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Partially Restrained Beam-Column Connections in Reinforced Concrete Structures

Engastamento Parcial de Ligações Viga-Pilar em Estruturas de Concreto Armado



G. M. S. ALVA [a]
alva_gerson@yahoo.com.br

M. A. FERREIRA [b]
marcelof@ufscar.br

A. L. H. C. EL DEBS [c]
analucia@sc.usp.br

Abstract

This paper presents the results of experimental tests on reinforced concrete beam-column connections carried out at the Engineering School of São Carlos – USP. The experimental results concentrate on relative rotations between beam and column, which are important parameters for the evaluation of the moment-rotation curves and the characterization of semi-rigid behavior of the connections. The experimental results were obtained from tests on specimens of edged beam-column connections. The influence of concrete compressive strength and joint transverse reinforcement ratio on relative rotations was investigated. The experimental results were compared with results obtained from the theoretical model proposed by Ferreira [1] for partially restrained connections, which provided an evaluation of the percentage of restriction obtained by the types of monolithic connections considered in this study.

Keywords: *: concrete structures; beam-column connections; semi-rigid connections; structural analysis.*

Resumo

Neste trabalho são apresentados os resultados experimentais de ensaios em ligações viga-pilar de concreto armado realizados na Escola de Engenharia de São Carlos – USP. Os referidos resultados experimentais concentram-se nas rotações relativas entre os elementos viga e pilar, importantes para a determinação da relação momento-rotação das ligações e da caracterização do comportamento semi-rígido das mesmas. Os dados experimentais foram obtidos a partir de ensaios de ligações viga-pilar de extremidade. A influência da resistência à compressão do concreto e da taxa de armadura transversal na região dos nós de pórtico sobre as rotações relativas foi analisada. Os resultados experimentais foram confrontados com o modelo teórico proposto por Ferreira [1] para ligações com engastamento parcial, permitindo uma avaliação da porcentagem de engastamento alcançada pelas tipologias de ligações monolíticas abordadas neste trabalho.

Palavras-chave: *estruturas de concreto; ligações viga-pilar; ligações semi-rígidas; análise estrutural.*

[a] *Universidade Federal de Santa Maria, Departamento de Estruturas e Construção Civil, alva_gerson@yahoo.com.br, Endereço: Avenida Roraima 1000 Cidade Universitária CEP: 97105-900, Santa Maria-RS, Brasil;*

[b] *Universidade Federal de São Carlos, Departamento de Engenharia Civil, marcelof@ufscar.br. Endereço: Campus São Carlos Rodovia Washington Luís (SP-310), km 235, CEP 13565-905. São Carlos - SP – Brasil;*

[c] *Universidade de São Paulo – Escola de Engenharia de São Carlos, Departamento de Engenharia de Estruturas, analucia@sc.usp.br. Endereço: Campus São Carlos Rodovia Washington Luís (SP-310), km 235, CEP 13565-905. São Carlos - SP – Brasil.*

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1. Introduction

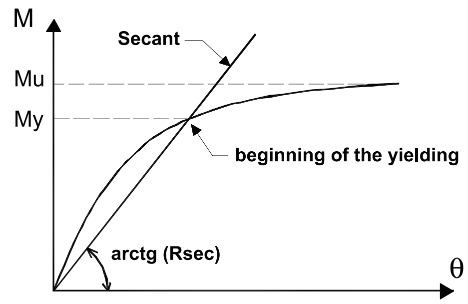
1.1 Initial considerations

In current structural analysis for cast in situ concrete structures, beam-column connections are considered as fully rigid (monolithic), and therefore there are not any relative rotations between the beam and column elements. However, real beam-column connections can present a partially restrained behavior, wherein their flexural stiffness can be identified from the beam-column relative moment-rotation relationship. Because of this semi-rigid behavior, beam-column connections can affect the moment redistribution along concrete structures. Therefore, knowing the degree of restriction obtained by beam-column monolithic connections can help designers to carry out a more accurate structural analysis.

1.2 General justifications of the study

The consideration of semi-rigid behavior of beam-column connections takes on greater importance in the light of growing development, commercialization and utilization of new computer systems applied to the engineering of structures. The understanding and correct evaluation of parameters associated to the stiffness of beam-column connections cannot be neglected by professionals who use such systems, for the semi-rigid behavior of connections influences the behavior of the whole structure, and more specifically, the internal force redistribution along the elements and displacements and global stability of the structure, associated to second-order effects.
Within the designing practice there has been some misunderstanding between the moment redistribution coefficient and the partial restriction coefficient. The first is related to moment redistribution, which is caused by cracking in concrete and by yielding of longitudinal reinforcement in the support region, and possesses values limited by the Brazilian Code (ABNT NBR 6118) [2] for linear analyses with redistribution. The latter is not directly related to the type of structural analysis (linear, linear with redistribution, plastic, non-linear), but rather with the most realistic representation of the conditions of connection between beam and column, in other words, the condition between the fixed and pinned condition. This misunderstanding has become evident from the utilization of commercialized computer systems that allow reductions in negative beam moments.
The complexity of beam-column connection studies can be attributed to the great number of variables that influence the structural behavior of beam-column connections, which are essentially related to reinforcement details, the geometry of the structural elements connected, the intensity of applied loads and the steel and concrete strength. In addition, it should be emphasized that the connections cannot be seen as merely a node in the structure, but as belonging to the region of discontinuity (D-region), a region where the hypothesis of Bernoulli is not valid (linear distribution of deformations). Because of the geometric discontinuity, the stretch of the beam end adjacent to the column and the beam-column joint are considered regions of discontinuity, whose behavior influences the moment-rotation of the connections.
The influence of the beam-column joints on the structural response of the connections can be manifested in a limitation of the resistance achieved and in the anchorage conditions supplied to the tensile reinforcement of the beam. Among the number of variables that influence beam-column joints, the most important are concrete compressive strength and joint transverse reinforcement, because they are directly related to joint shear strength – a parameter that governs the capacity of the connection.
Another variable that can affect beam-column joint behavior and which deserves more attention is the normal force of the column. Experimental investigations and numerical simulations with reinforced concrete exterior beam-column connections carried out by Haach [3] indicate that the increase in normal force reduces the joint capacity, making the failure of the connections more brittle, but on the other hand, improving the anchorage conditions of tensioned beam bars.

Figure 1 – Secant stiffness of beam-column connection



1.3 Justifications on the use of the model proposed by Ferreira [1]

Experimental proof of the analytical models that aim to determine connection deformability is a determining factor for such models to be applied safely in the evaluation of connection stiffness in design. Furthermore, comparisons with experimental results may allow for better calibration of the proposed analytical models and the inclusion of new and important variables in connection deformability. In this sense, the work of Soares [4], Ferreira [5] and Miotto [6] can be cited, in which are found the results of experimental investigations and applications of analytical models involving beam-column connection deformability in precast concrete structures.
Among the theoretical models that propose to represent the moment-rotation relation of semi-rigid connections in design procedures, the analytical model proposed by Ferreira [1] can be cited. According to his model, the relative rotation between beam and column is a result of two deformation mechanisms. The first is associated with the elongation of beam tensile reinforcement correspondent to the anchorage length in the column. The second is associated with bending deformations of the beam end in the region of discontinuity, in a stretch whose length depends on the height of the beam section. The theoretical model also contemplates the physical nonlinearity caused by concrete cracking. Analyses of connection deformability using the above-mentioned model can also be found in the work of Kataoka et al. [7], Catoia et al. [8], Nóbrega, Ferreira and Hanai [9], Ferreira et al. [10] and Ferreira and El Debs [11].

Important parameters for the evaluation of connection stiffness can be obtained from the moment-rotation curve (figure 1). Among these, the end fixity factor can be cited, which informs the designer as to how close the connection is to the fully rigid or ideally pinned condition allowing an evaluation of the percentage of restriction achieved (or the coefficient of partial restriction). This can be determined following equations 1 and 2:

End fixity factor:

$$\alpha_R = \frac{1}{1+\dfrac{3(EI)_{sec}}{R_{sec}L_{ef}}} \tag{1}$$

where

R_{sec} is the secant stiffness of the connection obtained from the moment-rotation curve;

L_{ef} is the effective span between the supports;

$(EI)_{sec}$ is the secant stiffness of the beam, according to Brazilian Code: ABNT NBR 6118 [2].

Coefficient of partial restriction:

$$\frac{M_E}{M_{ENG}} = \frac{3\alpha_R}{2+\alpha_R} \tag{2}$$

where

M_E is the bending moment mobilized in the beam end;

M_{ENG} is the fixed end moment.

The end fixity factor presented in equation 1 is derived from the method presented by Monforton and Wu [12] for structural analysis of frames with semi-rigid connections and is also used by ABNT NBR 9062 [13]. Additional details on the coefficient of partial restriction presented in equation 2 can be found in [5].

The obtainment of the moment-rotation curve using the theoretical model proposed by Ferreira [1] is attractive in design situations due to its simplicity and because it requires easily understood input parameters for structural engineers.

Figure 2 – Geometry and reinforcement layout of tested specimens (dimensions in mm)

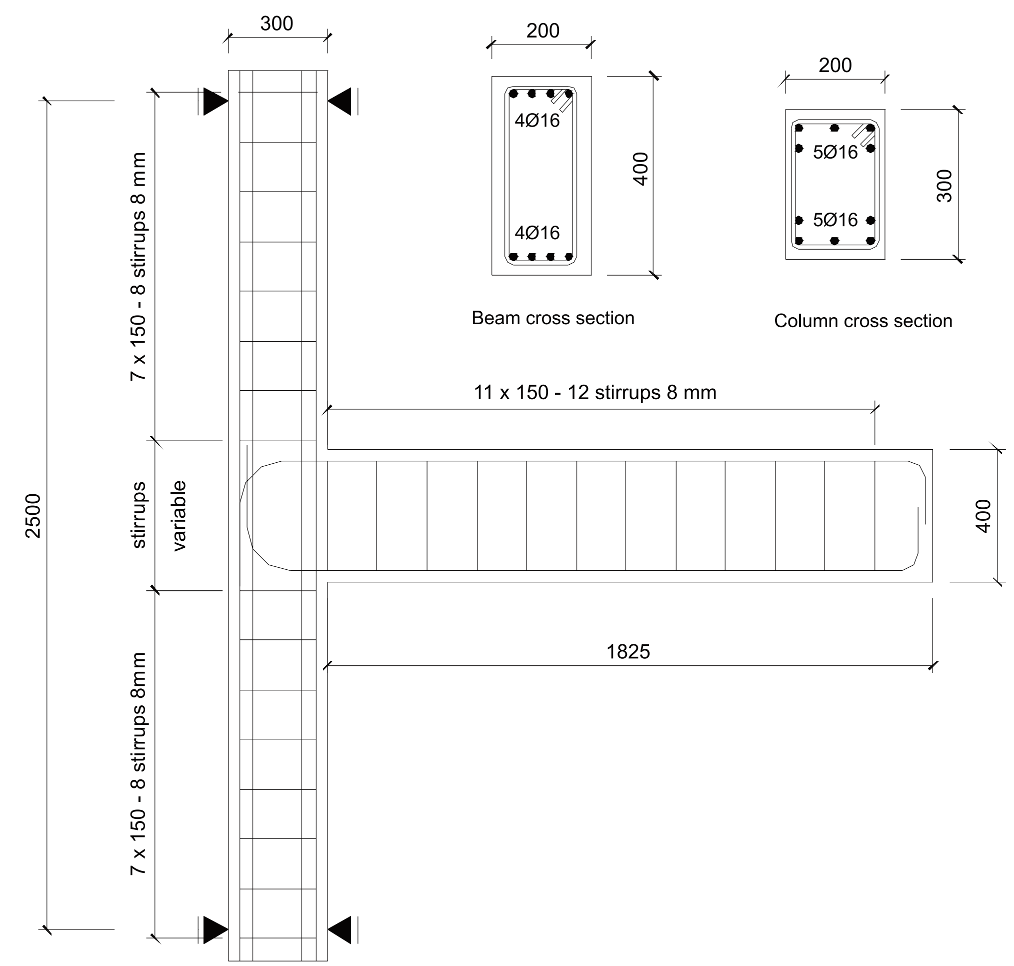
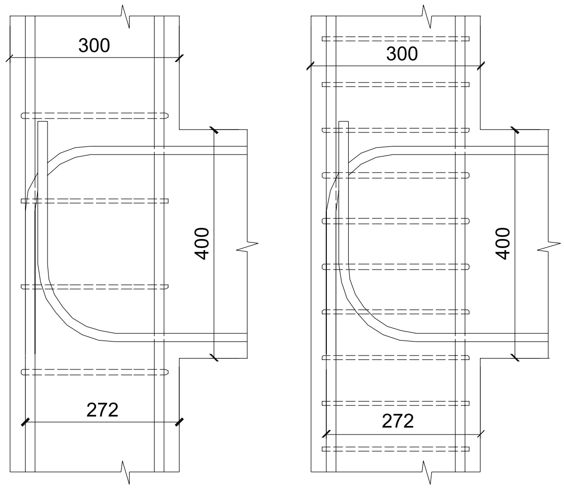


Figure 3 – Reinforcement details of the beam-column joints (dimensions in mm)



LVP2 and LVP4 LVP1, LVP3 and LVP5

The good correlations between theoretical results and experimental results obtained in a study at the Engineering School of São Carlos (University of São Paulo) provide a good indication that the model proposed by Ferreira [1] allows the estimation of the percentage of partial restriction achieved by reinforced concrete beam-column connections, including monolithic connections.

1.4 Objectives of the study

This paper presents the experimental data for the moment-rotation curve of monolithic beam-column connections. Initially, the experimental data are analyzed in order to determine the influence of the compressive strength of concrete and the transverse reinforcement available in the beam-column joint region. Subsequently, the experimental results are compared with the analytical model proposed by Ferreira [1] for the determination of the moment-rotation curve for semi-rigid connections in general. Finally, numerical examples are given employing the theoretical model, in order to evaluate the percentage of restriction achieved in exterior beam-column monolithic connections with spans and materials commonly used in buildings.

2. Experimental Results from Alva [14]

The experimental results shown in this paper were obtained by Alva [14], with the main objective of evaluating the behavior of reinforced concrete beam-column joints under cyclic loads. After extensive and careful analysis of the results obtained, the experimental data from the moment-rotation curves for which the cyclic character of the loads exerted little or no influence on the connections were used.

2.1 Characteristics of the physical models

The physical models tested consisted of exterior beam-column connections, without slabs, with beams with cross sections of 20x40 cm and columns with cross sections of 20x30 cm. The five connections tested (LVP1 to LVP5) presented the same longitudinal and transversal reinforcement, except in the beam-column joint region, where two 8 mm stirrups were placed on LVP2 and LVP4 and four 8 mm stirrups were placed on the remaining connections (see figures 2 and 3).

2. Materials

For the characterization of the concrete, tests were carried out on 15x30 cm cylinder specimens. For the characterization of the steel, tensile tests were carried out on bar samples for each diameter. The main mechanical properties of interest for this study are presented in table 1.

2.3 Test setup

All five beam-column connections were tested in the horizontal position, parallel to the reaction slab. The connections were positioned in a self-equilibrated reaction structure, with lateral supports at both ends

Table 1 – Mechanical properties of steel and concrete

Specimen	f_c (MPa)	E_{cs} (MPa)	$f_{ct,sp}$ (MPa)	f_y (MPa)
LVP1	40,43	27.902	2,87	630
LVP2	44,18	29.579	3,30	594
LVP3	23,89	25.093	1,95	594
LVP4	24,62	25.235	2,08	594
LVP5	25,91	25.487	2,20	594

f_c: concrete compressive strength; E_{cs}: secant modulus of elasticity of concrete; $f_{ct,sp}$: tensile strength of concrete (split test); f_y: yield strength of steel

Figure 4 – Test specimen on the self-equilibrated reaction structure

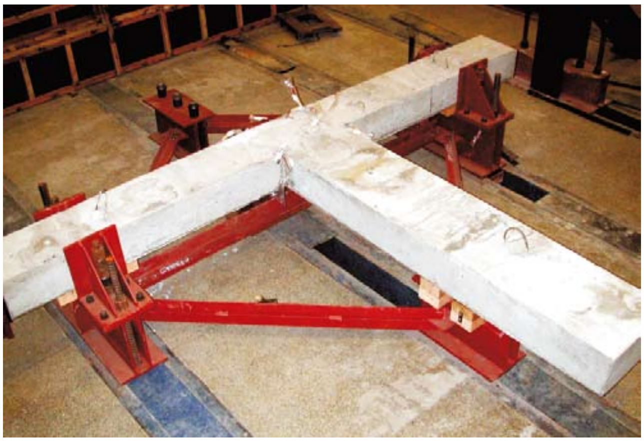
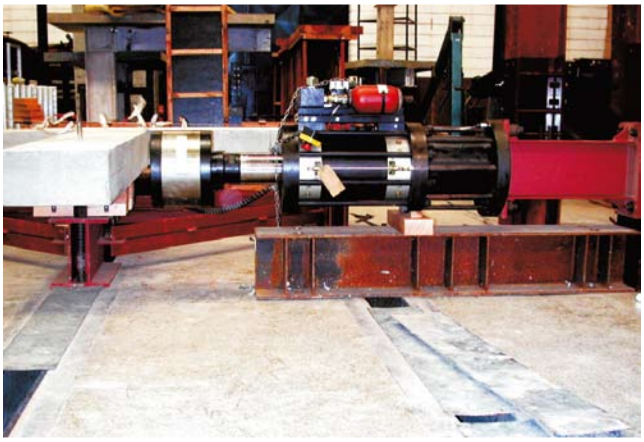


Figure 5 – Detail of the actuator set to apply concentrated loads to the beam



of the column. A hydraulic jack was used to apply a non-dimensional normal force (pre-load) of approximately 0.15. A servo-controlled actuator was also used to apply concentrated forces at the free end of the beam, generating moments in the beam-column connection. Figures 4 to 6 illustrate the test setup of the physical models and the equipment used in the application of force. Additional details of the test setup and the reaction structure can be found in Alva [14].

2.4 Instrumentation

A total of 10 displacement transducers were placed at diverse positions of the connections. Displacement transducers T3 and T4 (Kyowa, with a 10 mm stroke) were utilized with the specific objective of evaluating the relative rotation between beam and column, as shown in figure 7.

The relative rotation between beam and column can be determined from the expression:

$$\theta = \frac{\delta_3 - \delta_4}{H} \tag{3}$$

Figure 6 – Equipments for load application and static equilibrium of the connections

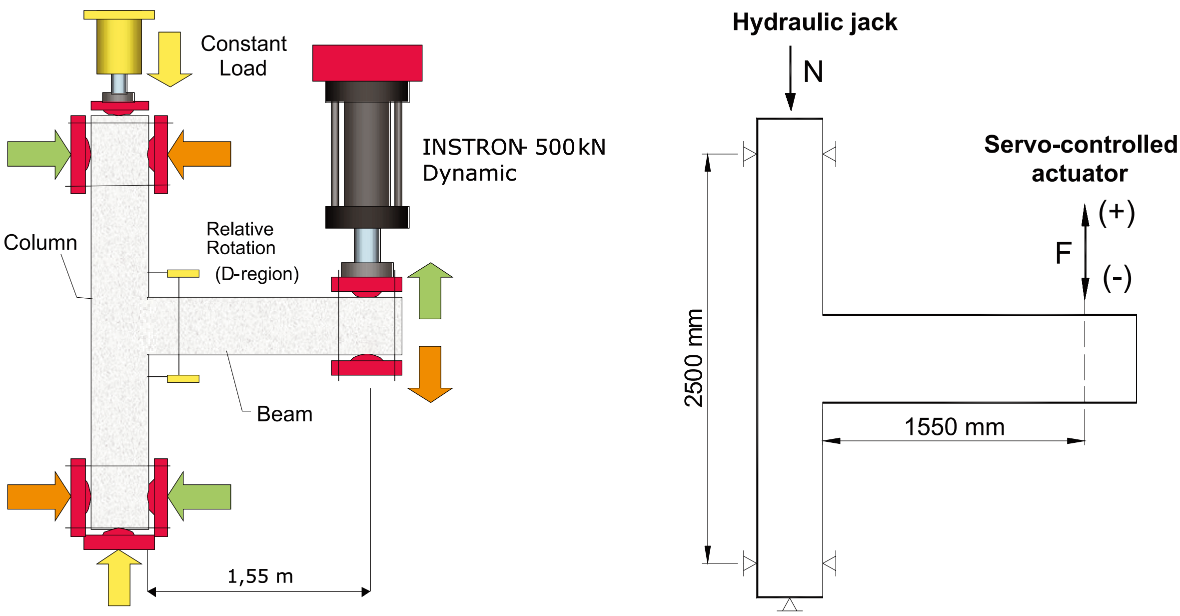
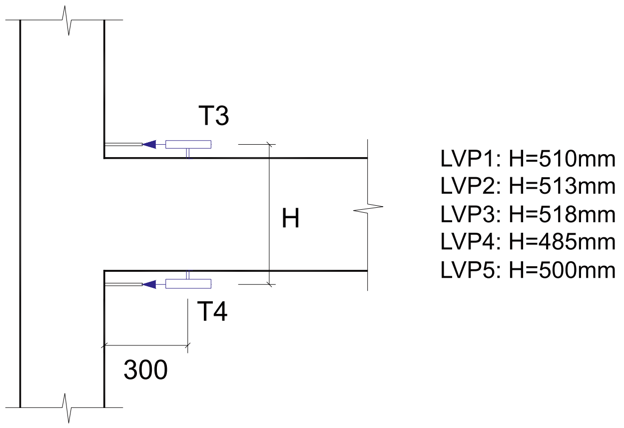


Figure 7 – Position of displacement transducers used to evaluate the relative rotations

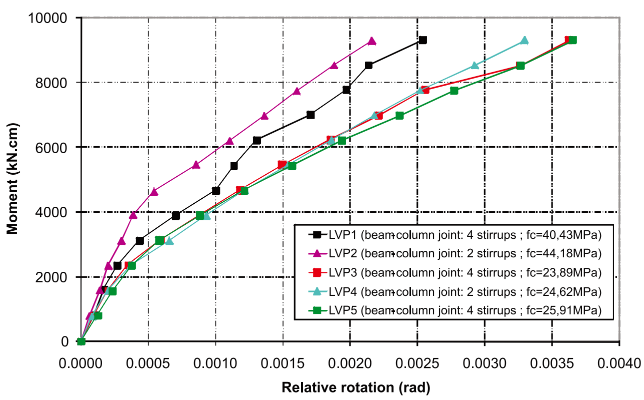


where δ_3 and δ_4 are the displacements measured by transducers T3 and T4 and H is the distance between the transducer axes (see figure 7).

2.5 Loading applied to the connections

The loading was applied to the connections in stages. In the first stage, for all connections, increasing alternate forces were applied to the exterior beam as follows: -10kN, +10kN; -20kN, +20kN; -30kN, +30kN; -40kN, +40kN; -50kN, +50kN; -60kN, +60kN. The amplitude of the forces reached in this stage produced a maximum moment in the beam to the order of 60% of the yielding moment, corresponding to approximately the stress level achieved in the service range.
For LVP1, the second stage of loading consisted of displacement cycles in the inelastic range and near to the rupture, in an attempt to simulate the effects produced by moderate seismic loads. The experimental data utilized for the analysis of the moment-rotation curves for LVP1 are limited to the first stage of loading and to the first semi-cycle of displacement, with a maximum force applied of -110 kN.
For LVP2, the second stage of loading consisted of application of 1000 cycles of alternate load with amplitudes of -60kN and +60kN and finally, as a last stage, the connection was led to rupture, with a maximum force applied of -131 kN. The cycles applied in the second stage produced an increase in relative rotations. However, the increase was not considered significant and did not negatively affect the comparisons with the theoretical model for monotonic actions evaluated in this study.
For connections LVP3 and LVP4, the second stage consisted of alternate loading with increasing amplitudes at every 10 kN, with 3 cycles for each amplitude: 3 cycles of -70kN and +70kN, 3 cycles of -80kN and +80kN, and so forth, until reaching a sudden drop of resistance characterizing failure of the connection. The experimental data utilized in the moment-rotation curves for LVP3 and LVP4 are limited to the first stage of loading and the first semi-cycle of amplitude of -70kN.
For LVP5, loading was applied similarly to that for LVP2. However, in the second stage there was a significant increase in relative rotations due to the cyclic character of loading. Thus, the the experimental data for LVP5 will be limited to the first stage only.
Analysis of the influence of concrete compressive strength
In order to analyze the influence of concrete compressive strength on the moment-rotation curve, the connections with the same joint transverse reinforcement were compared. Thus, the following pairs of connections were compared: LVP1 and LVP3, LVP1 and LVP5 and, finally, LVP2 and LVP4 (see figure 8).
The analysis of the curves presented in figure 8 indicates that the connections constituted of concrete with lower compressive strength are less rigid, i.e. they present greater relative rotation for the same actuating moment. To a certain degree, this finding was expected, given the correlation between concrete compressive strength and concrete tensile strength – a parameter which affects the values for the cracking moment and the effective moment of inertia of the beam (physical nonlinearity). These parameters of concrete strength are also important in the bond-slippage behavior of the beam reinforcement, which can generate relative rotation by bar slippage.

Figure 8 – Experimental moment-rotation curves: analysis of compressive strength of concrete



2.6 Analysis of the influence of joint transverse reinforcement

In order to analyze the influence of joint transverse reinforcement on the moment-rotation curve, connections constituted by concrete with approximately equal compressive strength were compared. Thus, the following sets of connections were compared: LVP3 (23.89MPa), LVP4 (24.62MPa) and LVP5 (25.91 MPa); and LVP1 (40.43MPa) and LVP2 (44.18MPa), despite the 9.2% difference in compressive strength.
For these comparisons, the results from the first stage of loading, which were common to all the connections, were used.
The comparative graphs in figure 8 for connections LVP3, LVP4 and LVP5 indicate that the amount of joint transverse reinforcement did not exert significant influence on the relative rotations for this stress level, whose order of greatness can be compared to that reached in the service range.

Table 2 – Global behavior at stage 2 (negative actions) – Specimens LVP3 and LVP4

Cycle	Load (kN)	Moment (kN.cm)	LVP3 Beam-column relative rotation (rad)	LVP4 Beam-column relative rotation (rad)
1	70	10850	0,0041	0,0046
2	70	10850	0,0045	0,0050
3	70	10850	0,0047	0,0052
4	80	12400	0,0057	0,0066
5	80	12400	0,0064	0,0083

The tendency of the curves presented for connections LVP1 and LVP2 in figure 8 and the slight difference in the values of relative rotations for these connections also reveal that the influence of transverse reinforcement was of little significance for this stress level.
On the other hand, for loadings nearer to the maximum capacity of the connection, the influence of the joint transverse reinforcement seems to be more significant, as indicated by the values presented in table 2. These observations indicate the need for further studies and comparison of experimental results, in order to better understand the influence of this variable on the percentage of restriction in monolithic reinforced concrete connections.

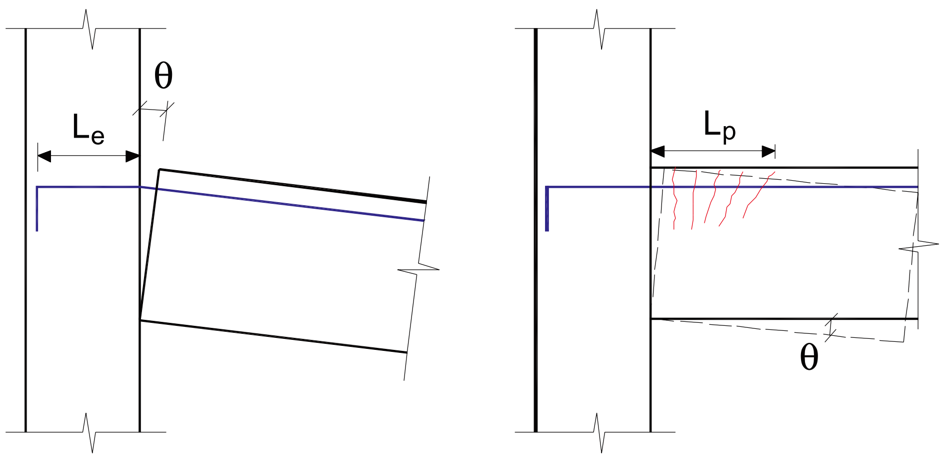
3. Theoretical model proposed by Ferreira [1]

According to the model proposed by Ferreira [1], the relative rotation between the cross sections of the beam and the column is a result of two mechanisms of deformation: i) elongation of beam tensile reinforcement correspondent to the anchorage length in the column; ii) bending deformations of the beam end in the region of discontinuity (D-region).
Ferreira [1] presents a general expression that allows the determination of the moment-rotation curve in beam-column connections until the yielding moment ($0 < M < M_y$), taking into account both deformation mechanisms presented in figure 9:

$$\theta = \left[\left(\frac{M}{E_{cs} I_{eq}}\right) \cdot L_p + \left(\frac{\sigma_s}{E_s d}\right) \cdot L_e\right] \cdot \left(\frac{M}{M_y}\right)^{0.5} \quad (4)$$

where
M_y is the yielding moment (yielding of the tensioned longitudinal reinforcement of the beam);

Figure 9 – Deformation mechanisms in exterior beam-column connections



Elongation of beam bars along the length Le

Bending deformations along the length Lp (D-region)

L_p is the length of the connection region, which is associated with the height of the beam section;
L_e is the embedded length, or the available anchorage length;
d is the effective depth of the beam;
E_s is the elasticity modulus of the steel;
E_{cs} is the secant modulus of elasticity of the concrete;
I_{eq} is the effective moment of inertia of the beam, considering the crack formation, determined by:

$$I_{eq} = \left(\frac{M_r}{M}\right)^3 \cdot I_I + \left[1 - \left(\frac{M_r}{M}\right)^3\right] \cdot I_{II} \quad (5)$$

where
M_r is the cracking moment;
I_I is the moment of inertia of the uncracked transformed section of the beam;
I_{II} is the moment of inertia of the cracked transformed section of the beam;
σ_s is the stress on the tensioned reinforcement of the beam, which

$$\sigma_s = \frac{M}{A_s z_{eq}} \quad (6)$$

can be obtained by:
A_s is the area of tensioned reinforcement;
z_{eq} is the lever arm between the tension and compression forces, determined by:

$$z_{eq} = d - \frac{1}{3}\left[x_I \cdot \left[1 - \left(\frac{M}{M_y}\right)^{0.5}\right] + x_{II} \cdot \left(\frac{M}{M_y}\right)^{0.5}\right] \quad (7)$$

where x_I and x_{II} are the depth of the neutral axis in the uncracked and cracked transformed section of the beam, respectively.

3.1 Comparison with experimental results of Alva [14]

Figure 10 shows the comparison between the experimental results obtained by Alva [14] and the results obtained with the theoretical model proposed by Ferreira [1]. For all connections, the value of the embedded length L_e was assumed to be 27.2 cm. The length of the connection region was assumed to be the value of the effective depth of the beam (L_p = 36.6 cm).
A comparative analysis of the graphs in figure 10 reveals that, in general, the model proposed by Ferreira [1] provides quite satisfactory results in the representation of the connection moment-rotation curve.
However, for connections LVP3, LVP4 and LVP5, the theoretical model gave a more rigid response than the experimental response. This may be due to a lower concrete compressive strength when compared to connections LVP1 and LVP2. In the first loading stage, although the same force was applied to all connections, the mechanical deterioration in the region of the beam-column joint was greater in the connections with lower concrete compressive strength. This increased deterioration affected the anchorage conditions of the longitudinal bars of the beam, and, consequently, exerted influence on the moment-rotation curves.
A deformation mechanism not contemplated in the model proposed by Ferreira [1] is the slippage of the longitudinal bars of the beam in the region of anchorage length. Consideration of this mechanism may likely improve the results of the theoretical-analytical model presented here.

3.2 Numerical examples: evaluation of the coefficient of partial restriction in monolithic beam-column connections using the theoretical model.

Given the good correlation between the theoretical results of the Ferreira model [1] and the experimental results presented in this paper, a series of numerical simulations of monolithic exterior beam-column connections was carried out. The main objective of the simulations was to quantitatively evaluate the percentage of partial restriction achieved in these connections with the Ferreira model [1], considering spans and structural materials commonly employed in buildings of reinforced concrete.
Reinforced concrete beams with a 20cm x 40cm rectangular section, effective depth equal to 36.0 cm and spans of 4.0 m and 4.8 m were simulated in an attempt to represent typical values used for the relation between the section depth and the span of the beam (1/12 to 1/10). Class C20 and C25 concrete were utilized as well as CA-50 steel for the longitudinal reinforcement.
Columns with dimensions between 30 cm and 80 cm in the direction of the beam axis were used in the simulations, with embedded lengths (L_e) between 27 cm and 77 cm (allowing a reinforcement cover of 3.0 cm). The longitudinal reinforcements used in the example are between the minimum value prescribed by the Brazilian Code, ABNT NBR 6118 [2] and the value that corresponds to the limit between domains 3 and 4 of the Ultimate Limit State in terms of normal stresses, according to ABNT NBR 6118 [2]. In a real design situation, the longitudinal reinforcements could be estimated from a linear analysis of rigid joints.
To facilitate the identification of the variable parameters in these simulations, the beam-column connections were divided into four groups, as shown in table 3.
Figure 11 illustrates the variation in the coefficient of partial restriction as a function of the increase in the rate of tensioned beam reinforcement, represented by the mechanical rate, in order to take

$$\omega = \frac{A_s f_y}{A_c f_c} \quad (8)$$

into account the strength of the concrete and steel materials:
where
f_y is the yield strength of the steel;
A_c is the concrete gross section area;
f_c is the concrete compressive strength.

In evaluating the partial restriction coefficient, design values for material strength in the Ultimate Limit State were considered. Thus, the material partial safety factors were equal to 1.4 for concrete and 1.15 for steel in the reinforcements.
The immediate conclusion upon analysis of figure 11 is that the percentage of restriction achieved in the connections increases with the increase in the rate of tensioned longitudinal reinforcement. Moreover, it can be seen that the partial restriction coefficient tends to reach a maximum value. In all curves, this maximum value corresponds to design situations at the limit between domains 3 and 4. The maximum values were between 0.83 and 0.87.

It is important to note that the Brazilian Code (NBR 6118 item 14.6.4.3) prescribes that in regions of support of beams or connections, the beam sections should be dimensioned for the Ultimate Limit State, respecting ductility conditions. The partial restriction coefficients for the rates of reinforcement corresponding to the condition of ductility (in this case, for a ratio of x/d equal to 0.5) gave values of between 0.73 and 0.82 for beams with spans of 4.0 m and values of 0.76 and 0.85 for beams with spans of 4.8 m, within the intervals of lengths L_e utilized and of the two classes of concrete compressive strength chosen.

Figure 10 – Moment-rotation curves: experimental vs. proposed model by Ferreira (1)

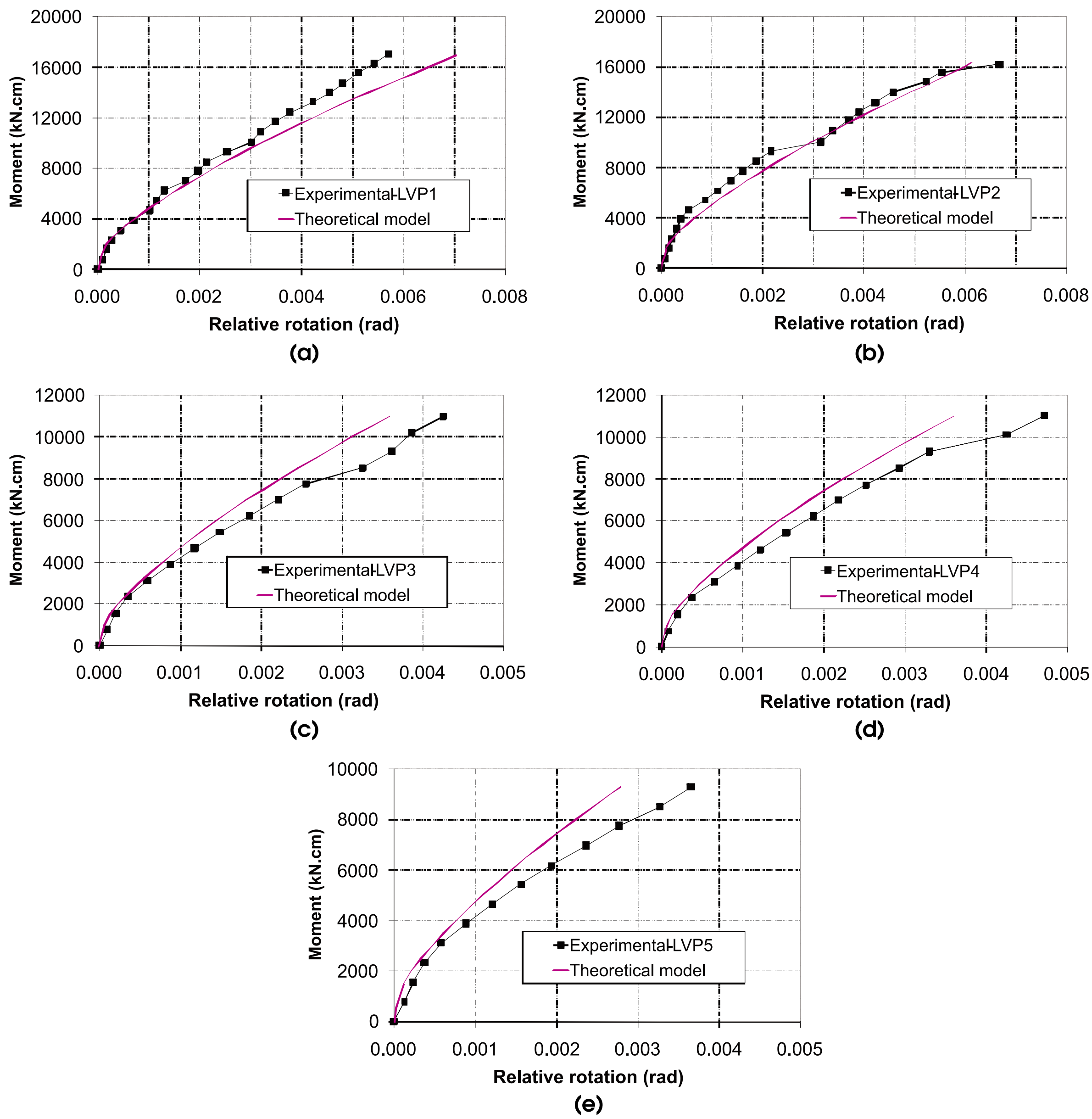
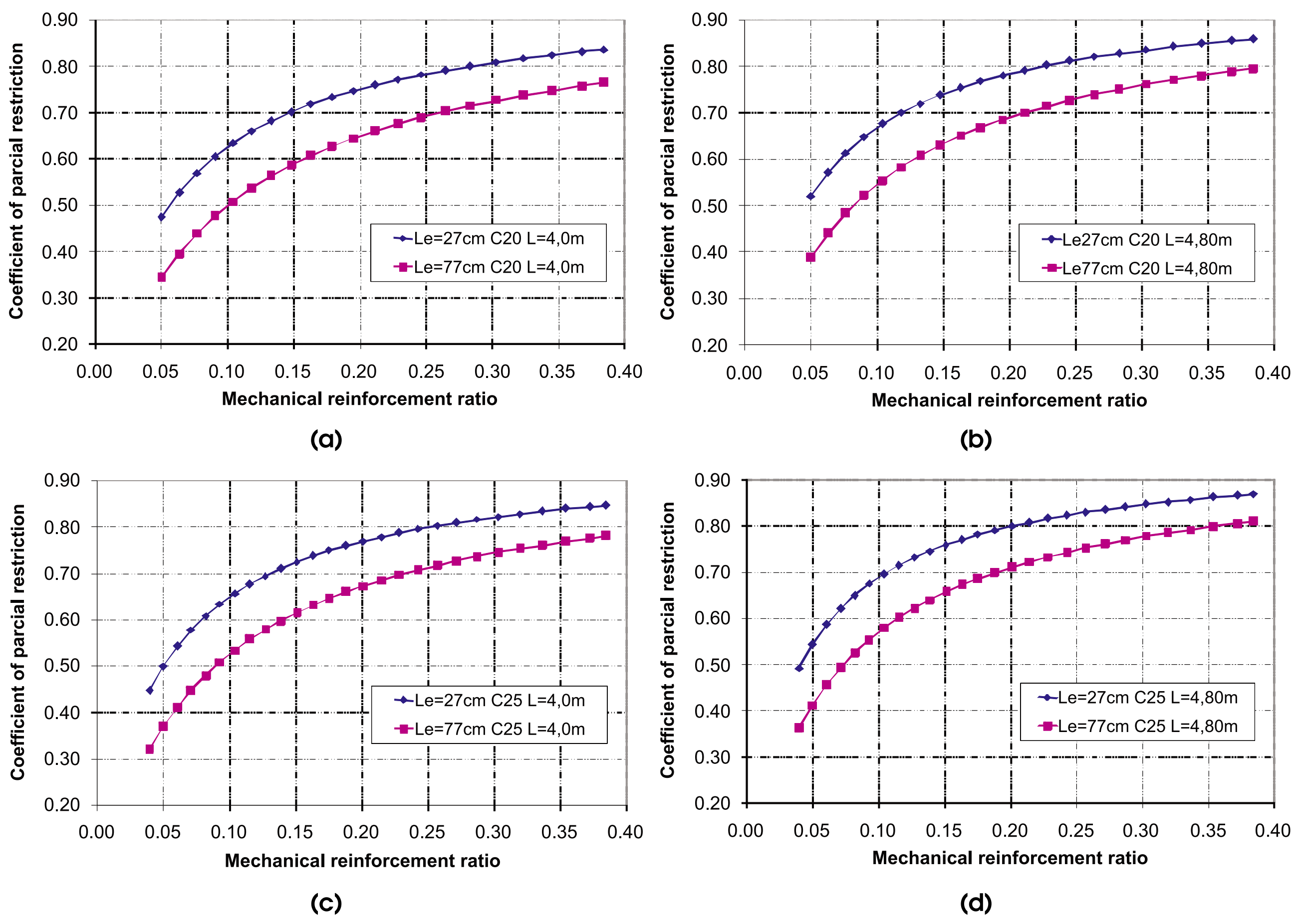


Table 3 – Groups of connections analyzed in numerical simulations

Group	Span of beam	Concrete	Column dimensions in the direction of the beam axis	Tensile reinforcement of the beam
1	4,00 m	C20	Variable: 30cm to 80cm	Minimum area to that corresponds to the limit between domains 3-4
2	4,00 m	C25	Variable: 30cm to 80cm	Minimum area to that corresponds to the limit between domains 3-4
3	4,80 m	C20	Variable: 30cm to 80cm	Minimum area to that corresponds to the limit between domains 3-4
4	4,80 m	C25	Variable: 30cm to 80cm	Minimum area to that corresponds to the limit between domains 3-4

Figure 11 – Coefficient of partial restriction: connections of the groups 1 and 4



4. Conclusions

Based on the experimental analysis of the 5 monolithic reinforced concrete beam-column connections tested by Alva [14], the following affirmations can by mentioned:

- The concrete compressive strength of the beam and column significantly influenced the moment-rotation curve of the connections. In fact, the concrete compressive strength is an important parameter in the mechanical behavior of beam-column joints, the region where beam reinforcements are inserted. In addition, it should be underlined that both concrete tensile strength (a property that is closely linked to cracking) and the bond-slippage behavior of the longitudinal reinforcement are related to the concrete compressive strength.

Comparing the experimental results presented in this paper and in Ferreira [7] with the theoretical results of the model proposed by Ferreira [1], the following affirmations can be mentioned:

- The theoretical model demonstrates good potential for the obtainment of relative moment rotation curves in monolithic reinforced concrete beam-column connections, which are useful in the evaluation of the percentage of partial restriction and in the refinement of the structural analysis.
- The theoretical response of the analytical model could be improved by including the deformation mechanism associated with slippage of beam longitudinal reinforcements.
- The percentage of restriction achieved experimentally for connections LVP1 and LVP2 was of 82% and 77%, respectively. For connections LVP3 and LVP4, considering only moments close to 70 per cent of the yielding moment, the percentage of restriction achieved was of 78% and 76%, respectively. The percentages of restriction were calculated as a function of the end fixity factory, obtained from the secant stiffness of the connection. In this case, the test connections attempted to simulate a beam with span of 4.5 m.

The series of numerical examples developed in this study utilizing the theoretical model of Ferreira [1] allowed the following affirmations:

- The percentage of restriction achieved in the beam-column connections, evaluated by the partial restriction coefficient, increases with the increase in the rate of tensioned beam reinforcement and with the increase of concrete compressive strength.
- The maximum partial restriction coefficients did not exceed a value of 0.87. The maximum values for this coefficient correspond to beam dimensions at the limit between domains 3 and 4.
- Considering ductility conditions in supports in accordance with the Brazilian Code ABNT NBR 6118 [2] in the design of beams, the values for the partial restriction coefficients were between 0.73 and 0.85 for the highest limit allowed for the position of the neutral axis.
- The numerical examples serve as an orientation in terms of typical values for the partial restriction coefficient in monolithic reinforced concrete connections. In a real design situation, these coefficients could be obtained from an initial estimation, considering the connections as perfectly rigid (rigid joints) in the structural analysis.

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6. References

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