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# Development and application of a flexible numerical model to evaluate the safety of room-and-pillar mines

## Abstract

The design of room-and-pillar mines relies on the correct estimation of the safety indicator of the underground structures formed by the excavation of drifts. Thus, the study of the geomechanical behavior is of vital importance. The mathematical models play an important role in the identification of problematic areas and allows different configurations to be tested in a safe manner. This article presents the development and application of a flexible and automatic routine to quantify the safety of room-and-pillar mines in terms of the safety factor of pillar and room convergence. The commercial software package FLAC3D was used to implement a computation routine in FISH language that can automatically represent the main characteristics of the geomechanical conditions, lithology and geometric features of a room-and-pillar project in a fast and accurate manner. A case study was performed on a manganese ore mine in order to demonstrate the capabilities of the developed routine.

**keywords:** mining, room-and-pillar safety, numerical modeling, ore recovery.

## 1. Introduction

Room-and-pillar (R&P) mining is a method of extracting the material of a series of crossing rooms (i.e. horizontal linear openings) to leave behind pillars of ore. When the rock mass can be considered highly competent, pillars are usually much smaller than the rooms and the weakest rock mass between the roof, ore layer, and floor will command the opening instability. According to Idris *et al.* (2015), a pillar can be defined as the in situ rock mass between two or more underground openings. Pillars can be composed entirely of ore or ore and waste when the ore body has a small thickness relative to the excavation height. In some cases, this type of mining may need ground support systems such as cable bolting or confinement ribs.

During the design stage, it is very important to define the pillar dimensions that would assure the highest extraction ratio, while still ensuring the room's stability considering all unit operations. The optimum size of pillars that guarantees stable rooms depends on the active load on the pillar and its strength and stiffness as well as on the carrying capacity and stiffness of the roof rock mass.

The stability of pillars and rooms

should be guaranteed for each element of the underground structures defined by the opening. Traditional strength-based pillar design requires the pillar stress and strength to be estimated. The estimation of the pillar strength can be done by empirical methods (Hedley and Grant, 1972; Lunder and Pakalnis, 1997; Potvin *et al.*, 1989; Salamon and Munro, 1967; Von Kimmelmann *et al.*, 1984) or numerical load tests (Martin and Maybee, 2000). Alternatively, safety may be inferred directly from numerical methods by means of limit strains (Alber and Heiland, 2001; Idris *et al.*, 2015; Napa-García and Navarro Torres, 2017), averaging the strength-to-stress ratio over the pillar's section (Hoek and Brown, 1980), or by using the strength reduction method (SRM) (Navarro-Torres *et al.*, 2011; Tulu *et al.*, 2015). The safety factor (FS) of the pillar is then calculated by dividing the pillar strength by the average acting pillar stress, typically estimated using the tributary area approach. The definition of an acceptable FS depends on the tolerable risk against failure (Bullock, 2011). The safety of rooms can be evaluated using empirical methods (Ghasemi *et al.*, 2012; Mark, 2016; Melo *et al.*, 2014;

Nickson, 1992; Potvin, 1988; Weiss *et al.*, 2004), room convergence (RC) (Mark and Gauna, 2017; Zingano *et al.*, 2007), or some type of mechanical safety factor against tensile/shear failure (Canbulat and Merwe, 2009; Hoek and Brown, 1980; Sherizadeh and Kulatilake, 2016).

Both features, the FS and RC, can be automatically obtained from a numerical model to be used as input parameters for further decision analysis. The FS of pillars can be estimated using the SRM implemented in commercial software packages such as FLAC3D (Itasca Consulting Group INC, 2012). The safety of rooms can be estimated measuring the RC and considering a convergence limit given that a failure event is commonly characterized by a specific target limit value (absolute or relative) of the sum of the inward displacements of the roof and floor. The room convergence of the roof and floor can be measured at its critical position, which is the intersection of two orthogonal rooms for regular geometries. Thus, this article describes the development of a flexible model built with the commercial software package FLAC3D and its application to a real case study on a manganese ore mine.

## 2. Materials and methods

According to Coulthard (1999), numerical modeling can assist geotechnical engineers in designing underground excavations and support systems. If extensive geological and geotechnical data are available, then deformations, stability, and support loads can be comprehensively predicted by numerical stress

### 2.1 Constitutive model description

In computational modeling, two failure criteria are commonly used: the Mohr–Coulomb (MC) model and generalized Hoek–Brown (HB) model with

where  $K$  is the slope of the line representing  $\sigma'_1$  as a function of  $\sigma'_3$  and  $\sigma_{cm}$

analyses. If not, the model can still be used to perform parametric studies and provide insight into the possible range of responses of a system given the likely ranges for various parameters. Brandani (2011) discusses the main objectives of numerical methods regarding room-and-pillar design as follows:

- Estimating the load capacity or maximum permissible load on pillars;
- Developing a geometric configuration appropriate to the estimated stress field;
- Determining the failure modes;
- Performing retro-analysis of rock mass properties based on the instrumentation data.

elastoplastic behavior (Navarro Torres, 2003). With regard to the failure criteria, the MC criterion can be expressed as the following function of the effective

$$\sigma'_1 = K\sigma'_3 + \sigma_{cm} \quad (1)$$

is the uniaxial compressive strength (UCS) of the rock mass.  $K$  depends on

$$k = \frac{1 + \sin \phi}{1 - \sin \phi} \quad (2)$$

major principal stress  $\sigma'_1$  for different values of the confining principal minor effective stress  $\sigma'_3$  as follows (Hoek and Brown, 1980):

the friction angle  $\phi$  of the rock mass, estimated as:

The  $\sigma_{cm}$  of the rock mass is calculated using the following function of the cohesion  $c$  and friction angle  $\phi$

$$\sigma_{cm} = \frac{2c \cos \phi}{1 - \sin \phi} \quad (3)$$

Hoek and Brown (1980) aimed to represent the failure of a rock mass

through an empirical criterion based on the major and minor principal stresses.

The original HB criterion was defined as follows:

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left( m_i \frac{\sigma'_3}{\sigma_{ci}} + s \right)^{0.5} \quad (4)$$

where  $\sigma_{ci}$  is the UCS of the intact rock; and  $m_i$  and  $s$  are material constants of the rock mass. For intact rock,  $s=1$ .

When there is no lateral confinement ( $\sigma'_3=0$ ) and  $s=1$  (intact rock), the uniaxial tensile strength of a rock or rock

mass is determined by considering  $\sigma'_1=0$  and  $\sigma'_3=-\sigma'_t$ . This results in (Hoek e Brown, 1980)

$$\sigma_t = 0.5 \sigma_{ci} [m_i - (m_i^2 + 4s)^{0.5}] \quad (5)$$

Later, Hoek *et al.* (2002) presented the generalized HB criterion, which includes not

only results from a laboratory but also values obtained from observed failures of fractured

rock masses in a nonlinear manner. This nonlinear criterion can be described as

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left( m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^a \quad (6)$$

$$m_b = m_i \exp \left( \frac{GSI - 100}{28 - 14D} \right) \quad (7)$$

$$s = \exp \left( \frac{GSI - 100}{9 - 3D} \right) \quad (8)$$

$$a = \frac{1}{2} + \frac{1}{6} [\exp(-GSI/15) - \exp(-20/3)] \quad (9)$$

where  $GSI$  is the Geological Strength

Index, and  $D$  is the perturbation factor

(Hoek *et al.*, 2002).

The deformability of a rock mass can be estimated by using the Hoek and Diederichs (2006) formulation:

$$E_{rm} = E_i \left( 0.02 + \frac{1 - D/2}{\exp[(60 + 15D - GSI)/11]} \right) \quad (10)$$

where  $E_i$  is the modulus of deformation of the intact rock and it is given by

$$E_i = MR \sigma_{ci} \quad (11)$$

where  $MR$  is the reduction modulus.

## 2.2 Modeling routine

The large amount of information necessary to build a robust numerical model and the need to evaluate diverse mining situations requires the development of a flexible tool that can represent this generality. Therefore, a flexible and robust routine was written in FLAC3D using the FISH language for automatic safety analysis of R&P mining. FLAC3D was chosen because of its “driven-command” functionality, which makes it intrinsically very flexible.

The numerical model consisted in only one pillar and half room size at each side of the pillar. The routine considers all layers to be horizontal and of constant thickness and pillars to be regularly distributed on a horizontal section. Pillars are rectangular, and rooms are equidimensional and orthogonally positioned. Some geometric details of the model, such as the approximate zone target sizes, can be specified. In addition, the excavation process can be considered to be an instantaneous deletion process or progressive core softening process.

The rock mass strength can be described by either MC or HB failure criteria. The entire lithology and rock mass mechanical properties are read from a text file, and the model is built above, below, and at the mining level by considering the

depth, thickness and material properties.

In the absence of measurements, the horizontal stress was considered to come from an elastic lateral stress coefficient because the ore horizon is placed at the middle high of a mountain. Thus, the lateral boundaries as well as the bottom of the model were restricted against normal displacements. The *in situ* stress state was estimated by initializing the vertical stresses by gravity and then cycling the model up to equilibrium.

Next, the rooms were instantaneously excavated. All of the mechanical state variables can also be explored and analyzed with the FLAC3D graphical/command capabilities. Displacements were measured after 20 000 cycles to differentiate between stable and unstable conditions. The  $RC$  is measured at the intersection of two perpendicular rooms. The reported  $FS$  corresponds to that obtained with the SRM (Dawson *et al.*, 1999) in FLAC3D (any other  $FS$  estimation technique might be used). After excavating, the main outputs of the model are the  $FS$  and  $RC$ .

Although the routine is capable of modeling any rectangular number of pillars (number of pillars along the x-axis  $\times$  number of pillars along the y-axis), the symmetry of the conceptual model enables

the use of only one pillar to represent a mining panel of similar characteristics with infinite number of pillars and rooms in both directions. However, this consideration is considered to be accurate enough for large panels. This routine does not consider non-symmetric conditions. The latter case needs to be studied in detail with a specific numerical model.

The implemented routine follows the next steps:

1. Filling input data files relative to strength parameters, and list of lithologies and thickness above and below the ore layer;

2. Set general model parameters such as failure criteria, number of lithologies, number of rock layers above and below the ore layer, room and pillar dimensions, mesh parameters, and excavation type;

3. Read rock mass parameters and lithology distribution;

4. Build geometry of the model;

5. Set mechanical properties into the numerical model;

6. Apply boundary conditions (displacement restrictions);

7. Calculate *in situ* stress state;

8. Run displacements and  $RC$  estimation;

9. Restore *in situ* stress state and run  $FS$  estimation;

## 3. Case study

A case study was performed on a pilot area of a manganese ore mine in Brazil. The studied area presents a regular orthogonal grid of linear 5 m wide rooms which led to the formation of approximate square-section pillars of 15 m side. These dimensions represent an ore recovery of 44%, which would leave 56% of the available

quantity behind. Thus, there is interest in increasing the recovered amount in a safe manner. Therefore, the main objective of this study was to evaluate the safety of the R&P mining with the current configuration in order to judge if further exploration is feasible. The capabilities of the presented flexible routine were explored by evaluating

the stability conditions of the current configuration of the mine as a function of the rock mass strength. In order to consider the geomechanical quality of several layers in the numerical model, significant classification needed to be performed. The data were based on the lithologic sequence of the adjacent rocks of the manganese ore layer.

### 3.1 Numerical modeling and simulation

The numerical model of the case study mine was built in FLAC3D con-

sidering 44 lithologies. Four rock types alternated above the mining level (approx-

mately 150-m), and a 50-m-thick layer of homogeneous rock below the mining

level, as shown in Figure 1. The MC failure criterion was used, and the mining process

was considered to be instantaneous.

#### 4. Results and discussion

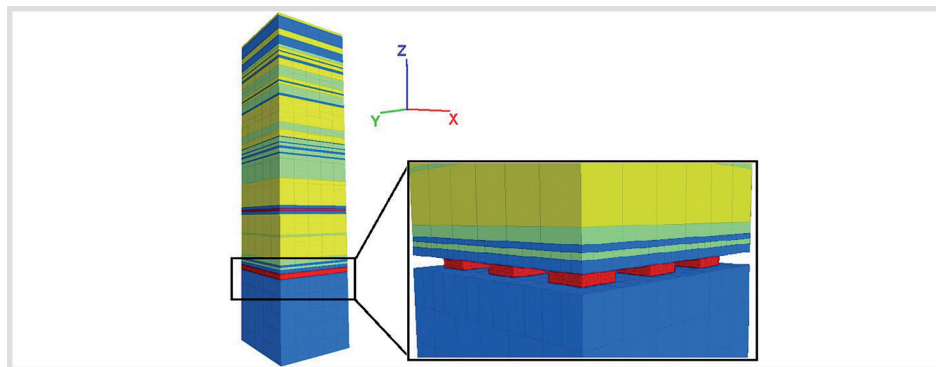
##### 4.1 Model calibration

As the conditions of the depth of the current mining panel of the manganese layer, lithology above the target level, quality of blasting with explosives ( $D = 0.8$ ), and intact rock strength are known, the resistance parameters to be used in the numerical model may be estimated based on Hoek *et al.*'s formulation

(2002). In this case, the working stress interval was estimated using the geostatic vertical stress and the HB to MC equivalence was that of underground openings.

Table 1 presents the parameters of the layer obtained by using Eq. (1) to (11). The variability present in the input data conditions a large uncer-

tainty propagation in the geomechanical parameters of the numerical model. Given that the UCS of the intact rock presents the main variability, a sensitivity analysis is needed to be performed on this parameter. Therefore,  $FS$  and  $RC$  were considered as functions of the UCS of the intact rock.



Parameter	Value
$a$	0.5
$m_b$	1.07
$s$	0.02
$\sigma_c$ [MPa]*	30-115

\*Disperse laboratory results ranging from 30 to 115 MPa

Figure 1  
Lithology of the numerical model of nine pillars with details of the manganese layer.

Table 1  
Geomechanical parameters of the manganese rock mass.

The variability in the UCS of the intact rock imposes a dispersion in the MC parameters. In fact, a co-dependency is present, as shown in Figure 2 (left). The rock mass cohesion varied from 5 to 21

MPa in an approximately linear fashion, and the friction angle varied nonlinearly from 34.5° to 36.3° approximately. Table 2 presents the discrete variation of the parameters described above. In ad-

dition, the variability in the UCS of the intact rock imposes a dispersion on the modulus of deformability  $E_{rm}$ . Figure 2 (right) shows the dependency relationship between the UCS and  $E_{rm}$ .

UCS $\sigma_c$ MPa	Friction angle $\phi$ degree	Cohesion C MPa	Modulus $E_{rm}$ GPa
30	34.2	5.37	7.06
45	35.1	8.06	10.60
60	35.5	10.76	14.13
80	35.9	14.37	18.83
100	36.2	17.98	23.55
115	36.3	20.69	27.08

Table 2  
Dependency of the friction angle, cohesion, and modulus of deformability with respect to the UCS of the intact rock.

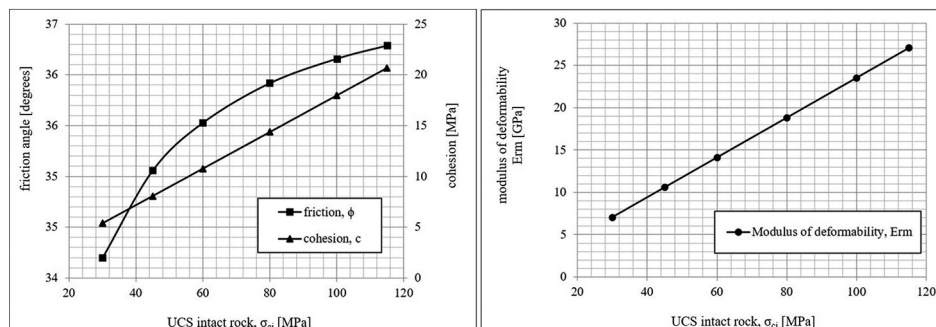


Figure 2  
MC parameter and  $E_{rm}$  dependency with the UCS of the intact rock.

## 4.2 Safety evaluation of the current mine conditions

The rock mass strength was evaluated at three discrete strength levels (scenarios): minimum (30 MPa), medium (60 MPa), and maximum (115 MPa). Stress-strain analyses showed vertical stress of approximately 8 MPa at the pillar core and 15 MPa at the corners of the cross-section for the three evaluated scenarios. Vertical displacements at

the roof were more sensitive to pillar strength/stiffness presenting values of 5 to 10 mm for maximum to minimum resistance level. On the other hand, vertical displacements at the floor remained approximately constant at the value of 5 mm for all resistance levels. Figure 3 presents the vertical stress distribution at the pillar middle cross-section and the vertical displacement

at a vertical section, for the minimum resistance scenario.

In all cases, the stress distributions and displacements were similar for the three scenarios. This behavior suggests that the pillar material was mostly elastic and far from generalized yielding in all analyzed resistance states, i.e., the pillar integrity indicated a considerable safety margin.

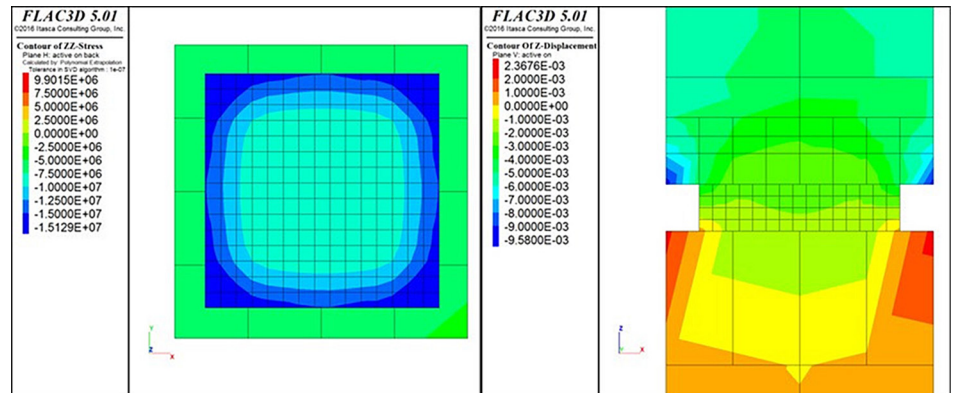


Figure 3  
Vertical stress and vertical displacement:  
15 m × 15 m pillar, minimum resistance.

Complete safety analyses (stress-strain plus SRM calculation) yielded results relative to the *FS* and *RC* of the current state of the mine. Figure 4 presents the *FS* and *RC* according to the UCS of the intact rock. The *FS* increased with UCS from 3.7 to 9.3 in a linear manner. On the other hand, the *RC* decreased nonlinearly from 18 mm to 14 mm.

The SRM (automatic) method was used to assess the *FS*; the values obtained for the *FS* were considered compatible with empirical formulations (Navarro-Torres *et al.*, 2011). The variability in the

*FS* obtained from the simulations of this case study for a UCS of 30 to 115 MPa for the material and slenderness ratio of 0.23 resulted in a *FS* of 3.71–9.34, which can be considered highly safe given that typical minimum required *FS* use to vary between 1.3 and 2.0.

The *RC* varied from 18 mm to 14 mm, which represents a non-dimensional convergence of 0.5% to 0.4%. These values can be considered to be within satisfactory roof behavior; according to practical standards, some minor problems may begin to appear at

a convergence of 1% and higher (Hoek and Marinos, 2000; Schubert *et al.*, 2003; Singh and Goel, 1999; Solak, 2009; Tonon, 2012).

The developed model showed satisfactory and promising results but still needs to be improved/calibrated with data to be obtained from the analysis of rock samples and instrumentation to better understand the current property values of the materials in the model. After more refined knowledge of the model material parameters is obtained, this new model may be used to optimize the ore recovery process.

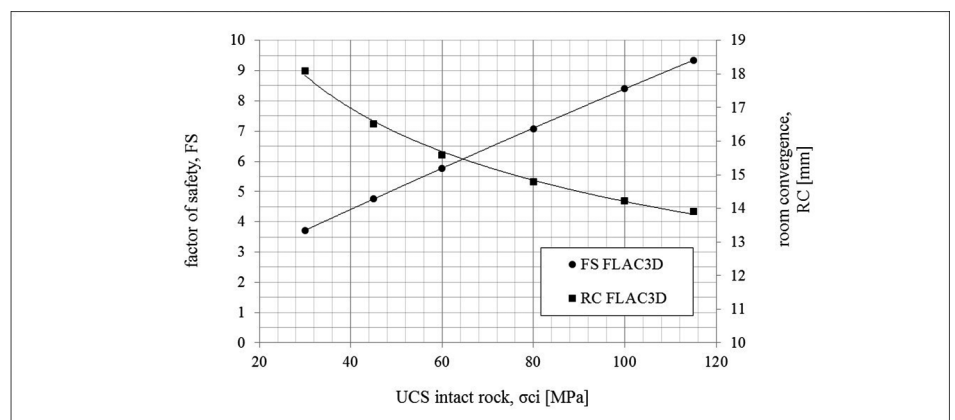


Figure 4  
Variation in the *FS* and *RC*  
according to the strength of the intact rock.

## 5. Conclusions

A three-dimensional numerical model was developed with innovative characteristics, including excellent flexibility and geomechanical and geometric adaptability to different scenarios, to

consider the variability of the operational and geomechanical conditions for a manganese ore mine. The developed methodology can incorporate important features of the rock mass as the lithology

and behavior of rocks. It can be applied to a single pillar as well as to an unlimited number of pillars, which allows the simulation of large areas of mining panels and various scenarios. In the case study, the



current available data led to the conclusion that the mining pilot area is highly safe because the SF was between 3.7 and 9.3, even though the minimum SF is usu-

ally 1.5. In addition, the RC was less than 18 mm, which represents stable behavior according to literature. The use of new information from a laboratory and in situ

instrumentation will allow the model to be calibrated and hence reduce the variability in the response to increase the reliability of the results for future applications.

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