Optimum Design of truss structures considering nonlinear analysis and dynamic loading using metaheuristic algorithms

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Abstract

The structural optimization of trusses is a complex problem that can be affected by many different factors. In this research, the authors investigated the optimization of trusses performing a geometric nonlinear analysis under dynamic loading, using two different metaheuristic algorithms: the Genetic Algorithm (GA) and Particle Swarm Optimization (PSO). The objective function was to minimize the weight of the structure. A number of benchmark test problems for spatial trusses considering a geometric nonlinear analysis, and dynamic loading are analyzed to verify the performance of the optimization algorithms. The results showed that both algorithms were able to find efficient solutions to the optimization problem and suggest that the choice of the optimization algorithm can have a significant impact on the performance of the optimization process.

Keywords: trusses; nonlinear geometric analysis; dynamic loading; optimization; metaheuristics algorithms.

1. Introduction

Ezugwu *et al.* (2021) define optimization as a set of methods from operational research, artificial intelligence, computer science, and machine learning used to improve processes in almost every industry. In structural engineering, optimization techniques help engineers to develop more economical and efficient designs, and to automate the process.

The optimization procedure begins with a problem formulation, where the design variables, objective functions, and constraint functions are defined. The structure configuration is then automatically modified by changing the variables until the best solution is found.

Probabilistic methods, especially metaheuristics, are well-suited for problems with robust solutions. Metaheuristics is a type of optimization technique that can provide approximate best solutions for problems that are difficult to optimize using deterministic techniques (Lagaros *et al.*, 2022). Metaheuristics have been used to optimize a variety of engineering problems, including steel frame structures, dampers displacement, machining of aluminum hybrid composites, composite bridges cost and environmental impact, dam displacement monitoring, and beam damage prediction. In the field of truss structures, Bigham and Gholizadeh (2020) studied the topology optimization of domes and Jawad *et al.* (2021) researched the size and layout optimization of trusses.

Bioinspired metaheuristics are the most popular ones found in optimization literature. They are commonly based on the metaphor of ecological interaction between living organisms and their behavioral patterns. The genetic algorithm (GA) and particle swarm optimization (PSO) are the first ones to attract interest in widespread research on optimization methods (Ezugwu *et al.*, 2021).

The Genetic algorithm (GA), developed by Holland (1975), is inspired by Darwin's theory of natural selection. From an initial population, new solutions are generated through genetic operators (crossover, mutation) to replace the existing solution. In literature, several studies involving GA application to optimize some aspects of structures can be found, such as cost (Arpini *et al.*, 2022; Benzo *et al.*, 2022; Korus *et al.*, 2021; Netto *et al.*, 2023), environmental impact (Arpini *et al.*, 2022; Benzo *et al.*, 2022), design (Huang *et al.*, 2020), and topology (Khodzhaiev and Reuter, 2021).

Particle swarm optimization (PSO), developed by Kennedy and Eberhart, (1995), is a metaheuristic algorithm that imitates the behavior of birds (particles) in search of food (best solution). Each particle has a position and velocity in the search space, which is iteratively updated according to its personal best and global best positions (Mahapatra *et al.*, 2022). Similar to GAs, PSO has been used to optimize various aspects of structures, including design (Abo-Bakr *et al.*, 2021; Jarrahi *et al.*, 2020; Sharma and Ganguli, 2021), and environmental impact (Arpini and Alves, 2022).

The efficiency of the optimization process is significantly affected when a structure is subjected to dynamic loading. If a nonlinear analysis is also included, the process becomes even more difficult due to the extensive analytical process, which leads to a large computational cost. However, when a nonlinear analysis is performed, it is possible to improve the analytical simulation and provide a more realistic prediction of the structure's behavior (McGuire *et al*., 2000).

Large-scale truss structures, such as domes and lattice towers, are often very slender and can experience large displacements. The impact of these geometric changes on the structural response can be captured by considering geometric nonlinear analysis. Additionally, if the material behavior changes due to deformation, a material nonlinear analysis must also be considered (Shi *et al*., 2015).

The structural nonlinear analysis is well established in literature (Pacoste and Eriksson, 1995; Rodrigues *et al*., 2019, 2021; Yang and Kuo, 1994). Moreover, studies involving structural optimization of trusses considering geometric nonlinearity have already been presented over the last years (Kameshki and Saka, 2006; Koohestani, 2012; Li and Khandelwal, 2016; Ma *et al*., 2022; Madah and Amir, 2017, 2019; Mai *et al*., 2022; Tort *et al*., 2016). However, studies that include both geometric nonlinear analysis and dynamic loading, using a mathematical programming algorithm, have been little explored (Alfouneh and Tong, 2018; Fu *et al*., 2018; Martinelli and Alves, 2020a, 2020b; Martins *et al*., 2021)

Therefore, in this article, an optimization problem is formulated to analyze spatial truss structures considering geometric nonlinear analysis with dynamic loading. The optimization problem is solved using two algorithms: the Genetic Algorithm (GA) and the Particle Swarm Optimization (PSO). A comparative analysis with benchmark problems is performed to verify the efficiency of the algorithms.

2. Optimization problem formulation

The optimization problem aims to define the cross-sectional areas of the bars that minimize the final weight of the structure, by imposing constraints on nodal displacements and axial stresses. The design variables that set the optimization problem are the cross-sectional area of each bar of the structure, included in vector **A**:

$$\mathbf{A}=\{A_1,A_2,\ldots,A_{bn}\} \tag{1}$$

where bn is the total number of bars.

These variables were considered discrete, determine the geometry of the steel bar sections and may assume area values from a catalog of tubular structural profiles.

The objective function calculates the total weight of the structure by the sum of the weight of each truss bar:

$$Minimize\ f=\sum_{i=1}^{nb}\rho A_i L_i \tag{2}$$

where ρ is the specific mass of the steel, A_i is the cross-sectional area of bar i, and L_i is the length of bar i. The constraints imposed to the optimization problem are:

$$\frac{\sigma_{T_{max}}}{\sigma_{T_{lim}}}-1\le 0 \qquad \frac{\sigma_{C_{max}}}{\sigma_{C_{lim}}}-1\le 0 \qquad \frac{U_{max}}{U_{lim}}-1\le 0 \qquad \frac{N_{C,Sd}}{N_{C,\,crit}}-1\le 0 \tag{3}$$

where σ_{Tmax} and σ_{Cmax} are the maximum values of tensile and compressive axial stresses, respectively; σ_{Tlim} and σ_{Clin} are the allowable limit values of tension and compression, respectively; U_{max} is the maximum absolute value of nodal displacement suffered by the structure; U_{lim} is the allowable limit value for nodal displacements; $N_{C,crit}$ is the resistant compressive force defined according to ABNT NBR 16239:2013, and $N_{C,Sd}$ the applied compression design load . All values of stress and displacement are obtained considering a geometric nonlinear analysis with a dynamic load.

When a reference example is not used, the limit of the nodal displacement and the axial compression force are calculated based on values present in the Brazilian standard ABNT NBR 8800:2008. The allowable limit values of tension and compression are defined by:

$$\sigma_{C_{lim}}=\sigma_{T_{lim}}=\frac{f_y}{\gamma_{a1}} \tag{4}$$

where f_y is the yield stress and γ_{a1} is the resistance weighting coefficient equal to 1 in a serviceability combination.

It is highlighted that the constraints given by Eq. (3) require a nonlinear dynamic analysis for each iteration of the optimization process. This analysis was performed via Ansys software using the LINK180 element and the Newmark method for solving the dynamic problem described in Eq. (5).

$$[\mathbf{K}]\{\boldsymbol{d}\}+[\mathbf{C}]\{\dot{\boldsymbol{d}}\}+[\mathbf{M}]\,(\{\ddot{\boldsymbol{d}}\})=\boldsymbol{F}(\boldsymbol{t}) \tag{5}$$

Where [**K**] is the stiffness matrix of the structure, which depends on the cross-sectional area of the bar, [**M**] is the mass matrix of the structure, which also depends on the cross-sectional area of the bar, [**C**] is the damping matrix, which when considered is a linear combination of the former two. On the right-hand side of the equation, we have the time-dependent loading F(t). Figure 1 shows how the optimization process works.

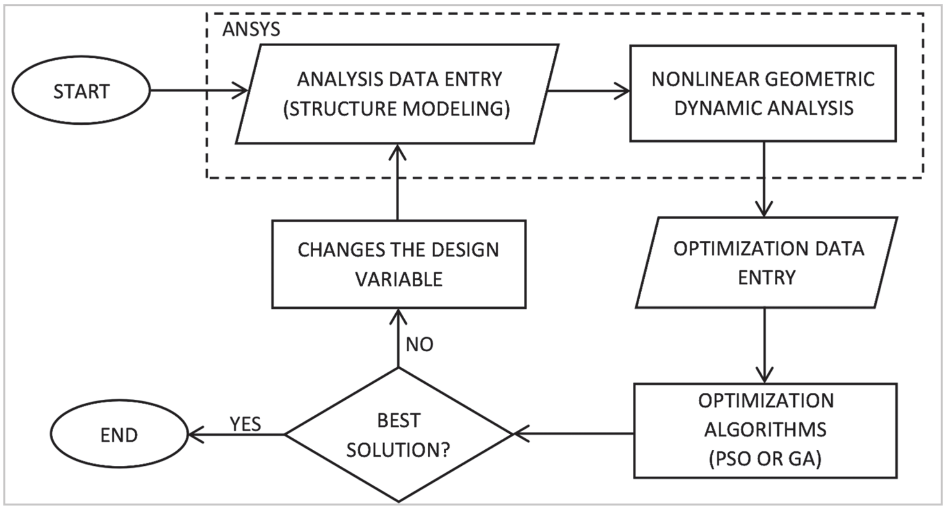


Figure 1 - Scheme of optimization process.

2.1 Implementation of optimization routines

The PSO algorithm was chosen for solving the optimization problem due to its ease of computational implementation and its robustness in searching for the optimal solution. Additionally, the GA from the Matlab Platform toolbox was used to validate the solutions proposed by both algorithms. For the implementation of PSO, the velocity function proposed by Shi and Eberhart (1998) was used based on the initial proposition of PSO by Kennedy and Eberhart (1995), where the inertia factor w was inserted as described in Eq. (6).

$$v_{id} = w \cdot v_{id}(t) + c_1 \cdot rand(p_{id}(t) - x_i(t)) + c_2 \cdot Rand(p_{gd}(t) - x_i(t)) \quad (6)$$

Where c_1 and c_2: acceleration constants, corresponding to the individual and global best given by; $\phi_1 = \phi_2 = 2.05$; $\phi = \phi_1 + \phi_2$; $c_1 = c_2 = \chi\ \phi_1$ rand and Rand: randomizing functions in the interval [0,1]; v_{id}: velocity of particle i; x_{id}: position of particle i; p_{id}: position of the i–th particle with the best value of the objective function obtained from previous iterations; p_{gd}: position of the i–th particle with the best value of the objective function among all particles in the population.

$$x = \frac{2\kappa}{\left| 2 - \phi - \sqrt{\phi^2 - 4\phi} \right|} \quad (7)$$

Where κ was adopted to be equal to 1, and the inertia coefficient is updated iteratively at each new generation, initially given by $w_{i-1} = c$; $w_{damp} = 0.99$; and $w_i = w_{i-1} \times w_{damp}$

The optimization routines were implemented using the following parameters for each algorithm. They were determined after a series of preliminary analyses to arrive at the population size for each algorithm, tolerance factors, as well as the number of generations. These parameters are:

- Particle Swarm Optimization (PSO)
  - °Adaptive Penalties method (APM) (Barbosa and Lemonge, 2008)
  - °Maximum number of iterations: 50
  - ° Population size: 50 individuals
  - ° Tolerance: 10^{-6}
- Genetic Algorithm (GA)
  - °Maximum number of generations: 100
  - °Crossover factor: 0.8
  - °Mutation factor: random [0,1]
  - °Population size: 200 individuals

3. Numerical results

This section explores two numerical examples of truss structures. For each one, a linear analysis and a geometric nonlinear analysis was performed, both with dynamic load without considering damping parameters. Additionally, an optimization of the same problem was performed 10 times for each proposed algorithm. Two types of loads were considered in the dynamic analyses, as shown in Figure 2. The solutions were obtained with discrete variables using the tube catalog from the Vallourec manufacturer.

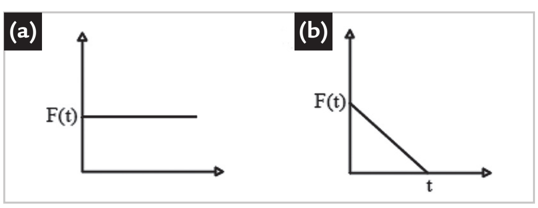


Figure 2 - Types of loads for dynamic analyzes: (a) constant; and (b) linear.

3.1 24-Bar dome truss

The first problem analyzed is a space truss with 24 members, as illustrated in Figures 3 and 4. The structure has a cross-sectional area $A = 6.45 \times 10^{-4}$ m², a specific mass $p = 2760$ kg/m³ and a Young's modulus $E = 68992$ MPa, for all members. It was subjected to a linear dynamic load with $F(t) = 8900$ N and $t = 0.01$ s, as seen in Figure 2(a), applied downwards at the middle point (node 1). The problem was previously analyzed by Martinelli and Alves, (2020a) by an updated Lagrangian formulation combined with the Newmark method, adopting Newton-Raphson type iterations with a time increment $\Delta t = 0.000156$ s. The results were compared to validate the geometric nonlinear dynamic analysis without damping parameters.

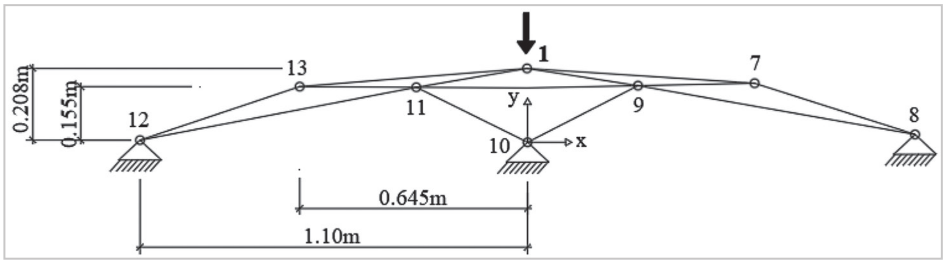
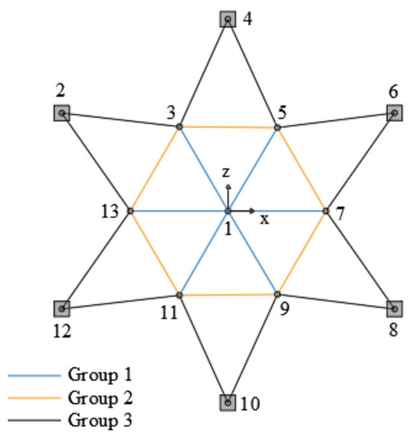


Figure 3 - 24-bar dome truss.



Element	Node 1	Node 2
1	1	3
2	1	5
3	1	7
4	1	9
5	1	11
6	1	13
7	3	5
8	5	7
9	7	9
10	9	11
11	11	13
12	13	3

Element	Node 1	Node 2
13	2	3
14	2	13
15	4	3
16	4	5
17	6	5
18	6	7
19	8	7
20	8	9
21	10	9
22	10	11
23	12	11
24	12	13

Figure 4 - 24-bar dome truss characteristics.

The results for the displacement of node 1 on the y-axis are presented in Figure 5 for both linear and nonlinear analysis. The results show a strong agreement with the reference. Besides that, the nonlinear analysis resulted in a displacement about 19% (or 1 mm) greater than the linear analysis. This indicates that the loading is causing an additional displacement in the structure that can only be observed through nonlinear analysis.

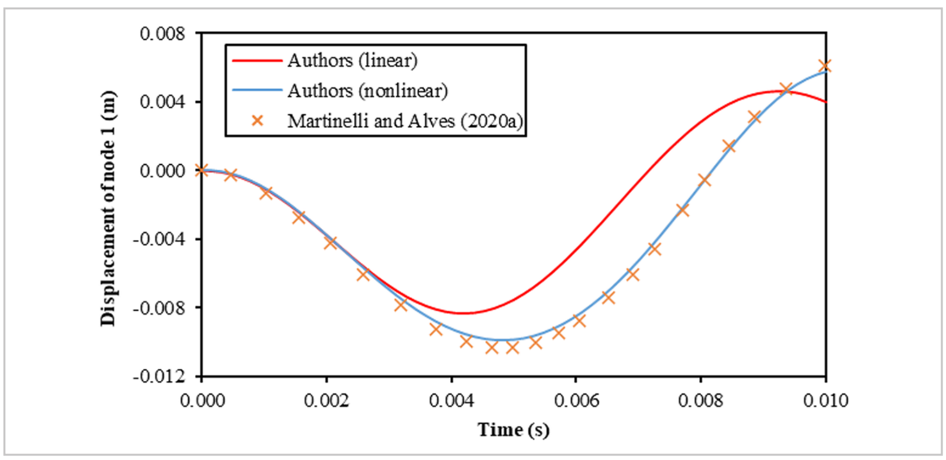


Figure 5 - Displacement of the middle point on the y-axis for 24-bar dome truss.

For the optimization, the structure was divided into three groups of bars, as shown in Figure 4, representing each design variable. The problem was also previously studied by Martinelli and Alves, (2020a), who used the Sequential Quadratic Programming (SQP) and Interior Point (IP) algorithms to minimize the truss weight. The properties of the structure were modified to $p = 7850$ kg/m³ and $E = 200$ GPa, while the dynamic load intensity increased, being $F(t) = 356$ N and $t = 0.1$ s in Figure 2(b). A time increment $\Delta t = 0.0001$ s was considered. The constraints imposed were 0.007 m for the nodal displacement limit, and

227 MPa for the tension limit and compression stresses. To maintain consistency with the proposal by Martinelli and Alves, (2020a), the constraint related to the critical load was not considered in this optimization analysis. Table 1 shows the results obtained, being RSD the relative standard deviation.

The differences related to the reference result are due to the use of continuous variables, while in this study discrete variables were considered. Despite this, the total weight in the results was very close, with the genetic algorithm (GA) presenting the best solution and a better RSD value for the nonlinear analyses. On the other hand, both algorithms found the same solution in the linear analysis. However, GA presented a better RSD value again, showing itself as a more robust algorithm in both cases.

Table 1 - 24-bar dome truss optimization results.

	Linear Analysis		Nonlinear Analysis		
	PSO	GA	PSO	GA	SQP/IP
A_1 (cm²)	171.00	171.00	195.00	191.00	193.58
A_2 (cm²)	171.00	171.00	171.00	195.00	177.42
A_3 (cm²)	15.20	15.20	25.70	15.50	22.31
U_{max} (m)	0.00624	0.00624	0.007	0.007	0.007
σ_{Tmax} (MPa)	88.19	88.19	94.36	106.97	91.27
σ_{Cmax} (MPa)	218.74	218.74	139.50	224.34	161.02
Best weight (kg)	1155.01	1155.01	1306.09	1291.17	1296.68
Mean weight (kg)	1194.40	1185.01	1358.02	1311.91	-
RSD (%)	5.43	4.59	6.86	1.81	-

The maximum value of displacement occurred in node 1 in the same direction of the load for both analyses. However, in the optimization with linear analysis, the displacement was smaller, as expected. The maximum stresses occurred in element 13, for both tension and compression, but in different instants. In Figure 6 is presented the displacement of node 1 and Figure 7 is observed the stresses over time of element 13 described on Figure 4.

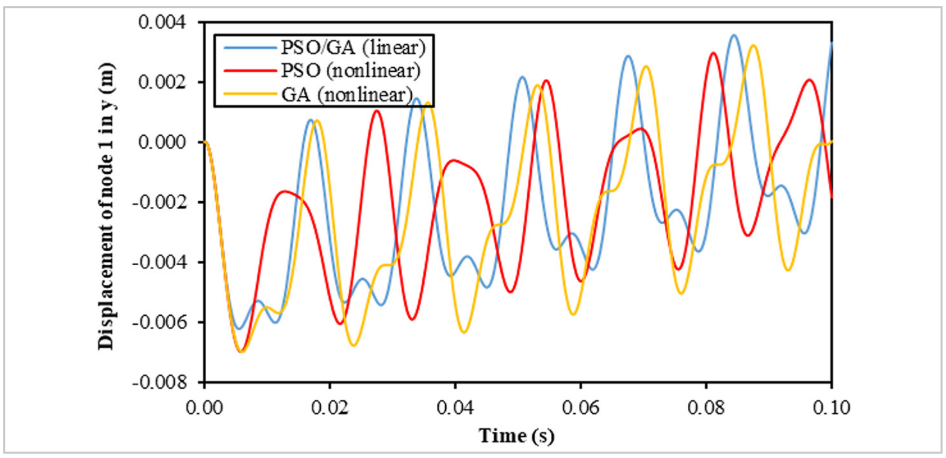


Figure 6 - Nodal displacement of 24-bar dome truss optimization.

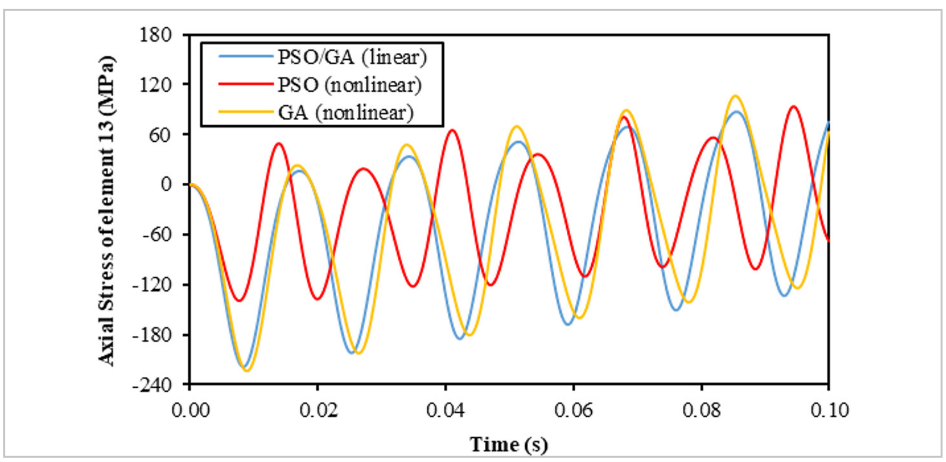


Figure 7 - Bar axial stress of 24-bar dome truss optimization.

As can be observed in the curves of Figures 6 and 7, there is a different behavior for PSO and GA due to the different solutions provided by them according to Table 1. The convergence curve's *total weight versus objective function evaluations* of each algorithm is presented in Figure 8. The structures optimized considering nonlinear analysis had a greater weight than the linear one. This can be justified, since the first one presented greater displacements, and consequently, larger cross-sections.

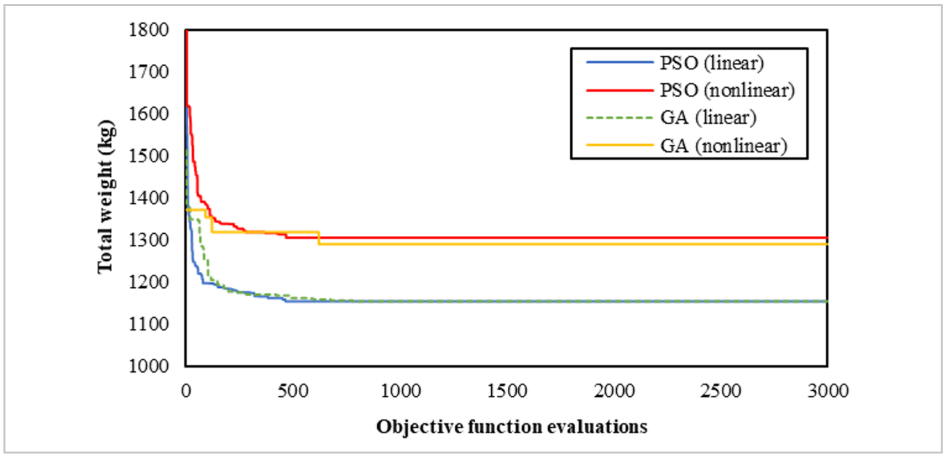


Figure 8 - Convergence curve of algorithms for 24-bar dome truss.

A relevant analysis about the constraint functions is seen in Figure 9, where a graph with their normalized values shows that the nodal displacement is the constraint responsible for commanding the optimization for all algorithms. In the case of the GA, the compressive stress was also very close to the limit in both analyses, indicating the good performance of the algorithm.

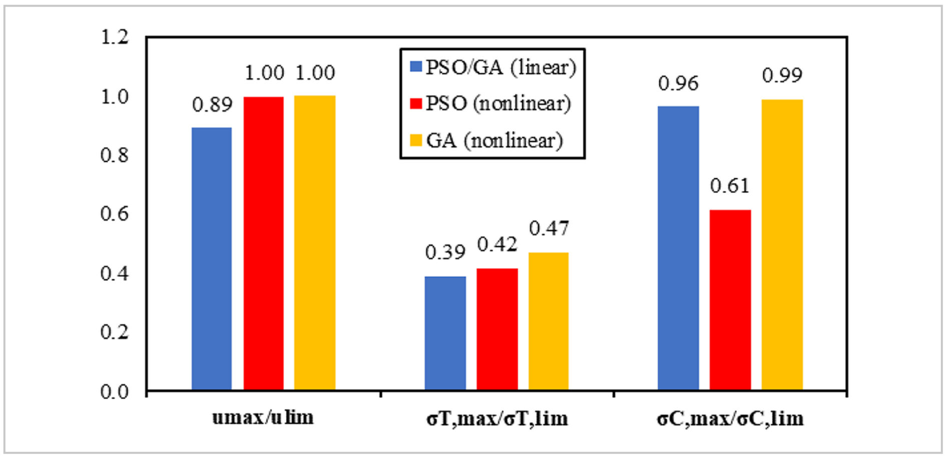


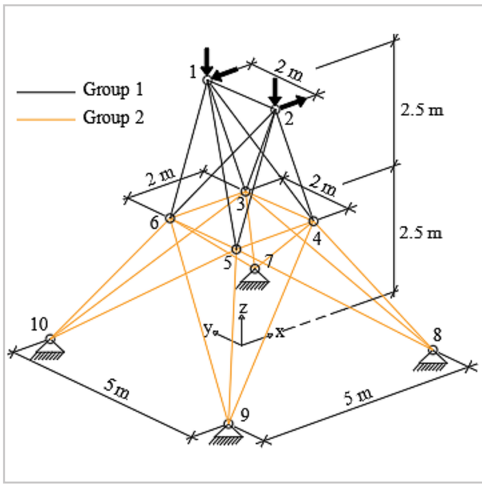
Figure 9 - Normalized constraints of 24-bar dome truss.

3.3 25-Bar Tower Truss

A new problem was now proposed, based on Mai *et al.*, (2022a), wherein the 25-bar tower truss in Figure 10 was subjected to the load seen in Figure 2(b), being t = 0.1 s and $F(t)$ assuming the values presented in Table 2. A time increment of Δt = 0.0001 s was adopted. First, a geometric nonlinear dynamic analysis was made, where the structure has a cross-sectional area A = 1 cm², a specific mass p = 7850 kg/m³ and a Young's modulus E = 200 GPa, for all members. The applied load is represented in Table 2.

Table 2 - Loading condition for the 25-bar tower truss.

Node	F_x (t)	F_y (t)	F_z (t)
1	-800 kN	0	-800 kN
2	800 kN	0	-800 kN



Element	Node 1	Node 2
1	1	2
2	1	4
3	2	3
4	1	5
5	2	6
6	2	4
7	2	5
8	1	3
9	1	6
10	3	6
11	4	5
12	3	4

Element	Node 1	Node 2
13	5	6
14	3	10
15	6	7
16	4	9
17	5	8
18	4	7
19	3	8
20	5	10
21	6	9
22	6	10
23	3	7
24	4	8
25	5	9

Figure 10 - 25-bar tower truss characteristics.

The displacements obtained for node one are seen in Figure 11, for both linear and nonlinear analyses.

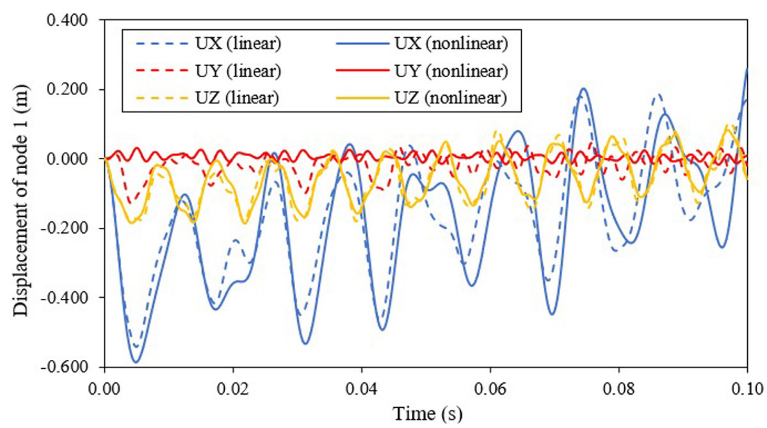


Figure 11 - Displacement of the nodes one for 25-bar tower truss.

Similarly, the two algorithms proposed were used to minimize the truss weight and the results are in Table 3. The effect of nonlinearity did not significatively impact the PSO final weight when compared to the linear analysis for this load. However, the algorithm showed itself to be more robust, as it presented better values for weight and RSD compared to GA.

Table 3 - 25-bar tower truss optimization results.

	Linear Analysis		Nonlinear Analysis	
	PSO	GA	PSO	GA
A_1 (cm²)	50.6	53.1	50.6	53.6
A_2 (cm²)	36.0	36.0	36.0	36.0
U_{max} (m)	0.0123	0.0119	0.0123	0.0118
σ_{Tmax} (MPa)	235.98	234.20	235.93	233.60
σ_{Cmax} (MPa)	341.94	340.99	342.22	341.04
Best weight (kg)	2666.71	2718.10	2666.71	2728.38
Mean weight (kg)	2703.73	2818.26	2729.25	2815.34
RSD (%)	2.89	12.53	4.80	12.63

In Figure 12 the displacement of node 1 is presented, and in Figure 13, the curves with the axial stresses over time of element 18 and 19 are seen, respectively presented on Figure 10. In some instants the nonlinear analysis showed greater displacements, and in the others, the linear analysis did. The maximum compressive stress occurred equally in elements 18 and 21, and maximum tension stress occurred in elements 19 and 20. Although the results were different between the algorithms, the maximums occurred in the same place and instant.

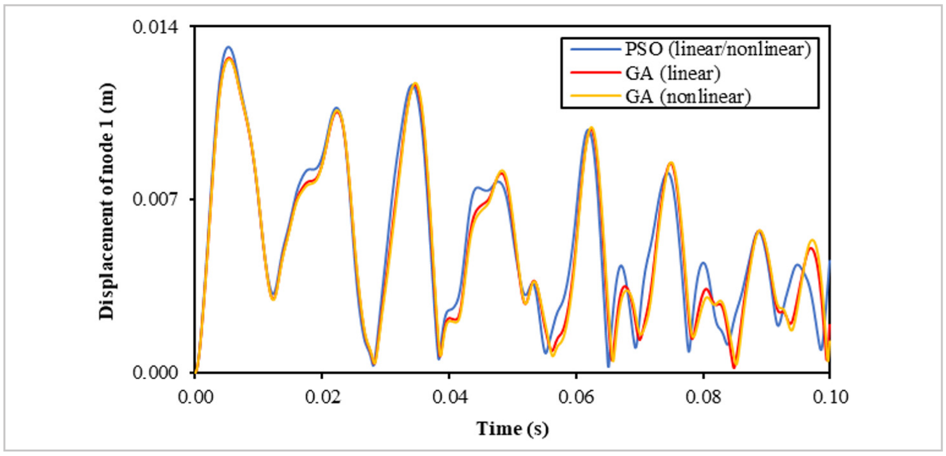


Figure 12 - Nodal displacement of 25-bar tower truss optimization.

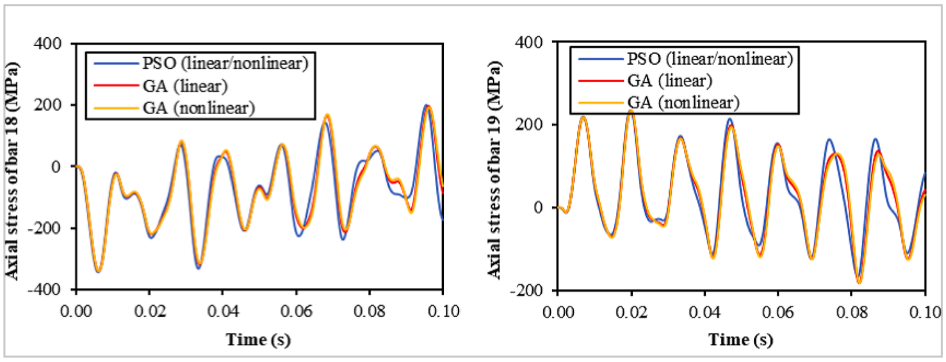


Figure 13 - Bar axial stress of 25-bar tower truss optimization.

Figure 14 shows the convergence curves of *total weight versus objective function evaluations* of each algorithm. The structure optimized by PSO had the same final result in both analyses, while by GA, the weight was greater in the nonlinear one.

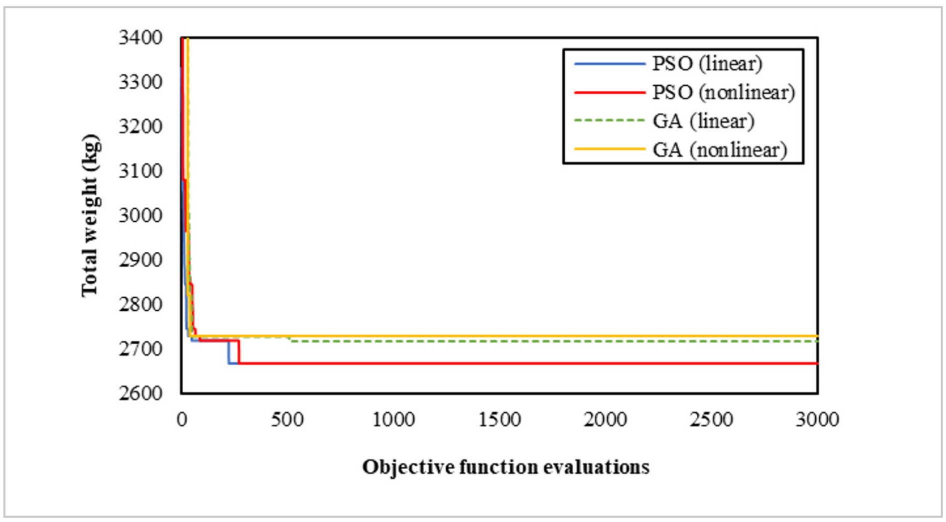


Figure 14 - Convergence curve of algorithms for 25-bar dome truss.

As can be observed in Figure 14 and according to Table 3, the structure did not exhibit a strong nonlinear behavior for the applied loading, leading to the same results as the linear analysis. Despite the identical results, for the nonlinear analysis, the solution via PSO converged from the 300th evaluation of the objective function, while for the linear analysis, it was at the 250th. evaluation. According to the graph and also demonstrated in Table 3, PSO presented better solutions compared to GA for both linear and nonlinear analysis.

The normalized values of the constraint function are shown in Figure 15, where the nodal displacement, the compressive stress and the axial compressive load are responsible for controlling the optimization similarly for all algorithms.

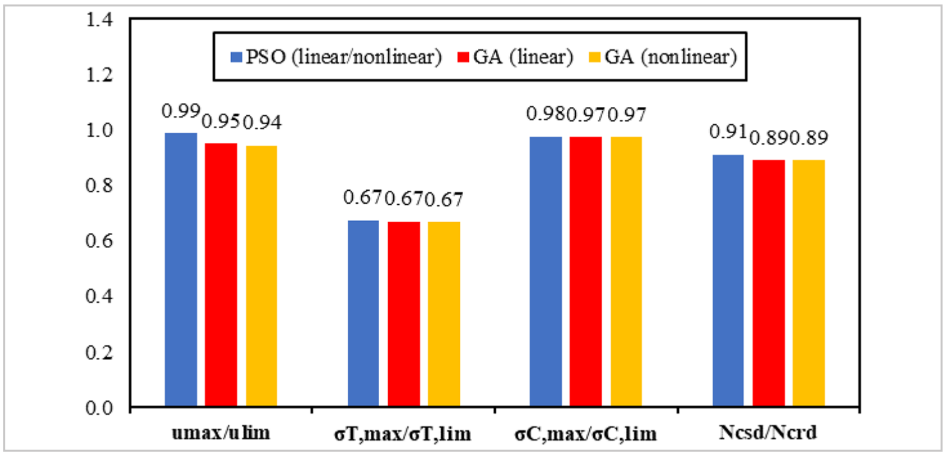


Figure 15 - Normalized constraints of 25-bar tower truss.

3. Conclusions

This study proposes a formulation of an optimization problem to analyze truss structures considering geometric nonlinear analysis with dynamic loading. The optimization problem is solved using two algorithms: the Genetic Algorithm (GA) and the Particle Swarm Optimization (PSO). First, a comparative analysis with benchmark problems is performed to verify the efficiency of the algorithms when compared with mathematical programing algorithms. Then, a new analysis is proposed to add references for this type of problem, due to the lack of such references in current literature. The results show that the algorithms run efficiently. In general, when the optimization process is done considering a nonlinear analysis, the greater the displacements, the heavier the structure. Moreover, the results indicate that the behavior of the algorithms depends on the situation. Regarding the constraints, displacement has been the main factor that has guided the optimizations, but in some cases, the stresses and axial force also had a considerable influence.

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