

Physical and geometric non-linear analysis using the finite difference method for one-dimensional consolidation problem

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Abstract

This article presents a numerical model based on the finite difference method for the physical and geometric non-linear analysis of a one-dimensional consolidation problem regarding a saturated, homogeneous and isotropic soil layer with low permeability and high compressibility. The problem is formulated by adopting the void ratio as the primary variable, considering a Lagrangian movement description. The physical non linearity is introduced on the formulation by the constitutive law defined as effective stress and permeability void ratio functions. Based on this numerical model, a computational system named AC-3.0 was developed, which has been verified and validated in terms of the temporal variation of the void ratio distribution throughout the soil layer, by comparing the numerical results with analytical and numerical solutions found in literature for some specific scenarios. Knowing the void ration distribution, it is possible to obtain secondary variables such as: superficial settlement, effective stress and excess of pore water pressure. The importance of the non-linear formulation is highlighted for the analysis of problems related to material presenting high compression and a very high initial void ratio.

keywords: non-linear geometric analysis, finite difference method, Lagrangian formulation, physical non-linearity, self-weight consolidation problem.

1. Introduction

Consolidation is a physical phenomenon that can occur in porous media and is related to the temporal dissipation of the excess of pore water pressure generated by an external force (body and/or surface loads), causing displacement, strain, and stress field variations throughout the soil layer. This problem was initially studied by Terzaghi, in 1923, who formulated the classical consolidation theory (Schiffman, 2001). Terzaghi adopted some simplified hypotheses, such as: the soil layer's material is homogeneous and saturated; the solid particles and the water are incompressible; the water flow and the deformation occur only in a vertical direction; the strains are small and time-independent; the Darcy Law is valid; and, the permeability coefficient and the volumetric compressibility modulus does not vary during the process.

Although these simplified hypoth-

eses are unreal for some scenarios, the Terzaghi's theory, by its mathematical relative simplicity, provides very good results for some practical engineering problems. Nevertheless, in some engineering practical situations, such as mining tailings deposits/dams and the dredging process, it is necessary to adopt a more precise theory involving physical and geometric non-linearity, since the tailings (ormud) generated by the mining plant and dredging activities are very soft and present a high initial void ratio distribution when placed in the field.

Bartholomeeusen *et al.* (2002) highlight the importance of using a non-linear constitutive law in order to analyze the consolidation problem related to the dredging and embankment of soft soil. Yao *et al.* (2002) highlight the importance of studying this phenomenon in the sense

of optimizing the storage capacity and the consolidation time reduction of the high amount of tailings generated by the mining and civil construction industries. In this context, as well highlighted by Abu-Hejleh *et al.* (1996), the computational models are valuable tools for the mining reclamation process, since they provide the storage capacity of the tailings disposal along with the estimated duration of the consolidation process (Almeida *et al.* 2005). Penna and Oliveira Filho (2012) highlight the importance of the slope stability of the tailings retaining structures during their constructive phase.

Herein, the governing differential equation and the finite difference equations of the non-linear, physical and geometric, one dimensional consolidation problem taking into account body force effects are presented. The

numerical model generated was implemented into a computational system named AC (Analysis of Consolidation) in its third version. The previous ver-

sion was developed by Nogueira (2002) and Ferreira and Nogueira (2004). This computational system is verified with the analytical solution provided by Gibson

et al. (1967) and Xie and Leo (2004); and, with other numerical solutions presented by Yao and Znidarcic (1997) and Batholomeeusen *et al.* (2002).

2. Non-linear consolidation problem's governing equation

The theory of the non-linear one dimensional consolidation problem presented by Gibson *et al.* (1967) considers the following hypotheses: the porous media is considered as homogeneous, saturated and normally consolidated; solid particles and water are incompressible; the deformation of the media is due to solid particle rearrangement followed by the flow of the water into

pores; the volume of the solid remains constant during the process; there is no restriction related to the magnitude of the deformation; the strain and the water flow occur in a vertical direction; the process is induced by the body and/or surface monotonic loads; the strain is time-independent; the permeability coefficient and the volumetric compressibility vary during the process with the

void ratio variation.

By considering the above hypotheses and adopting a Lagrangian movement description, one can obtain the following non-linear differential partial equation, which represents the conservation and balance of the porous media mass (Gibson *et al.*, 1967), in terms of the reduced coordinate (z) and the time (t):

$$\frac{-\partial}{\partial e} \left(\frac{k(\gamma_s - \gamma_w)}{\gamma_w(1+e)} \right) \frac{\partial e}{\partial z} - \frac{\partial}{\partial z} \left(\frac{k}{\gamma_w(1+e)} \frac{d\sigma'}{de} \right) \frac{\partial e}{\partial z} - \left(\frac{k}{\gamma_w(1+e)} \frac{d\sigma'}{de} \right) \frac{\partial^2 e}{\partial z^2} = \frac{\partial e}{\partial t} \quad (1)$$

where e is the void ratio; γ_s is the self-weight of solid particles; γ_w is the self-weight of water; k and σ' are, respec-

tively, the permeability coefficient and the effective stress which is defined as a void ratio function named of constitu-

tive laws. The reduced coordinate (z) is defined in terms of the Lagrangian coordinate (a) as:

$$z(a) = \int_0^a \frac{1}{1+e(a,0)} da \quad (2)$$

where (a,0) represents the initial void ratio distribution, which in relation to the solid

particle volume is between the fixed referential and a generic point. Thus, Equation

(1) can be rewritten in a compact form as follows:

$$g_3 \frac{\partial e}{\partial z} + g_2 \frac{\partial^2 e}{\partial z^2} = \frac{\partial e}{\partial t} \quad (3)$$

in which:

$$g_3 = g_1 + \frac{\partial g_2}{\partial z} \quad (4)$$

$$g_1 = - \left(\frac{\gamma_s - \gamma_w}{\gamma_w} \right) \frac{d}{de} \left(\frac{k}{(1+e)} \right) \quad (5)$$

$$g_2 = - \frac{k}{\gamma_w(1+e)} \frac{d\sigma'}{de} \quad (6)$$

are functions which depend on the constitutive laws and their derivatives related to the void ratio. The constitutive laws are

defined by field and laboratory observation (Silva and Azevedo, 1999). Many constitutive laws can be found in literature.

Based on the Hydraulic Consolidation Test (HCT), Liu and Znidarcic (1991) suggested the following constitutive laws:

$$K = Ce^D \quad (7a)$$

$$\sigma' = (e/A)^{1/B} - Z_L \quad (7b)$$

or

$$e(\sigma') = A(\sigma' + Z_L)^B \quad (7c)$$

for which A, B, C, D, and Z_L are constitutive parameters determined by curves

adjusted for the procedure. From Equation (7b) it is possible to obtain:

$$\frac{d\sigma'}{de} = \frac{1}{AB} \left(\frac{e}{A} \right)^{(B-1)/B} \quad (7d)$$

Xie and Leo (2004), on the other hand, suggested the following ones:

$$k = k_{00} \left(\frac{1+e}{1+e_{00}} \right)^2 \tag{8a}$$

$$\sigma' = q'_0 - \frac{1}{m_v} \ln \left(\frac{1+e}{1+e_{00}} \right) \tag{8b}$$

or
$$e = 1 - (1 + e_{00}) \exp [m_v (\sigma'_0 - \sigma')] \tag{8c}$$

where σ'_0 is the previous effective stress on the soil layer; m_v is the volumetric deformability modulus on confined compression

(made constant during the process); e_{00} is the initial void ration on the soil layer's surface; and, k_{00} is the permeability coef-

ficient related to the initial void ratio for the soil layer's surface. From Equation (8b), it is possible to obtain:

$$\frac{d\sigma'}{de} = - \frac{1}{m_v (1+e)} = \frac{1}{a_v} \tag{8d}$$

where a_v is the compressibility coefficient (made constant during the process).

Equation (3) can be written in a normalized form as suggested by Gibson

et al. (1981) by doing:

$$Z = z/H_r \tag{9}$$

$$T = (g_{200}/H_r^2)t \tag{10}$$

$$H_r = H_0/(1+e_{00}) \tag{11}$$

where Z and T are, respectively, the normalized height and time; z is the height of a generic point on the soil layer defined in terms of reduced co-ordinated according to the Equation

(2); H_r is the initial thickness of the soil layer in the reduced coordinate system; H_0 is the initial thickness of the soil layer; g_{200} is the function defined in Equation (6) corresponding to the

initial void ration on the soil layer's surface (e_{00}). Using the definition on Equations (9) to (11), one can obtain the following normalized governing equation:

$$N \frac{\partial e}{\partial Z} + M \frac{\partial^2 e}{\partial Z^2} = \frac{\partial e}{\partial T} \tag{12}$$

where

$$N = (g_3/g_{200})H_r \tag{13}$$

$$M = g_2/g_{200} \tag{14}$$

Equation (12) must obey the initial condition, $e(Z,0) = e(\sigma'(Z,0))$, where:

$$\sigma'(Z,0) = \sigma'_0 + (\gamma_s - \gamma_w)H_r(1-Z) \tag{15}$$

The second term of Equation (15) represents the effective surcharge of the soil layer

weight on the Z point. In addition, Equation (12) must obey the following boundary con-

dition on the surface or top of the soil layer ($Z=1$), considered as a drained boundary:

$$e(1,T) = e(\sigma'(1,T)) = e(\sigma'_0 + \Delta q) \tag{16}$$

in which Δq is the incremental load applied instantaneously on the surface and kept constant during the process.

For the base of the soil layer ($Z=0$), the condition depends on the drainage. If the base is considered as a drained

boundary, the problem is identified as double drained where:

$$e(0,T) = e(\sigma'(0,T)) = e(q'_0 + \Delta q + (\gamma_s - \gamma_w)H_r) \tag{17}$$

For a single drained condition, when the base is considered as an impermeable boundary, one can have

$$\frac{de}{dZ}(0,T) = (\gamma_w - \gamma_s)H_r \frac{1}{d\sigma'/de} \tag{18}$$

Once the void ratio distribution (primary variable) is obtained by solving Equation (12), one can obtain the

secondary variables such as: settlement, total and effective stress, and pore water pressure (total, initial and excess).

The settlement $S(T)$ can be obtained by executing:

$$S(T) = -H_r \int_0^1 [e(Z,0) - e(Z,T)] dZ \tag{19}$$

The total stress distribution $\sigma(Z,T)$ can be obtained with:

$$\sigma(Z,T) = \underbrace{\sigma'_0 + \Delta q + \gamma_w H_w}_{\sigma(Z,0)} - \gamma_s H_r Z - \gamma_w H_r \int_0^Z e(Z,T) dZ \tag{20}$$

where H_w is the water thickness above the soil layer surface. The effective stress can be obtained by adopting

one of the constitutive laws presented Equation (7a) or (8a). Once the total and effective stresses are known, the

pore water pressure (p) can be obtained according to the Terzaghi's principle, by using:

$$p(Z,T) = \sigma(Z,T) - \sigma'(Z,T) \tag{21}$$

The hydrostatic pore water pressure $p_{ss}(Z,T)$ is given by:

$$p_{ss}(Z,T) = \gamma_w \left[H(T) + H_w(T) - H_r Z - H_r \int_0^Z e(Z,T) dZ \right] \tag{22a}$$

where

$$H(T) = H_r - S(T) \tag{22b}$$

is the soil layer's height at the reduced coordinate. Finally, the excess of pore water pressures is obtained by doing:

$$\Delta p(Z,T) = p(Z,T) - p_{ss}(Z,T) \tag{23}$$

3. Finite difference equations of the non-linear consolidation problem

The non-linear second order differential partial equation which represents the non-linear consolidation problem (Equations 1, 3 and 12), is considered as a high complexity resolution differential partial equation. For some simplified situations, it is possible to obtain an analytical solution such as the solution presented by Xie and Leo (2004). However, some situations having complex geometry, loading and boundary conditions indicate the use of some approximation technique which transforms the differential equation into an algebraic equation system. By doing this, it

is possible to obtain at least an approximated solution for the complex problem. Many different methods can be defined depending on the adopted discretization technique.

The finite difference method (FDM), adopted herein, divides the domain problem into a finite number (n_{point}) of discrete points named nodal points (k point). The set of nodal points is the nodal point mesh. For each nodal point, the derivative parcels of the differential partial equation are changed by the variation rate of the primary variable.

As Equation (12) involves a

time derivative, the normalized time (T) is divided into finite increments (ΔT) and a time march is defined by the θ constant, which localizes the time function into the time interval from T to $T+\Delta T$ and defines the kind of algorithm: $\theta=0.0$ for the explicit algorithm, $\theta=0.5$ for the implicit algorithm (Crank-Nicholson), and $\theta=1.0$ for the purely implicit algorithm. Using a central difference approximation, it is possible to obtain the following approximation for the first and second order of the space derivative which appears in the governing equation:

$$\left(\frac{de}{dZ} \right)_{Z,T+\Delta T} \cong \theta \left(\frac{e_{Z+\Delta Z,T+\Delta T} - e_{Z-\Delta Z,T+\Delta T}}{2\Delta Z} \right) + (1-\theta) \left(\frac{e_{Z+\Delta Z,T} - e_{Z-\Delta Z,T}}{2\Delta Z} \right) \tag{24a}$$

$$\left(\frac{d^2e}{dZ^2} \right)_{Z,T+\Delta T} \cong \theta \left(\frac{e_{Z+\Delta Z,T+\Delta T} - 2e_{Z,T+\Delta T} + e_{Z-\Delta Z,T+\Delta T}}{\Delta Z^2} \right) + (1-\theta) \left(\frac{e_{Z+\Delta Z,T} - 2e_{Z,T} + e_{Z-\Delta Z,T}}{\Delta Z^2} \right) \tag{24b}$$

Applying the approximation described by Equations (24) into

Equation (12), one can obtain the following algebraic equation system

on a generic k nodal point within the problem domain:

$$B_1 e_{k+1,T+\Delta T} + B_2 e_{k,T+\Delta T} + B_3 e_{k-1,T+\Delta T} = A_1 e_{k+1,T} + A_2 e_{k,T} + A_3 e_{k-1,T} \quad (25)$$

where:

$$A_1 = (1-\theta) (NR_1 + MR_2) \quad (26a)$$

$$A_2 = [1 - 2M(1-\theta)R_2] \quad (26b)$$

$$A_3 = (1-\theta) (-NR_1 + MR_2) \quad (26c)$$

$$B_1 = \theta (-NR_1 - MR_2) \quad (26d)$$

$$B_2 = \theta (1 + 2M\theta R_2) \quad (26e)$$

$$B_3 = \theta (NR_1 - MR_2) \quad (26f)$$

and:

$$R_1 = \Delta T / 2\Delta Z \text{ and } R_2 = \Delta T / \Delta Z^2 \quad (27a,b)$$

For the point on the top of soil layer contour, the boundary condition equation (k=npoin; Z=1), considered as a drained (Equation 16) is given by:

$$B_{1 \text{ topo}} e_{\text{npoin}-1,j+1} + B_{2 \text{ topo}} e_{\text{npoin},j+1} = A_{1 \text{ topo}} e_{\text{npoin}-1,j} + A_{2 \text{ topo}} e_{\text{npoin},j} + CC_{\text{ topo}} \quad (28)$$

where:

$$A_{1 \text{ topo}} = A_{2 \text{ topo}} = B_{1 \text{ topo}} = 0.0 \quad (29a)$$

$$B_{2 \text{ topo}} = 1.0 \quad (29b)$$

$$CC_{\text{ topo}} = e_{\text{npoin}} = e(q'_0 + \Delta q) \quad (29c)$$

For the point on the base of soil layer (k=1; Z=0) the boundary condition equation is given by:

$$B_{1 \text{ base}} e_{1,j+1} + B_{2 \text{ base}} e_{2,j+1} = A_{1 \text{ base}} e_{1,j} + A_{2 \text{ base}} e_{2,j} + CC_{\text{ base}} \quad (30)$$

For the drained contour (Equation 17a):

$$A_{1 \text{ base}} = A_{2 \text{ base}} = B_{2 \text{ base}} = 0.0 \quad (31a)$$

$$B_{1 \text{ base}} = 1.0 \quad (31b)$$

$$CC_{\text{ base}} = e_1 = e(q'_0 + \Delta q + (\gamma_s - \gamma_w)H_r) \quad (31c)$$

For the impermeable contour (Equation 18):

$$A_{1 \text{ base}} = 1 - 2(1-\theta)MR_2 \quad (32a)$$

$$A_{2 \text{ base}} = 2(1-\theta)MR_2 \quad (32b)$$

$$B_{1 \text{ base}} = 1 + 2\theta MR_2 \quad (32c)$$

$$B_{2 \text{ base}} = -2\theta MR_2 \quad (32d)$$

$$CC_{\text{ base}} = (NR_1 - MR_2) [2\Delta Z (\gamma_w - \gamma_s)H_r][1/(d\sigma/de)] \quad (32e)$$

Equations (25), (28) and (30) can be arranged into a compact form by making:

$$BE^1 = AE^0 + C \quad (33)$$

where:

$$\mathbf{E}^0 = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_k \\ \vdots \\ e_{npoin-1} \\ e_{npoin} \end{bmatrix}_0, \quad \mathbf{E}^1 = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_k \\ \vdots \\ e_{npoin-1} \\ e_{npoin} \end{bmatrix}_1 \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} CC_{base} \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ CC_{topo} \end{bmatrix} \quad (36)$$

$$\mathbf{A} = \begin{bmatrix} A_{1base} & A_{2base} & 0 & 0 & 0 & 0 & 0 \\ A_1 & A_2 & A_3 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & A_1 & A_2 & A_3 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & A_1 & A_2 & A_3 \\ 0 & 0 & 0 & 0 & 0 & A_{1topo} & A_{2topo} \end{bmatrix} \quad (37)$$

$$\mathbf{B} = \begin{bmatrix} B_{1base} & B_{2base} & 0 & 0 & 0 & 0 & 0 \\ B_1 & B_2 & B_3 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & B_1 & B_2 & B_3 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & B_1 & B_2 & B_3 \\ 0 & 0 & 0 & 0 & 0 & B_{1topo} & B_{2topo} \end{bmatrix} \quad (38)$$

Due to the non-linear nature of the constitutive laws, Equation (33) is also non-

linear. Then, an iterative process is necessary in order to obtain the equation solu-

tion. This study adopts the Picard Method (Pereira, 2017) as the solution strategy.

4. Verification examples

Three examples are now presented. The first example refers to the non-linear analyses of the consolidation problem due to the body and surface load as suggested by Xie and Leo (2004). The second example considers a specific scenario of a mining

industry related to a material with initial void ratio distribution (Yao and Znidarcic, 1997). The last one was proposed by Bartholomeeusen *et al.* (2002) and is related to a consolidation problem due to the self-weight force of a sediment produced by a

dredging process. The results presented by Xie and Leo (2004) are analytical, while the results presented by Yao and Znidarcic (1997) and Bartholomeeusen *et al.* (2002) are numerically obtained by different computational systems.

4.1 Example 1

This first example deals with the consolidation problem in the context of the physical and geometric non-linearity of a saturated, homogeneous, isotropic and soft soil layer. The initial thickness of the soil layer is 10m. The soil layer presents an initially uniform distribution of effective stress

(q'_0) of 10kPa and is subject to a uniform surface load of 100kPa instantaneously applied and kept constant throughout the time limit. This soil layer is subject to a doubled drainage boundary condition. The following material parameters were considered: m_v of 0.004kPa^{-1} ; e_{00} of 3.0; k_{00} of 10^{-9}m/s ; γ_s of 27.5 kN/m^3

and γ_w of 10kN/m^3 . The initial thickness in the reduced coordinate is equal to 2.5m according to Equation (11), while Equations (6), (9) and (10) respectively provide: $g_z(e_{00})=1.5625 \times 10^{-9}\text{m}^2/\text{s}$, $Z=0.4z$ and $T=2.5 \times 10^{-10}t$. According to the constitutive law indicated by Equations (8c), one can obtain:

$$e = 4\exp[0.004(10 - \sigma')] - 1.0 \quad (39)$$

The initial condition in terms of the void ratio is obtained by Equation (39)

according to the following distribution of the effective stress along the soil layer:

$$\sigma' (Z,0) = 10 + 43.75 (1-Z) \quad (40)$$

According to this equation, the initial void ratio on the base and top of the soil layer is respectively given by 2.36 and 3.0. According to the Equations (16) and (17) the effective stress at the top (Z=1) and base (Z=0) of soil layer is, respectively, 110kPa and 153kPa, leading

to a boundary condition in terms of the void ratio of 1.68 and 1.25, respectively.

Figure 1 presents the numerical results obtained with the AC-3.0 system by adopting a finite difference mesh with 21 nodal points and the implicit algorithm ($\theta=1.0$) with the real time

increment (Δt) of 9.855×10^6 s. The tolerance for Picard's algorithm was 10^{-5} . Figure 1 presents the time variation of the void ratio distribution, while Figure 1b presents the time variation of the average degree of consolidation defined as:

$$U_m (T) = \left(1 - \frac{\Delta p_m (T)}{\Delta p_m (T = 0^+)} \right) \times 100\% \quad (41)$$

where:

$$\Delta p_m (T) = \sum_{k=1}^{npoin} \Delta p (Z_k, T) \quad (42a)$$

$$\Delta p_m (T = 0^+) = \sum_{k=1}^{npoin} \Delta p (Z_k, 0^+) \quad (42b)$$

for which, Δp is the excess of pore water pressure given by Equation (23); and $T=0+$

represents the time instant immediately after the instantaneous surface load application.

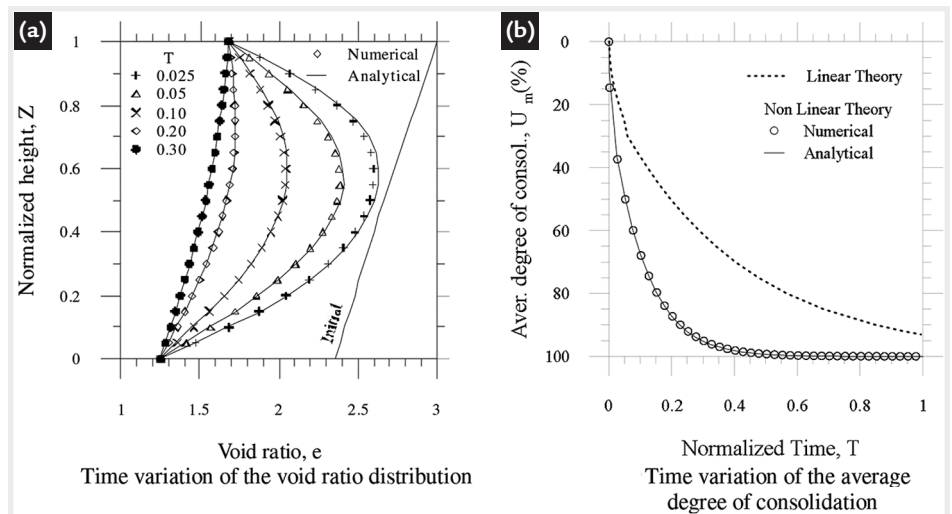


Figure 1 Example 1. Numerical results versus analytical results.

The numerical results are in a good agreement with the analytical solution presented by Xie and Leo (2004). This verifies the implementation of the AC-3.0 computational program. At the end of the process, the void ratio distribution tends

to a linear variation, such as the initial distribution. As can be seen in Figure 1b, more than 70% of consolidation occurs in a normalized time around 0.1, while according to the linear theory (Schiffman, 2001), this same average degree of

consolidation is observed at a normalized time of 0.4. The velocity of the consolidation process is higher when analyzed in the context of the non-linear theory than when analyzed in the context of the linear theory.

4.2 Example 2

The second example presents the consolidation analysis of a saturated, homogeneous, isotropic and soft soil layer with high initial void ratio uniform distribution subject to a self-weightload. The initial thickness of the soil layer is 17.85m. The uniform distribution of the initial void ratio of 32.42 is considered as the initial condition. The base of the soil layer is considered impermeable, so

the problem is analyzed adopting a single drainage boundary condition. The constitutive law (Equation 7) proposed by Liu and Znidarcic (1991) is adopted and the following material parameters were considered: $A=13.49$; $B=-0.319$; $Z_L=0.064$ kPa; $C=3.84 \times 10^{-12}$ m/s; and, $D=3.5$. Also, considered were: γ_s is equal to 26.58kN/m³; and γ_w is equal to 9.81kN/m³. This scenario was analyzed

numerically by the software Condes0 (Yao and Znidarcic, 1997).

The initial thickness for the reduced coordinate is equal to 0.534m according to Equation (11), while Equations (6), (9) and (10) respectively provide: $g_2(e_{00})=1.41 \times 10^{-11}$ m²/s, $Z=1.87z$ and $T=4.94 \times 10^{-11}t$. The boundary condition on top of the soil layer is 32.42 as the soil layer is only subject to a self-

weight load. However, the boundary condition on the base of the soil layer

$$\frac{d\sigma'}{de} = -0.232 \left(\frac{e}{13.49} \right)^{4.13} \quad (43)$$

is given, adopting Equations (7d), by:

Figure 2 presents the numerical results obtained in the AC-3.0 system by adopting a finite difference mesh with 61 nodal points and the implicit algorithm ($\theta=1.0$) with the real time increment (Δt) of 3.15×10^6 s. The tolerance of Picard's algorithm was 10^{-5} . Figure

2a presents the time variation of the void ratio distribution, while Figure 2b presents the variation in time of the soil layer thickness. The numerical results obtained by the AC-3.0 software are in good agreement with those of software Condes0. The final void ratio distribu-

tion is non-linear with the minimal value located at the base of the soil layer. No significant variation of the soil layer thickness is observed after 20 years. This information is very important in the field of tailings disposal in the mining process.

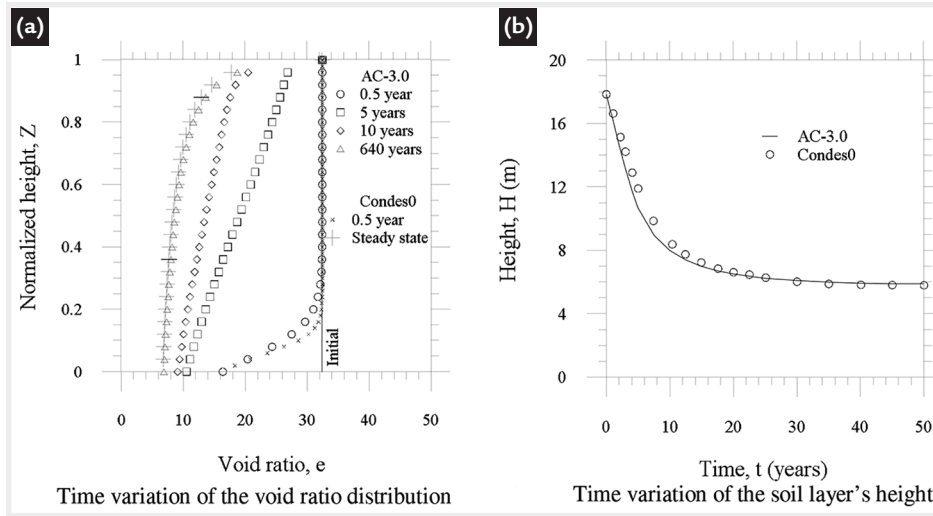


Figure 2 Example 2 – Numerical results: AC-3.0 versus Condes0.

4.3 Example 3

Bartholomeeusen *et al.* (2002) studied the consolidation problem for a sediment of the Schelde River (Antwerp, Belgium) provided by a dredging process. The consolidation problem was considered for only the self-weight load. Bartholomeeusen *et al.* (2002) adopted a different constitutive law in order to analyze its influence on the process. According to these authors, when compared to the large scale experimental data, the best numerical results were

obtained by adopting the constitutive law (Equation 7) presented by Liu and Znidarcic (1991).

The homogeneous, isotropic and saturated sediment was placed into a consolidation column with an initial thickness of 0.565m. The sediment layer is submitted to a single drainage from its top. The following material parameters were adopted: $\gamma_s=27.2\text{kN/m}^3$; $\gamma_w=10\text{kN/m}^3$; $A=1.69$, $B=-0.12$, $Z_L=0.046\text{kPa}$; $C=4.14 \times 10^{-9}\text{m/s}$; and, $D=6.59$. The initial thickness

of the reduced coordinate is equal to 0.16m according to Equation (11) by considering a uniform initial void ratio distribution of 2.45. Equations (6), (9) and (10) respectively provide: $g_2(e_{00})= 6.15 \times 10^{-9} \text{ m}^2/\text{s}$, $Z=6.25z$ and $T=2.40 \times 10^{-7}t$. The boundary condition on top of the soil layer is 2.45, as the soil layer is subject only to a self-weight load. However, the boundary condition on the base of the soil layer is given when adopting the Equations (7d), by:

$$\frac{d\sigma'}{de} = -4.93 \left(\frac{e}{1.69} \right)^{9.33} \quad (44)$$

Figure 3 presents the numerical results obtained by AC-3.0 system by adopting a finite difference mesh with 41 nodal points and the implicit algorithm ($\theta=1.0$) with the real time increment (Δt) of 8.6×10^3 s. The tolerance of Picard's algorithm was 10^{-5} . Figure 3a presents the time variation of the void ratio distribution, while Figure 3b presents the variation in time of the soil layer thickness. The numeri-

cal results obtained by the AC-3.0 software are in a good agreement with the numerical results presented by Bartholomeeusen *et al.* (2002) at the end of the consolidation process. No significant variation of the soil layer thickness is observed after 7 days. The final void ratio at the base of soil layer was found to be around 1.54, while on the top of this layer the void ratio was kept constant and equal to 2.45.

The time variation of the soil layer thickness obtained by AC-3.0 software (Figure 3b) agrees with the result presented by Bartholomeeusen *et al.* (2002) and the results provided by the software SVFLUX/SVSOLID GT (SoilVision Systems Ltda, 2017). In the context of a practical engineering problem, the numerical results are in a good agreement with the large scale experimental data.

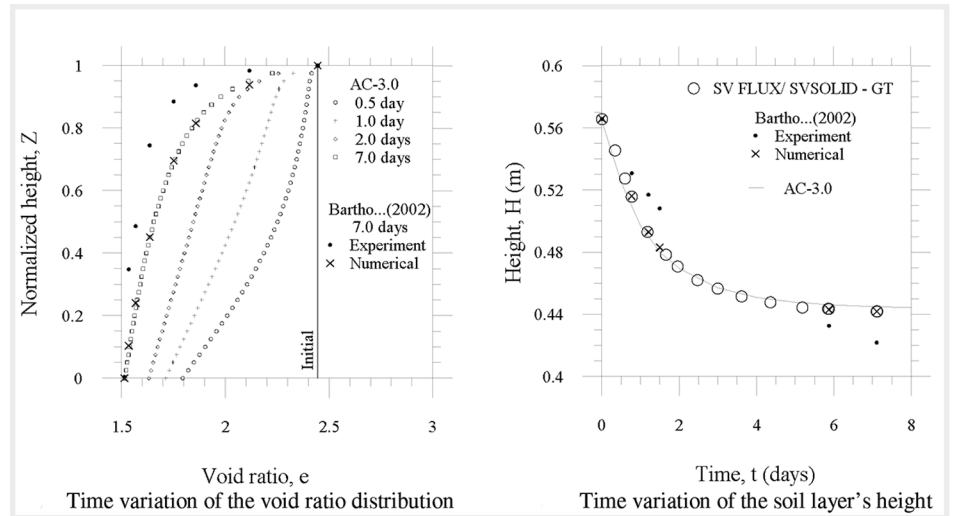


Figure 3
Example 3.
Numerical results comparison.

5. Conclusions

A soft soil layer that is highly compressible and with a high initial distribution void ratio subject to a consolidation process due to self-weight and/or surface load must be analyzed in the context of the physical and geometric nonlinearities. The physical nonlinear-

ity should take into account the constitutive law that plays a fundamental role in the numerical simulation. The numerical/computational model based on the FDM presented herein was verified by comparison with analytical and numerical solutions available in litera-

ture. Also, the model was validated by comparison with field data that is also available in literature. However, great effort must be made regarding constitutive law definition and field monitoring in order to improve the quality of the numerical simulation.

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