alpha}{\alpha+\beta}$	$V(\chi) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Gama	$f(\chi) = \frac{\lambda^\Gamma \chi^{\Gamma-1} e^{-\lambda\chi}}{\Gamma(\Gamma)} \chi > 0; \lambda > 0; \Gamma > 0$	$E(\chi) = \frac{\Gamma}{\lambda}$	$V(\chi) = \frac{\Gamma}{\lambda^2}$

$$\beta = \left(1 - \overline{X}\right)\left[\frac{\overline{X}\left(1 - \overline{X}\right)}{S_X^2}\right] \tag{9}$$

$$\alpha = \frac{\overline{X}\beta}{1 - \overline{X}} \tag{10}$$

$$Y = \frac{X - X_{\min}}{X_{\max} - X_{\min}} \tag{11}$$

where y is the normalized random variable x, x_{min} is the lowest value in the series, and x_{max} is the highest value.

Gamma distribution parameters were estimated by equations (12) to (14), according to Assis, Arruda, and Pereira (1996) and Silva *et al*. (2015). In the equations presented, r is the shape factor, λ is the scale factor, and A is the asymmetry coefficient of the gamma distribution.

$$A = \ln\left(\overline{X}\right) - \frac{1}{n}\sum_{i=i}^{n}\ln\left(X_i\right) \tag{12}$$

$$r = \frac{1}{4A}\left(1 + \sqrt{1 + \frac{4A}{3}}\right) \tag{13}$$

$$\lambda = \frac{r}{\overline{X}} \tag{14}$$

The gamma distribution does not admit null values, which is a limitation for the analysis of rainfall data in short time intervals. Disregarding these occurrences and working only with non-zero values result in the overestimation of the probable event for a given probability level. This issue is resolved by the concept of mixed distribution (SOUZA *et al*., 2019), in which the cumulative probability function F(x) is determined in two parts, according to equation (15):

$$F\left(X\right) = P_0 + \left[\left(1 - P_0\right)G\left(X\right)\right] \tag{15}$$

where, P_0 is the probability of occurrence of null values, obtained by the ratio between the number of zeros and the size of the dataset; G(x) is the cumulative probability function of the gamma distribution, whose values were obtained using functions available in the Microsoft Excel software.

RESULTS AND DISCUSSION

Figure 1 shows that the mean deficit between rainfall and ET_0 (full line) starts in mid-March and extends until mid-October, intensifying between July and September. From April to October, the accumulated deficit is 300.7 mm, and from July to September it is 192.5 mm (64.0%), with the highest deficit occurring in August (82.7 mm). The dotted curves in Figure 1 delimit 90% of the values in each month, with 5% above the top line and 5% below the bottom line. The greatest data variability occurs in January, February, and March.

Table 2 shows the decennial values. In Piracicaba, SP, Brazil, irrigation is normally developed from March to September, when the crops are in full vegetative development (SAAD *et al*., 2002). In this case, the irrigation systems should be sized to meet the water demand in August, when the greatest water deficiency is observed. Table 2 shows that the highest deficit between ET_0 and rainfall happens in August, totaling 118.2 mm of ET_0 and 28.5 mm of rainfall. The use of the first-order Markov Chain in the decennials over the three months indicated a high probability of dry days in all decennials. The period expected to have the greatest number of dry days in the analyzed city, indicated by the probability of a dry day [P(D)], is the second decennial of August, i.e., 9.3 days in 10 days. In any decennial in August, the chances of rainfall exceeding ET_0 are < 8.8%.

Figure 1 - Deficit between monthly rainfall and ET_0 in Piracicaba, SP, Brazil, using a 30-year historical series (1990–2019)

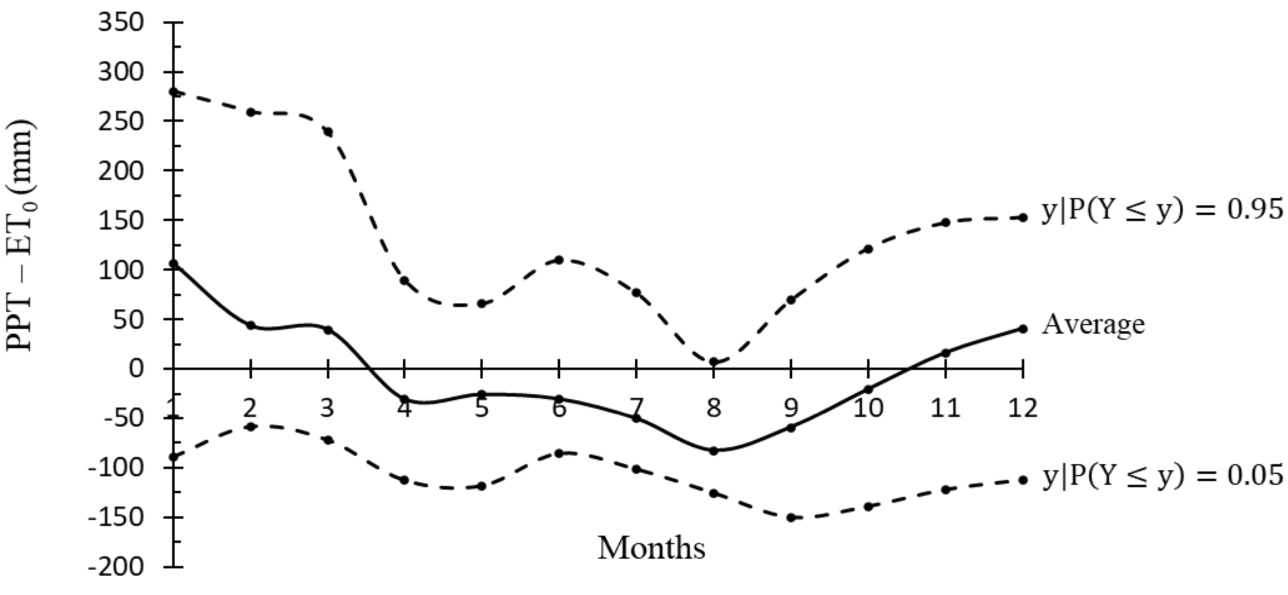


Table 2 - Initial probabilities for rainfall lower than ET_0 (dry day, D) and rainfall ≥ ET_0 (wet day, W) and the respective transition probabilities (or conditionals) in Piracicaba, SP, Brazil

Month	Period	ET0 (mm)	Rainfall (mm)	P(D)	P(W)	P(D/D)	P(W/D)	P(W/W)	P(D/W)
July	1–10	26.2	10.7	0.907	0.093	0.938	0.062	0.440	0.560
	11–20	28.0	12.3	0.913	0.087	0.948	0.052	0.375	0.625
	21–30	31.8	12.0	0.906	0.094	0.937	0.063	0.414	0.586
August	1–10	32.6	9.4	0.920	0.080	0.953	0.047	0.440	0.560
	11–20	36.2	7.5	0.930	0.070	0.961	0.039	0.556	0.444
	21–30	42.4	11.6	0.912	0.088	0.934	0.066	0.321	0.679
September	1–10	40.5	20.8	0.857	0.143	0.902	0.098	0.409	0.591
	11–20	42.2	21.7	0.857	0.143	0.885	0.115	0.325	0.675
	21–30	41.4	22.2	0.827	0.173	0.875	0.126	0.396	0.604

The analysis of the second decennial of August showed that if a dry day occurs at the beginning of this period, the probability of the following day also being dry [P(D/D)] is 0.961, against 0.039 [P(W/D)] of the day being wet. In this period, the probability of a given sequence of consecutive dry days can be obtained using an initial probability P(D) of 0.930 and the transition (or conditional) probability P(D/D) of 0.961. The probability of the first day of the period being dry is the initial probability P(D). The probability of each of the subsequent days in the period being dry is given by the transition probability P(D/D). Thus, the probability of a sequence of five dry days starting on any day during the second decennial of August is given by the product $P(D,D,D,D,D) = 0.930 \times 0.961^4 = 0.793$, indicating that the chances are nearly four to one for five consecutive dry days to occur during the first ten days of August. Thus, the chances of occurrence of six and seven consecutive dry days are 0.762 and 0.733, respectively.

ET_0 and rainfall estimates in the two fortnights of August, in the three decennials, and in the six quinquennials were analyzed using the K–S test to verify if they could be represented by the probability distributions analyzed at a significance level of 0.05. Rainfall data and ET_0 fitted best to the mixed gamma and log-normal distributions, respectively. Table 3 shows rainfall values and the parameters of the mixed gamma distribution for August according to fortnightly, decennial, and quinquennial periods obtained from the analyzed historical series. It also shows the mean values and the standard deviation over the period, and the probability of the mean rainfall being equaled or exceeded.

The wide range of variation in data, with standard deviations of more than the mean in all periods, indicates that data are widely dispersed, and the study of probability distributions is necessary. The mean value for August (28.5 mm) has a probability of 36% of being equaled or exceeded (return period of 2.8 years). The mean values in fortnights 1 and 2 are 10.7 and 17.8 mm with probabilities of being equaled or exceeded at 27 and 28%, respectively. The mean values in decennials 1, 2, and 3 are 9.4, 7.5, and 11.6 mm with probabilities of being equaled or exceeded at 23, 26, and 35%, respectively, with second decennials having the lowest rainfall. The probability of mean rainfall values is < 30% in the quinquennials, with the lowest rainfall in the third (1.4 mm).

The probable rainfall shown in Table 3 relates to the return period. Thus, the probabilities of 75 and 80% are associated with return periods of 1.3 and 1.25 years, respectively. Considering the probable rainfall of 5.1 mm at a probability of 75% in August, this value is expected to be equaled or exceeded once every 1.3 years, on average. In August, the rainfall is expected to be > 5.1 mm in three out of four years.

The 0.75 probability level is recommended for analysis of probable rainfall for irrigation design purposes (ANDRÉ; ANUNCIAÇÃO, 2017). At this probability level, the probable rainfall at fortnightly, decennial, and quinquennial periods is zero. In practice, the monthly mean value is usually used, but this is not a recommendable criterion, because there is a significant difference between the probable rainfall at a 0.75 probability level and the mean rainfall, as shown in Table 3. All the analyzed periods show a significant difference between the mean and the probable rainfall at a probability of 0.75. The means occur with low probabilities of being equaled or exceeded (between 0.18 and 0.39) and may underestimate the NID when used as a design criterion. The difference between the probable rainfall at a probability of 75% and the mean rainfall can be 1.24 mm day^{-1} in the fourth quinquennial.

Table 3 - Probable rainfall according to mixed gamma distribution, distribution parameters, and rainfall mean, and standard deviation for August based on fortnightly, decennial, and quinquennial periods in Piracicaba, SP, Brazil, with critical values at 0.05 significance ($D_{critical}$) using the Kolmogorov–Smirnov statistic and maximum calculated values (D_{sup})

P (-)	Probable rainfall, mm											
	Month	Fortnightly		Decennial			Quinquennial					
	August	1	2	1	2	3	1	2	3	4	5	6
0.05	92.1	52.5	66.0	51.6	38.5	43.8	32.7	24.3	7.7	37.5	24.7	32.8
0.10	69.9	34.0	47.4	29.8	24.0	31.3	17.3	14.0	5.0	20.9	16.1	23.2
0.15	56.8	23.7	39.7	20.3	17.5	26.1	9.7	8.1	3.3	12.3	10.6	18.5
0.20	47.6	17.1	33.1	12.8	12.0	22.5	4.5	4.2	2.1	6.6	6.4	13.9
0.25	40.3	12.3	27.6	7.2	7.6	18.7	2.0	1.6	1.1	3.0	3.9	9.8
0.30	34.6	8.6	22.9	4.3	4.9	14.9	0.3	0.0	0.1	0.6	1.1	7.1
0.35	29.6	5.8	18.7	2.4	3.1	11.6	0.0	0.0	0.0	0.0	0.0	4.8
0.40	25.3	3.7	15.2	0.1	1.0	9.2	0.0	0.0	0.0	0.0	0.0	1.9
0.45	21.5	2.1	12.1	0.0	0.7	7.4	0.0	0.0	0.0	0.0	0.0	1.2
0.50	18.2	0.9	9.1	0.0	0.0	5.3	0.0	0.0	0.0	0.0	0.0	0.0
0.55	15.1	0.2	6.0	0.0	0.0	2.6	0.0	0.0	0.0	0.0	0.0	0.0
0.60	12.3	0.1	2.7	0.0	0.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0
0.65	9.7	0.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.70	7.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.75	5.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.80	3.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.85	1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
r	1.28	0.54	1.34	0.50	0.68	1.39	0.43	0.77	1.23	0.63	0.97	1.28
$1/\lambda$	27.80	33.26	20.86	40.03	23.72	14.42	34.12	17.04	3.61	29.16	13.81	12.94
$\overline{X}$	28.5	10.7	17.8	9.4	7.5	11.6	5.4	3.9	1.4	6.2	4.5	7.1
Sx	29.1	22.9	21.5	22.9	14.5	16.1	16.0	10.7	2.9	13.9	10.2	12.8
Max	105.7	105.7	72.5	95.0	95.0	59.4	68.8	52.2	12.3	51.2	49.6	59.4
Min	0	0	0	0	0	0	0	0	0	0	0	0
$P(X \geq \overline{X})$	0.36	0.27	0.28	0.23	0.26	0.35	0.18	0.22	0.23	0.22	0.23	0.30
D_{sup}	0.074	0.130	0.090	0.078	0.076	0.049	0.107	0.059	0.042	0.069	0.073	0.062
$D_{critical}$	0.276	0.309	0.301	0.349	0.349	0.309	0.391	0.432	0.432	0.410	0.361	0.410

The analysis of rainfall probability, especially by month, shows that NID estimates for system design should not be based on a minimum rainfall value (for example, 90%) because it would result in an oversized design in most years. On the other hand, it should neither be based on the mean or maximum rainfall (for example, at 5%), as this would lead to an underestimation of NID. Values between 70–80% are recommended by André and Anunciação (2017) and Souza *et al*. (2019). Therefore, the probability level of 75% is the most appropriate (BERNARDO *et al*., 2019).

Research shows that rainfall time series data fit well to gamma distribution in different regions for monthly or shorter analysis periods (AMBURN; LANG; BUONAIUTO, 2015; ANDRÉ; ANUNCIAÇÃO, 2017; JERSZURKI; SOUZA; EVANGELISTA, 2015; SAMPAIO *et al*., 2007). ET_0 fits well to different probability distribution models such as the beta, normal, and log-normal (DENSKI; BACK, 2015; SILVA *et al*., 2014). Silva *et al*. (1998) reported that ET_0 fits well to the normal, log-normal, and beta distributions in periods of 30 or less days in Cruz das Almas, BA, Brazil. In Petrolina, PE, Brazil, Silva *et al.* (2015) found better ET_0 fit to normal distributions in periods of 30 or less days. Souza *et al.* (2019) reported better decennial ET_0 fit to the normal distribution in Pinhais, PR, Brazil.

Table 4 shows a sample of estimated ET_0 values for the 19 probability levels. For the same periods, ET_0 data fitted the log-normal distribution better than the other probabilistic models studied. Mean ET_0 values occur in the periods with probabilities close to 50%, with a probability of 53% of not being exceeded. At the probability level of 75% of no-exceedance, the ET_0 is 61.1 mm (4.07 mm day^{-1}) in the first fortnight of August. The probability of exceedance is 25%, with a return rate of four years.

In the fortnight of August, the ET_0 value will be ≤ 61.1 mm in three out of four years, on average. On the other hand, the return rate will be four years, i.e., the ET_0 of 61.1 mm is expected to be equaled or exceeded once every four years, on average. ET_0 values remain close to each other in the first and second fortnights (4.07 and 4.06 mm day^{-1}) at a probability of 75%, decreasing in the decennials from 3.91 to 4.13 mm day^{-1}, and increasing in the quinquennials from 3.88 to 4.25 mm day^{-1}, indicating that ET_0 is higher at the end of the month. The beginning of August corresponds to the mid-winter season in Piracicaba, SP, Brazil, with low rainfall. Temperature and solar radiation are also mild, resulting in relatively lower ET_0 than in later periods.

Table 4 - Maximum ET_0 expected using log-normal distribution and ET_0 mean and standard deviation for August by fortnightly, decennial, and quinquennial frequencies in Piracicaba, SP, Brazil, with critical values at a significance of 0.05 ($D_{critical}$) using the Kolmogorov–Smirnov statistic and maximum calculated values (D_{max})

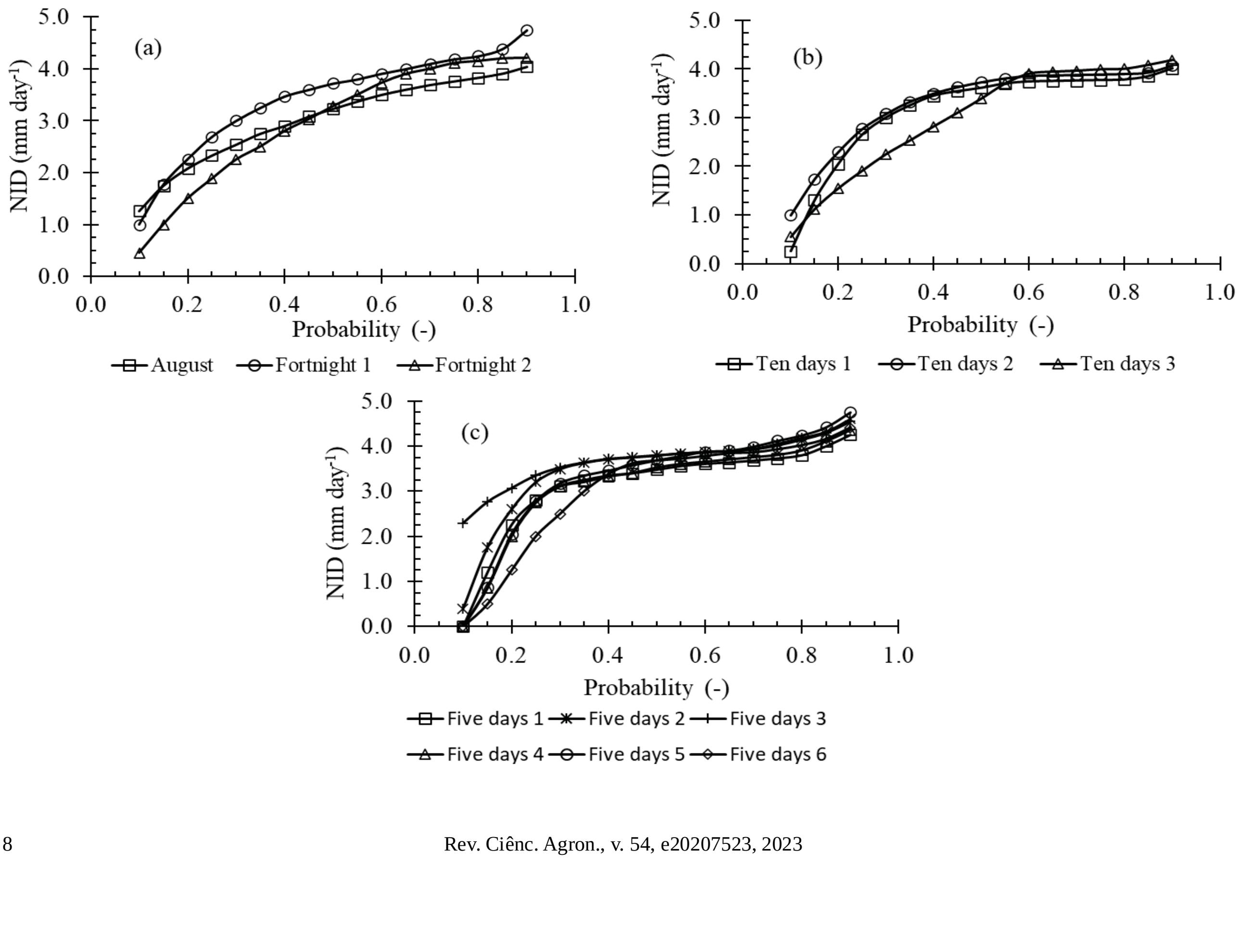
P (-)	Reference evapotranspiration (ET_0), mm											
	Month	Fortnightly		Decennial			Quinquennial					
	August	1	2	1	2	3	1	2	3	4	5	6
0.05	107.0	46.6	53.0	30.5	32.2	36.1	13.9	15.2	16.2	14.1	14.0	19.0
0.10	109.2	48.8	55.1	32.4	33.9	37.8	14.9	16.2	16.9	15.7	15.2	20.3
0.15	110.8	50.6	56.4	33.4	34.8	38.8	15.5	16.7	17.4	16.4	15.9	20.9
0.20	112.1	51.7	57.2	33.8	35.3	39.5	15.9	17.1	17.7	16.7	16.3	21.4
0.25	113.1	52.7	57.9	34.2	35.7	40.0	16.2	17.3	17.9	16.9	16.6	21.7
0.30	114.0	53.5	58.5	34.6	36.0	40.5	16.4	17.7	18.1	17.2	17.0	22.0
0.35	114.8	54.3	59.2	35.0	36.4	41.1	16.7	17.8	18.4	17.5	17.5	22.4
0.40	115.7	55.1	59.9	35.6	36.9	41.6	17.0	18.2	18.6	17.9	18.0	22.8
0.45	116.5	55.9	60.7	36.1	37.4	42.2	17.4	18.5	18.8	18.3	18.5	23.2
0.50	117.4	56.8	61.4	36.7	37.9	42.7	17.7	18.8	19.1	18.7	19.0	23.6
0.55	118.3	57.7	62.1	37.2	38.3	43.3	18.1	19.1	19.3	19.0	19.4	24.0
0.60	119.2	58.5	62.7	37.7	38.8	43.8	18.4	19.5	19.6	19.3	19.8	24.3
0.65	120.1	59.3	63.4	38.1	39.2	44.3	18.7	19.8	19.8	19.6	20.2	24.7
0.70	121.0	60.1	64.1	38.6	39.6	44.8	19.1	20.1	20.0	20.0	20.6	25.1
0.75	122.0	61.1	64.9	39.1	40.1	45.4	19.4	20.5	20.3	20.5	21.2	25.5
0.80	123.1	62.7	65.9	39.9	40.7	46.1	19.9	21.0	20.7	21.0	21.9	26.1
0.85	124.5	65.3	67.1	40.9	41.4	47.1	20.5	21.6	21.0	21.3	22.7	26.8
0.90	126.6	70.0	68.5	42.1	42.4	48.3	21.3	22.3	21.5	21.6	23.4	27.6
0.95	128.9	78.1	70.1	43.4	43.7	50.0	22.4	22.9	22.3	21.8	23.9	28.5
$\overline{X}$	118.7	57.2	61.5	37.1	38.3	43.3	17.9	18.9	19.2	18.8	19.1	23.8
Sx	6.6	7.5	5.1	2.2	3.3	4.0	2.4	2.3	1.8	2.2	3.0	2.8
Max	129.8	78.1	69.6	26.2	44.0	50.4	22.6	22.4	22.9	21.9	23.4	28.3
Min	106.1	46.6	49.7	15.8	29.4	33.4	11.9	13.3	15.3	10.6	11.4	16.5
$P(X \geq \overline{X})$	0.56	0.53	0.50	0.54	0.55	0.55	0.52	0.51	0.51	0.51	0.51	0.52
D_{sup}	0.113	0.196	0.112	0.095	0.061	0.077	0.078	0.177	0.071	0.180	0.160	0.087
$D_{critical}$	0.248	0.248	0.248	0.248	0.248	0.248	0.248	0.248	0.248	0.248	0.248	0.248

System sizing is based on the potential crop evapotranspiration (ET_{pc}), which is obtained for any given K_c value by multiplying the ET_0 values (Table 4) at a given probability level by the specified K_c. The recommended criteria for selecting the probability level should be based on an economic analysis, considering losses associated with reduced production quantity and quality due to water deficit and increased system costs to meet higher probability levels. High levels are usually selected for crops with higher economic value and more sensitive to water deficit (SAAD *et al*., 2002; SILVA *et al*., 2015). In supplementary irrigation, the economic viability of irrigation designs hardly justifies a probability level > 80%. In irrigation practice, the usual values adopted range from 50–75%, depending on economic implications (SOUZA *et al*., 2019).

Figure 2 shows maximum NID curves obtained at different ET_0 and rainfall probability levels for K_c = 1, i.e., with ET_{pc} equal to ET_0. In August, the maximum irrigation requirement at a probability of 75% was estimated at 3.77 mm day^{-1} (116.9 mm). Souza *et al.* (2019) highlighted that for the design of an irrigation system, the NID should be determined for monthly or shorter periods during the period of maximum demand for crop irrigation. The length of the period of analysis is vital. Determining the maximum demand in a very short period, of one or two days, for example, usually results in a high irrigation requirement (BERNARDO *et al*., 2019), oversizing the design. On the other hand, considering a longer period, for example monthly, usually results in a low irrigation requirement, and the irrigation design may be undersized.

Considering the case in which the period of the maximum crop demand occurs in August, and assuming a K_c of 1 at this stage, the design can be sized based on 15-, ten, or five-day demands. At a probability level of 75%, the NID values of the first and second fortnights are 4.07 (61.1) and 4.06 mm day^{-1} (64.9 mm), respectively. The NID values of decennials 1, 2, and 3 are 3.91, 4.01, and 4.13 mm day^{-1}, respectively. The NID values of quinquennials 1 to 6 are is 3.88, 4.10, 4.06, 4.10, 4.24, and 4.25 mm day^{-1}, respectively. The NID curves (Figure 2) for probability values between 70 and 80% show that the monthly series presents values slightly below the other series, indicating less accuracy with respect to estimates of daily mean from monthly values. The fortnightly, decennial, and quinquennial series, on the other hand, show closer mean values, indicating that the fortnightly period is adequate for an approximate estimation of NID.

Figure 2 - Maximum NID as a function of probability levels for K_c = 1 in (a) two fortnights, (b) three decennials, and (c) six quinquennials of August



The mean daily irrigation requirements differ slightly within each period in the two fortnights, three decennials, and six quinquennials. The lowest NID was obtained considering the monthly daily mean (3.77 mm day^{-1}). The daily mean obtained for August was 7.4% lower than the fortnightly mean, 6.2% lower than the decennial mean, and 8.3% lower than the quinquennial mean. Assuming that quinquennial estimates would be reasonable for the estimation of NID, the mean quinquennial value could be considered for irrigation design, i.e., 4.1 mm day^{-1}, which would be compatible with fortnightly and decennial means.

The use of mean rainfall and ET_{pc} (K_c = 1) values, which are commonly used for the estimation of NID, results in NID values of 90.2 mm for August, 46.5 mm for fortnight 1, and 43.7 mm for fortnight 2. These values are related to probabilities of 0.39, 0.33, and 0.39, respectively, which are lower than those obtained at a probability of 75%, which may lead to under sizing of the irrigation system.

CONCLUSIONS

Data analysis pertaining to the period 1990–2019 in Piracicaba, SP, Brazil, shows that:

(1) The NID should be estimated from ET_0 and rainfall data analyzed using log-normal and mixed gamma distribution, respectively;

(2) The sizing of irrigation design requires the analysis of historical series for August, using accumulated values in periods of 15, ten, or five days;

(3) For the case of August with K_c = 1, an irrigation design should be sized to meet a mean NID value of 4.1 mm day^{-1}, which is estimated as the mean value over periods of five, ten, or 15 days, at a probability of exceedance of 0.75 for rainfall and 0.25 for ET_0;

(4) The use of monthly mean rainfall and ET_0 values for the sizing of an irrigation system underestimates the NID by 26.6%, on average, compared to probable rainfall values of 0.75 and ET_0 values of 0.25 in five-, ten- or 15-day periods.

REFERENCES

ALLEN, R. G. *et al*. The ASCE Standardized Reference Evapotranspiration Equation. **ASCE-EWRI Task Committee Report**, 59 p., Jan. 2005.

AMBURN, S. A.; LANG, A. S. I. D.; BUONAIUTO, M. A. Precipitation forecasting with gamma distribution models for gridded precipitation events in eastern Oklahoma and northwestern Arkansas. **Weather and Forecasting**, v. 30, p. 349-367, 2015.

ANDRÉ, R. G. B.; ANUNCIAÇÃO, Y. M. T. A precipitação pluvial provável em Jaboticabal, São Paulo. **Agrometeoros**, v. 25, n. 2, p. 347-359, 2017.

ASSIS, F. N.; ARRUDA, H. V.; PEREIRA, A. R. **Aplicações de estatísticas à climatologia**: teoria e prática. Pelotas: Editora Universitária de Pelotas, 1996. 161 p.

BERNARDO, S. *et al.* **Manual de Irrigação**. 9. ed. Viçosa, MG: Editora UFV, 2019. 545 p.

BONAMENTE, M. **Statistics and analysis of scientific data**. New York: Springer Science, 2017. 348 p.

BRADLEY, A. P. ROC curve equivalence using the Kolmogorov-Smirnov test. **Pattern Recognition Letters**, v. 34, p. 470-475, 2013.

DENSKI, A. P. N.; BACK, A. J. Aderência de distribuições de probabilidade aos dados de evapotranspiração de referência decendial. **Revista Brasileira de Climatologia**, v. 17, ano 11, p. 27-40, 2015.

FERNANDES, R. D. M. *et al.* Probability distribution functions applied in the water requirement estimates in irrigation designs. **Revista Caatinga**, v. 32, n. 1, p. 189-199, 2019.

JERSZURKI, D.; SOUZA, J. L. M.; EVANGELISTA, A. W. P. Distribuição de probabilidade e movimento temporal da precipitação na região de Telêmaco Borba, Paraná. **Revista Brasileira de Ciências Agrárias**, v. 10, n. 1, p. 110-116, 2015.

LEB. Departamento de Engenharia de Biossistemas. **Série de dados climatológicos do Campus Luiz de Queiroz - Piracicaba, SP**. 2020. Disponível em: http://www.leb.esalq.usp.br/leb/postocon.html. Acesso em: 20 maio 2020.

MESQUITA, W. O.; GRIEBELER, N. P.; CORRECHEL, V. Sequência de dias chuvosos: fator de risco à colheita em Goiás. **Engenharia Agrícola**, v. 33, n. 3, p. 463-474, 2013.

MINUZZI, R. B. Probabilidade de transição e condicional para chuva mensal e extremos diários em Santa Catarina. **Revista Ceres**, v. 63, n. 6, p. 775-781, 2016.

MONTGOMERY, D.; RUNGER, G. **Applied statistics and probability for engineers**. 6. ed. Hoboken: John Wiley & Sons. 2014. 811 p.

SAAD, J. C. C. *et al.* Estudo da distribuição da evapotranspiração de referência visando o dimensionamento de sistemas de irrigação. **Irriga**, v. 7, n. 1, p. 10-17, 2002.

SAMPAIO, S. C. *et al.* Estimativa e distribuição de precipitações decendiais para o estado do Paraná. **Irriga**, v. 12, n. 1, p. 38-53, 2007.

SILVA, A. O. *et al*. Evapotranspiração de referência para dimensionamento de sistemas de irrigação em Petrolina, PE. **Brazilian Journal of Biosystems Engineerin**g, v. 9, n. 1, p. 30-38, 2015.

SILVA, F. C. *et al.* Distribuição e frequência da evapotranspiração de referência de Cruz das Almas, BA. **Revista Brasileira de Engenharia Agrícola e Ambiental**, v. 2, n. 3, p. 284-286, 1998.

SILVA, M. G. *et al*. Probabilidade de ocorrência de valores estimados de evapotranspiração de referência no Sertão do Ceará. **Engenharia na Agricultura**, v. 22, n. 6, p. 562-574, 2014.

SOUZA, J. L. M. *et al*. Precipitação e evapotranspiração de referência prováveis para o município de Pinhais-PR. **Revista Brasileira de Agricultura Irrigada**, v. 13, n. 4, p. 3512-3523, 2019.

WALLER, P.; YITAYEW, M. **Irrigation and Drainage Engineering**. Switzerland: Springer, 2016. 742 p.