

Modeling, Simulation and Identification for Control of Tandem Cold Metal Rolling

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This paper describes a modeling procedure for tandem cold metal rolling, including the linearization step and system identification for control. The tandem cold rolling process is described by a mathematical model based on algebraic equations developed for control purposes and empirical relations. A state-space model is derived and detailed analyses in open loop are presented, concerning the sensitivity with regard to the variations in process parameters and results for the application of a new subspace identification method are compared with classical methodologies. Therefore, this work intends to be a contribution for developments in new control strategies for tandem cold rolling process that offer the potential to reduce the design efforts, the commissioning time and maintenance in rolling mills. The preliminary results obtained with this model have shown reasonable agreement with operational data presented at literature for industrial cold rolling process.

Keywords: *rolling mill control, tandem cold rolling, process simulation, mathematical model, system identification*

1. Introduction

The tandem cold rolling of metal is a highly complex engineering process, with interactions of the multiple process variables, non-linearity and time delays, both extremely dependents of the operating conditions and the rolling mill speed. In general, the rolling mill automation technology deal with the control problem by division into sub problems, where both the adjust thickness and the interstand tension are independent¹, decentralized strategy. This classical strategy is employed and the results produced are acceptable, but fundamentals studies^{2,3} show the possibility of important progresses in product quality by the use of advanced control strategies.

The process controller design is essentially based on an adequate process model for control. A model for control, differently from a model for prediction is not characterized by great math complexity. The aim is to achieve a model with a good similarity to the process variable in the operating point neighborhood, and that be easily implemented computationally. The dominant idea⁴⁻⁷ is the process linearization and the state space modeling practice, usually recommended for MIMO systems, multiple-input multiple-output system.

As well as the modeling and simulation, the identification of the process is an essential step in development of new controllers. In industrial applications, in general, there

are complex plants, petrochemical⁸, iron and steel, etc. In general models for these plants are sought for in the standard black box state space linear form, since the main use for these models is to enable the design of a predictive controller.

The subspace identification method (SIM) is a good candidate for performing identification of industrial plants, since this method is tailored for obtaining MIMO black box linear state space models. SIM's can deal with open or closed loop operation, and in this last case a decorrelation preprocessing of the data is necessary, for avoiding a biased model.

The SIM do not require the use of any initial parameter estimate, and the model structure, namely the number of states, is also obtained from the data, by means of a singular value analysis. Further details about the SIM and the several alternative implementations, the reader is referred to the literature⁹.

Identification of general industrial processes was considered¹⁰, where the subspace identification method called N4SID was compared to the PEM (Prediction Error Method), with respect to computational complexity and prediction error value. From the outset, it must be stressed that the SIM and PEM are not competing methods, but complementary ones. For instance, initial order and system parameter estimates can be obtained by SIM, and then be given to the PEM as initial conditions.

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This work aims at showing the deduction of a model, tailored for control, for a tandem cold metal rolling of 5-stands, the dynamic simulation of this process and results with application of SIMF⁸, a new subspace identification method implementation that will be employed as basis for the research and development in new controllers. The results can enable advances in search of better quality for rolled products.

The main contributions of this paper are: a) a linear model for 5-tandem cold metal rolling mill in state-space representation is proposed, where the output vector contains not only the output thickness and interstand tension, but also the roll force; b) this model offers the possibility of performing disturbance analysis in process variables, such as mill thickness input, strip input tension on stand 1 and strip output tension on stand 5; c) interstand time delay is considered for a linear approximation, by using 4 first order lags in cascade and d) a new subspace identification method, called SIMF, was employed to obtain a state space model from input/output data and the result compared with the classical N4SID method, providing better performance, since the user has better control over the design parameters, which can hardly be automated when the system order is high, as in the present case.

With the above consideration in mind, the paper is organized as follows. In Section 2, the cold rolling process is mathematically described by classical equations and a state-space representation is derived. Then, in Section 3 some aspects of subspace identification are introduced. The results are shown in Section 4, where simulations for 3 operating points are performed and compared with the literature for validation. A 10th order black-box model is identified via SIMF. Finally, the concluding remarks are presented in Section 5.

2. Tandem Cold Rolling Process

In the flat sheet production, the desired thickness is an important specification to achieve. Therefore the steel

strip is passed through several stands in a tandem cold mill. Figure 1 illustrates this operation. The multiple stands cause considerable interaction of thickness and interstand tension. These variables interaction and the necessity to reach the dimensional specifications of steel strip give rise to a complex control problem. In order to control this process, output thickness and interstand tension are defined as controlled outputs. A state-space representation is presented here and can be used in the development of new control strategies.

2.1. Process modeling

In this process, the passage of the strip through the individual pairs of work rolls results in successive thickness reductions by compressive force applied to material in a small region, the roll bite shown in Figure 2. In rolling processes special attention is provided to strip deformation on roll bite. The mathematic model of the process, in this work, is composed by a set of algebraic equations greatly applied in industrial areas¹.

The strip deformation occurs by the movement of the work rolls, caused by position and speed actuators that were treated by a first order dynamic with time constant adjusted with industrial practice and literature suggestions¹¹.

The equations that compose this mathematical model are introduced later and Table 1 contains a symbol description used in this modeling. Below, the subscripts *in* and *out* mean the parameters values in input or output stand, respectively, *0* indicates parameter in operating point and *i* is associated to stand *i*.

The deformation modeling in roll bite is described by the specific roll force calculation. It is force by strip width in $\text{kN}\cdot\text{mm}^{-1}$ and the forward slip calculation, the fractional increase in the speed of the strip exiting the roll bite area, dimensionless parameter.

The specific roll force is approximated by the following expression, where weighting coefficients¹ are employed.

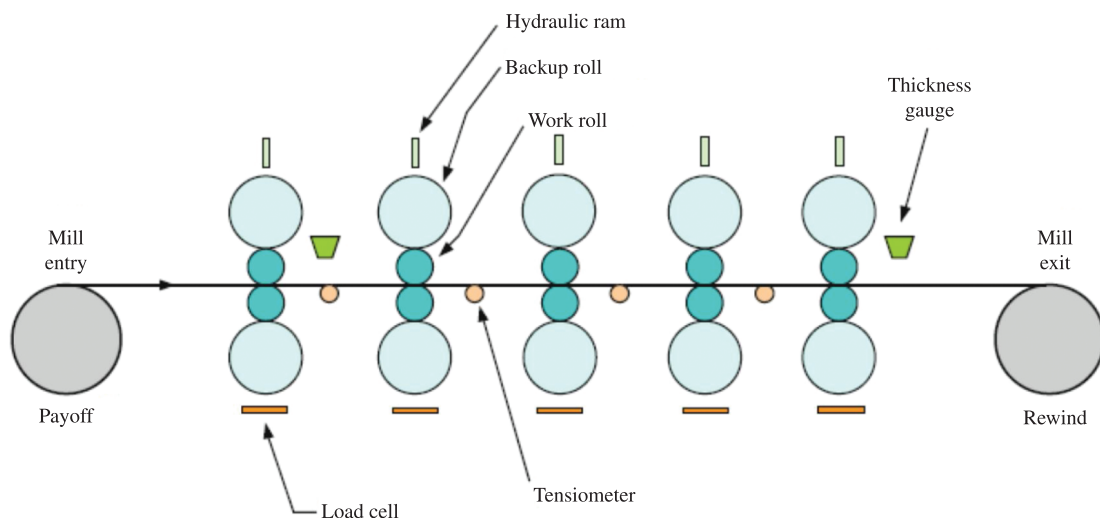


Figure 1. Typical 5-stand tandem cold mill¹¹.

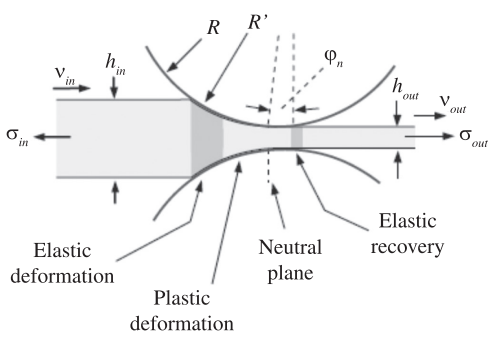


Figure 2. Schematic diagram for the material deformation in roll bite¹¹.

Table 1. Symbol description.

Symbol	Description
P	Specific roll force
k	Compressive yield stress
σ	Tension stress
R	Work roll radius
δ	Thickness reduction in stand
h	Thickness
μ	Coefficient of friction
H_a	Strip annealed thickness
E	Young's module of strip
f	Forward slip
V	Strip speed
L_0	Distance between adjacent mill stands
S	Roll gap
F	Total roll force
W	Strip width
M_m	Mill modulus
τ_d	Delay time
V_R	Roll speed
U_S	Roll gap control
U_V	Speed control
τ_S	Roll gap time constant
τ_V	Speed time constant
q, r	Auxiliar variables

$$P = \left((0,2k_{in} + 0,8k_{out}) - \left(\frac{2}{3}\sigma_{in} + \frac{1}{3}\sigma_{out} \right) \right) \sqrt{R'\delta} \left(1 + 4 \left(\sqrt{\frac{h_{out}}{h_{in}}} \exp \left(\frac{\mu\sqrt{R'\delta}}{0,28h_{in} + 0,72h_{out}} \right) - 1 \right) \right) \quad (1)$$

$$k_{in(out)} = 0,819 \left(0,002 + r_{in(out)} \right)^{0,27} \quad r_{in(out)} = \frac{H_a - h_{in(out)}}{H_a}$$

The hypothesis of the circular arc contact and the flattening work roll allows the use of Hitchcock's formula¹² for a deformed work roll radius, R' .

The forward slip f is approximated by

$$f = \left(\frac{\delta}{h_{out}} \right) \left(\frac{\frac{1}{2} \frac{h_{out}}{(0,28h_{in} + 0,72h_{out})} \sqrt{\frac{\delta}{R'}}}{4 \frac{h_{out}\delta}{(0,28h_{in} + 0,72h_{out})\mu R'}} + \frac{1}{4} \frac{h_{out} \left(\frac{\sigma_{out}}{k_{out}} - \frac{\sigma_{in}}{k_{in}} \right)}{\mu R' \sqrt{\frac{\delta}{R'}}} \right) \quad (2)$$

The interstand tension is obtained by application of the Hookes's law to a length of strip between successive stands, namely

$$\frac{\partial \sigma_{i,j+1}}{\partial t} \equiv \dot{\sigma}_{i,j+1} = \frac{E(V_{in,i+1} - V_{out,i})}{L_0}, \quad \sigma_{i,j+1}(0) = \sigma_{0,j,i+1} \quad (3)$$

A linear approximation for the mill stretch of the rolling stand is used for estimates of output thickness,

$$h_{out} = S + \frac{F}{M_m}, \quad \text{where } F = PW \quad (4)$$

The interstand time delay is approximated by

$$\tau_{d,i,j+1} = \frac{L_0}{V_{out,i}} \quad (5)$$

By considering the time delay between adjacent mill stands, the input thickness is modeled by

$$h_{in,i}(t) = h_{out,i-1}(t - \tau_{di,i-1}) \quad (6)$$

Taking into account the hypothesis of material continuity between adjacent stands and through the roll bite, (7) calculates all work roll peripheral speeds and strip input speeds on stands, once known the output strip speed of rolling mill.

$$V_{in} = V_{out} \left(\frac{h_{out}}{h_{in}} \right) \quad \text{where } V_{out} = V_R (f + 1) \quad (7)$$

The dynamic comportment of roll gap and roll speed are approximated by first orders relations,

$$\dot{S}_i = -\frac{S_i}{\tau_S} + \frac{U_{Si}}{\tau_S} \quad (8)$$

$$\dot{V}_{Ri} = -\frac{V_{Ri}}{\tau_V} + \frac{U_{Vi}}{\tau_V} \quad (9)$$

The set of equations described above are linearized and normalized. The linear approximation was performed with expansion on Taylor series. For normalization of any variable its value in the operating point was used, except in the case of the roll gap S , normalized by output thickness in operating point, $h_{out,i0}$. The following expression indicates a normalization procedure for a generic variable x , that is a fractional variation of a referred variable x in relation to its value in operating point x_0 .

$$\overline{\Delta x} = \frac{x - x_0}{x_0} \quad (10)$$

The calculation of coefficients in Equations 11 to 16 and 18 to 21 are detailed in another work¹³.

The roll force depends on the input and output thickness, input and output tension stress, friction, and material plastic characteristics. In this work, possibilities of variation in friction and material plastic characteristics will not be considered. The normalized and linearized variation of the specific roll force is,

$$\overline{\Delta P}_i = e_{2i} \overline{\Delta h}_{in,i} + (e_{1i} - 1) \overline{\Delta S}_i + e_{3i} \overline{\Delta \sigma}_{in,i} + e_{4i} \overline{\Delta \sigma}_{out,i} \quad (11)$$

The output thickness depends on the position of roll positioner actuator and specific roll force. In these conditions, the normalized and linearized variation in output thickness is obtained by following expression,

$$\overline{\Delta h}_{out,i} = e_{1i} \overline{\Delta S}_i + e_{2i} \overline{\Delta h}_{in,i} + e_{3i} \overline{\Delta \sigma}_{in,i} + e_{4i} \overline{\Delta \sigma}_{out,i} \quad (12)$$

Similarly to roll force, the forward slip will be considered dependent only of the input and output thickness and input and output tension stress.

For interstand tension between stand 1 and 2, the linearized and normalized equation is

$$\begin{aligned} \frac{d}{dt} \overline{\Delta \sigma}_{in,2} = & M_1 \overline{\Delta \sigma}_{in,2} + M_2 \overline{\Delta \sigma}_{in,3} + M_3 \overline{\Delta h}_{in,1} + M_4 \overline{\Delta V}_1 \\ & + M_5 \overline{\Delta S}_1 + M_6 \overline{\Delta h}_{in,2} + M_7 \overline{\Delta V}_2 + M_8 \overline{\Delta S}_2 + M_9 \overline{\Delta \sigma}_{in,1} \end{aligned} \quad (13)$$

For interstand tension between stand 2 and 3, the linearized and normalized equation is

$$\begin{aligned} \frac{d}{dt} \overline{\Delta \sigma}_{in,3} = & N_1 \overline{\Delta \sigma}_{in,2} + N_2 \overline{\Delta \sigma}_{in,3} + N_3 \overline{\Delta h}_{in,2} + N_4 \overline{\Delta V}_2 \\ & + N_5 \overline{\Delta S}_2 + N_6 \overline{\Delta h}_{in,3} + N_7 \overline{\Delta V}_3 + N_8 \overline{\Delta S}_3 + N_9 \overline{\Delta \sigma}_{in,4} \end{aligned} \quad (14)$$

For interstand tension between stand 3 and 4, the linearized and normalized equation is

$$\begin{aligned} \frac{d}{dt} \overline{\Delta \sigma}_{in,4} = & Q_1 \overline{\Delta \sigma}_{in,3} + Q_2 \overline{\Delta \sigma}_{in,4} + Q_3 \overline{\Delta \sigma}_{in,5} + Q_4 \overline{\Delta h}_{in,3} \\ & + Q_5 \overline{\Delta h}_{in,4} + Q_6 \overline{\Delta V}_3 + Q_7 \overline{\Delta V}_4 + Q_8 \overline{\Delta S}_3 + Q_9 \overline{\Delta S}_4 \end{aligned} \quad (15)$$

For interstand tension between stand 4 and 5, the linearized and normalized equation is

$$\begin{aligned} \frac{d}{dt} \overline{\Delta \sigma}_{in,5} = & R_1 \overline{\Delta \sigma}_{in,4} + R_2 \overline{\Delta \sigma}_{in,5} + R_3 \overline{\Delta h}_{in,4} + R_4 \overline{\Delta h}_{in,5} \\ & + R_5 \overline{\Delta V}_4 + R_6 \overline{\Delta V}_5 + R_7 \overline{\Delta S}_4 + R_8 \overline{\Delta S}_5 + R_9 \overline{\Delta \sigma}_{out,5} \end{aligned} \quad (16)$$

The normalized form for Equations 8 and 9, with $X = S$, for roll gap and $X = V$ for speed, is

$$\frac{d \overline{\Delta X}_i}{dt} = \frac{\overline{\Delta U}_{X_i}}{\tau_X} - \frac{\overline{\Delta X}_i}{\tau_X} \quad X_i(0) = 0 \quad (17)$$

For simulating the delay time between two successive stands in linear form, 4 first order lags in cascade were used. The time constant used in each lag is equal to quarter part of nominal delay time between referred stands, i.e.,

$$e^{-\tau_d s} \cong \frac{1}{\left(\frac{\tau_d}{4} s + 1\right)^4} \quad (18)$$

The input thickness in stand 2 is described by Equation 19, with auxiliary relations shown in Table 2.

$$\frac{d}{dt} \overline{\Delta h}_{in,2} = (\overline{\Delta q}_3 - \overline{\Delta h}_{in,2}) \left(\frac{4}{\tau_{d1-2}} \right) \quad (19)$$

The input thickness in stand 3 is described by Equation 20, with auxiliary relations shown in Table 2.

$$\frac{d}{dt} \overline{\Delta h}_{in,3} = (\overline{\Delta r}_3 - \overline{\Delta h}_{in,3}) \left(\frac{4}{\tau_{d2-3}} \right) \quad (20)$$

The Equations 21 and 22 are obtained of similar way¹³.

The input thickness in stand 4 is described by,

$$\frac{d}{dt} \overline{\Delta h}_{in,4} = (\overline{\Delta z}_3 - \overline{\Delta h}_{in,4}) \left(\frac{4}{\tau_{d3-4}} \right) \quad (21)$$

The input thickness in stand 5 is described by,

$$\frac{d}{dt} \overline{\Delta h}_{in,5} = (\overline{\Delta a}_3 - \overline{\Delta h}_{in,5}) \left(\frac{4}{\tau_{d4-5}} \right) \quad (22)$$

2.2. State-Space representation

The obtained linear model was represented in state-space form. This representation followed the format given by Equation 23.

Table 2. Auxiliary relations for interstand delay time (first stands).

Auxiliary relations for stand delay 1-2	Auxiliary relations for stand delay 2-3
$\frac{d}{dt} \overline{\Delta q}_1 = (\overline{\Delta h}_{out,1} - \overline{\Delta q}_1) \left(\frac{4}{\tau_{d1-2}} \right)$	$\frac{d}{dt} \overline{\Delta r}_1 = (\overline{\Delta h}_{out,2} - \overline{\Delta r}_1) \left(\frac{4}{\tau_{d2-3}} \right)$
$\frac{d}{dt} \overline{\Delta q}_2 = (\overline{\Delta q}_1 - \overline{\Delta q}_2) \left(\frac{4}{\tau_{d1-2}} \right)$	$\frac{d}{dt} \overline{\Delta r}_2 = (\overline{\Delta r}_1 - \overline{\Delta r}_2) \left(\frac{4}{\tau_{d2-3}} \right)$
$\frac{d}{dt} \overline{\Delta q}_3 = (\overline{\Delta q}_2 - \overline{\Delta q}_3) \left(\frac{4}{\tau_{d1-2}} \right)$	$\frac{d}{dt} \overline{\Delta r}_3 = (\overline{\Delta r}_2 - \overline{\Delta r}_3) \left(\frac{4}{\tau_{d2-3}} \right)$

$$\begin{cases} \dot{x} = Ax + Bu + D_{in}d & X(0) = 0 \\ y = Cx + D_{out}d \end{cases} \quad (23)$$

The state vector x , the control input u , the disturbance vector d and the output vector y must be defined. Based on the dynamic linear equations showed in section 2.1, these vectors are

$$x = [\overline{\Delta\sigma_{in2}} \cdots \overline{\Delta\sigma_{in5}} \quad \overline{\Delta h_{in2}} \cdots \overline{\Delta h_{in5}} \quad \overline{\Delta q_1} \quad \overline{\Delta q_2} \quad \overline{\Delta q_3} \quad \overline{\Delta r_1} \quad \overline{\Delta r_2} \quad \overline{\Delta r_3} \quad \overline{\Delta z_1} \quad \overline{\Delta z_2} \quad \overline{\Delta z_3} \quad \overline{\Delta a_1} \quad \overline{\Delta a_2} \quad \overline{\Delta a_3} \quad \overline{\Delta V_1} \cdots \overline{\Delta V_5} \quad \overline{\Delta S_1} \cdots \overline{\Delta S_5}]^T \quad (24)$$

$$u = [\overline{\Delta U_{V_1}} \cdots \overline{\Delta U_{V_5}} \quad \overline{\Delta U_{S_1}} \cdots \overline{\Delta U_{S_5}}]^T \quad (25)$$

$$d = [\overline{\Delta h_{in,1}} \quad \overline{\Delta\sigma_{in,1}} \quad \overline{\Delta\sigma_{out,5}}]^T \quad (26)$$

$$y = [\overline{\Delta h_{out1}} \cdots \overline{\Delta h_{out5}} \quad \overline{\Delta\sigma_{out1}} \cdots \overline{\Delta\sigma_{out4}} \quad \overline{\Delta P_1} \cdots \overline{\Delta P_5}]^T \quad (27)$$

The system then have 30 states, 10 control inputs, 3 disturbance inputs and 14 outputs. Hence the system matrix, A , is 30×30 , the input matrix, B , is 30×10 , the output matrix, C , is 14×30 , the input disturbance matrix, D_{in} , is 30×3 and the output disturbance matrix, D_{out} is 14×3 .

3. Subspace Identification

System identification concerns structure and parameters determination from experimental data. As far as the structure is concerned, there are two possibilities: a) grey box: balance relations can be used to obtain the system mathematical model, regardless of the data, and b) black box: both structure and parameters are obtained from the experimental data.

When linear black box models are required, the Prediction Error Method (PEM) approaches has difficulty with the MIMO case, in the sense that inadequate selection of the model structure and parameterization can lead to numerical problems, due to poor identifiability¹⁴. Besides, PEM is based on recursive optimization, hence can converge to a local minimum.

An efficient alternative for black box MIMO systems identification are the SIM, which are based on the realization theory. This approach tackles adequately the 3 main difficulties arising in the identification of MIMO systems: a) there is no parameterization problem, for the model structure is also obtained from the experimental data; b) the solution is non-recursive, since it is based on the solution of 1 or more SVD (Singular Value Decomposition) problems, and c) there is no need for an initial state estimate. For some examples of SIM implementations, such as CVA¹⁵ (Canonical Variate Analysis), N4SID¹⁶ (Numerical algorithm for Subspace State-Space System Identification) and MOESP¹⁷ (Multivariable Output-Error State-Space) see Qin⁹.

For performing system identification, consider the state space discrete time model with n states, m outputs and l inputs, written in the innovation form, i.e.,

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{K}\mathbf{e}(t) \quad (28)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) + \mathbf{e}(t) \quad (29)$$

which can also be rewritten in the predictor form as

$$\mathbf{x}(t+1) = \overline{\mathbf{A}}\mathbf{x}(t) + \overline{\mathbf{B}}\mathbf{u}(t) + \mathbf{K}\mathbf{y}(t) \quad (30)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) + \mathbf{e}(t) \quad (31)$$

For closed loop identification, representation (30)-(31) is convenient, since $\overline{\mathbf{A}}$ is stable, even if the original dynamic matrix A in (28) is unstable. For further details, see Qin⁹ (2006).

The identification of (30)-(31) via a subspace method, in open or closed loop, can be carried out as in reference 8 and amounts to the estimation of the dimension of the state space, namely, the value of n , and all the corresponding 5 matrices $\{A, B, K, C, D\}$ in (28)-(29). The matrix K in (30) corresponds to the steady state Kalman Filter gain, hence the identification method is robust to noise.

4. Results and Discussion

The operating point definition is necessary for the determination of a linearized model. For the validation of this model, three typical operating conditions of 5-stand tandem mill were adopted^{1,2,11}. Comparisons of the results obtained by these authors with results of this work are presented. For each simulated case, a disturbance in input thickness of stand 1 was applied and their effects evaluated in the output thicknesses, roll forces and interstand tensions.

4.1. Operating conditions and characteristics of the rolling mill/strip

The operating conditions of rolling mill are defined by choice of the operating point. In Table 3a are defined the three cases considered and Table 3b also shows the characteristics of the mill and strip used in the simulation.

4.2. Validation of the Linear Model

The variations in output thicknesses, roll forces and interstand tensions were registered after the application of a step with amplitude 2 in input thickness of stand 1, which means to apply a variation of +2% in its parameter. The results were obtained for each operating point and Table 4 compares the results obtained in this work (Model), with the of Bryant, Geddes and Pittner¹¹. The values in Table 4 are the average of the results obtained in the 3 operating points.

In general the results were satisfactory, as shown in Table 4. For output thickness, in Bryant¹, a reference in mill control, the results obtained by the linear model shows a good consistency relative to the initial stands. With relation to roll force, there is consistency in the stand 1 and the results of linear model are better in comparison with the others authors to stand 5. Some differences were detected in output tension stress, fact occurred in all works.

Several facts can be singled out as responsible for the differences observed: (a) the utilization of models with algebraic nature and empirical calculation of roll force and forward slip¹, where simplicity is wished. The Bryant's model is widely used in control systems of industrial mills. There are models with better precision for this calculates¹⁸.

Some effects can be added to the model for improving the results, such as the material elasticity in roll bite and strain rate; (b) the system linearization: The linear model offers a good response when input variations leave the system varying around the $\pm 5\%$ of the operating point, which is case in this work; (c) the friction modeling: In Pittner's model⁸ the friction is dependent on draft-diameter rate in work roll, speed roll and frictional characteristics. In this work the friction is constant. Finally, (d) the time delay interstand was considered constant and non-dependent of output speed of the stand.

4.3. Open loop identification

Since the model has variables with different time and amplitude scales, and also large order, the data is bound to be badly conditioned. This can be seen in 2 ways: a) the singular value analysis indicated a model with lower order than the true one, and b) the prediction capability is sensitive to the SVD threshold employed for solving the ensuing LS problems.

In Figure 3 it is shown the performance for the output $Y(14)$, roll force in stand 5, where SIMF is the subspace

Table 3. Operating points and characteristics of the rolling mill and strip.

Parameter	Operating point n.º		
	1	2	3
h_{in1} (mm)	3.56	2.36	1.78
h_{out1} (mm)	2.95	2.01	1.22
h_{out2} (mm)	2.44	1.52	0.79
h_{out3} (mm)	2.01	1.22	0.56
h_{out4} (mm)	1.68	0.97	0.38
h_{out5} (mm)	1.58	0.91	0.36
σ_{in1} (kN.mm ⁻²)	0.0	0.0	0.0
σ_{in2} (kN.mm ⁻²)	0.080	0.103	0.111
σ_{in3} (kN.mm ⁻²)	0.078	0.126	0.132
σ_{in4} (kN.mm ⁻²)	0.057	0.096	0.132
σ_{in5} (kN.mm ⁻²)	0.055	0.060	0.085
σ_{out5} (kN.mm ⁻²)	0.028	0.028	0.028

(a)

Parameter	Value
R (mm)	292
M_m (kN.mm ⁻¹)	3921
L_0 (mm)	4318
W (mm)	914
H_a / h_{in1}	1.095
E (kN.mm ⁻²)	207
ν	0.3
μ	0.04

(b)

Table 4. +2% Step change in stand 1 input thickness.

Variable	Source	Fractional change in variable % (Steady-State)				
		Stand 1	Stand 2	Stand 3	Stand 4	Stand 5
Output thickness	Bryant	1.8	1.8	1.8	1.8	1.7
	Geddes	2.32	2.13	1.90	2.14	2.21
	Pittner/Simaan	2.40	2.38	2.39	2.31	2.42
	Model	1.702	1.47	1.448	1.132	1.122
Roll force	Bryant	2.0	1.3	0.9	0.5	0.8
	Geddes	3.23	2.27	1.30	1.23	1.68
	Pittner/Simaan	2.17	1.67	1.35	1.09	1.57
	Model	1.702	1.47	1.448	1.132	1.122
Output tension stress	Bryant	-trace	-trace	+trace	-trace	-
	Geddes	+trace	1.7	4.6	-2.4	-
	Pittner/Simaan	1.2	0.7	0.4	8.8	-
	Model	-5.15	-5.24	-4.59	-5.14	-

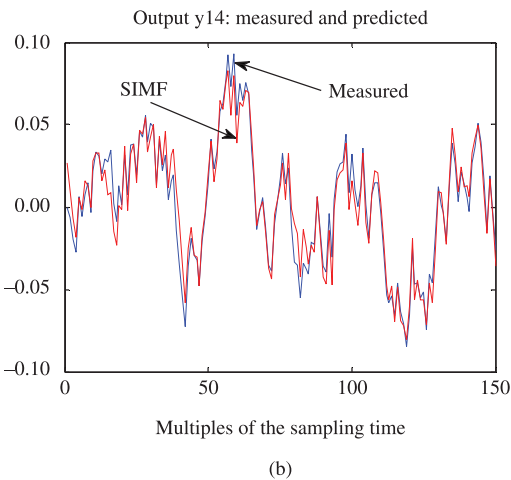
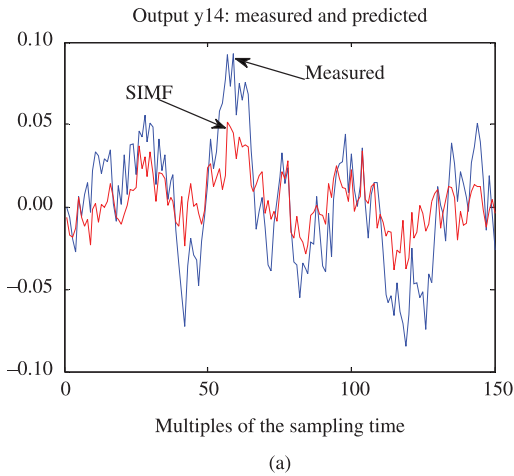


Figure 3. Output Y(14), roll force in stand 5: measured and predicted, for $\text{tol}=1\text{e-}9$ (a) and $1\text{e-}6$ (b), for 10 states.

identification method employed in this work, by considering a black-box model with 10 states.

From Appendix 1 it is concluded that the *SVD threshold selection* is critical for performance.

This occurs to all the outputs, as can be seen from Appendix 1, where the *Theil* coefficients are shown.

The ideal value for UC is 1, which means zero prediction error, and from the first table in Appendix 1, mainly for output 9, the result is far from the ideal one, when the tolerance is too small. In the second table, for a larger threshold, the results are adequate.

The bad data conditioning makes the automation of subspace methods a complicated task, as can be seen in Figure 4, where the SIMF is compared with the method N4SID-Matlab: even with the selection of CVA method and focus on prediction, good result is not obtained.

The dynamic matrices obtained by SIMF are shown in Appendix 2.

We now consider the case with 30 states. The result for output Y(14) is shown in Figure 5.

Compare the predicted value with those in Figure 3. The results for the correct order show very small prediction

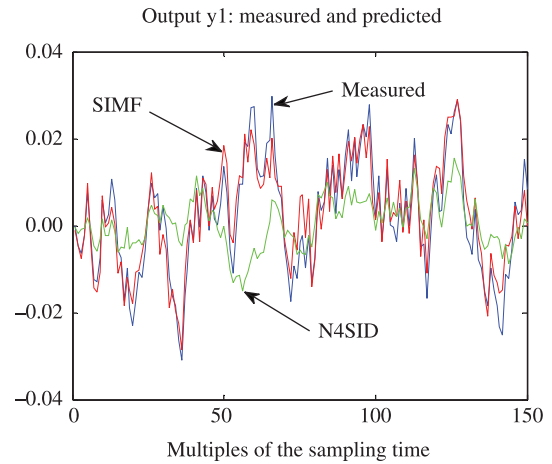


Figure 4. Performance comparison: SIMF and N4SID-Matlab.

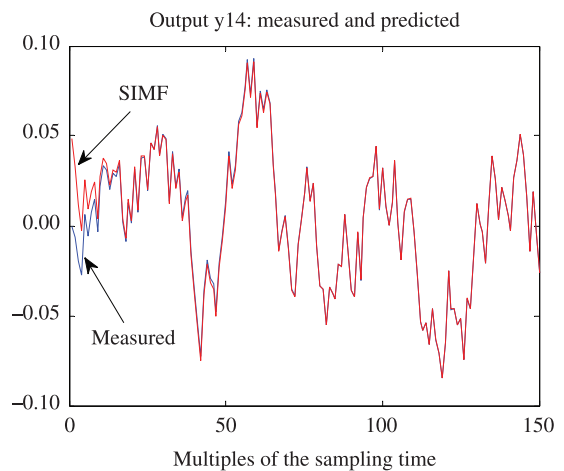


Figure 5. Output Y(14): measured and predicted values.

error, as expected, since the output data was generated with low noise content.

5. Conclusions

The model deduced here is tailored for control, useful for studies and researches on new control strategies applied to tandem mill. The obtained results are coherent and robust to design parameters. The variations due to the input disturbances have small amplitude and the linearized model seemed adequate.

The state-space modeling offered the correct treatment for a multivariable system, favoring the access to internal variables, to state variables, and not only to output variables.

The SIMF showed superior performance when compared with N4SID-MatLab, even with data bad-conditioned, since the user has control on some key design parameters, which cannot be easily automated, mainly when large order MIMO systems are considered.

This work has then proposed procedures for modeling and identification of rolling mills, which can enable more accurate simulation and the development of better control strategies. Experimental results, for both identification and control, will be presented elsewhere.

References

1. Bryant CF. *Automation of Tandem Mills*. London: British Iron Steel Institute; 1973.
2. Geddes EJM and Postlethwaite I. Improvements in Product Quality in Tandem Cold Rolling Using Robust Multivariable Control. *IEEE Transactions on Control Systems Technology*. 1998; 6(2):257-269. <http://dx.doi.org/10.1109/87.664192>
3. Hoshino I, Maekawa Y, Fujimoto T, Kimura H and Kimura H. Observer-based multivariable control of the aluminum cold tandem mill. *Automatica*. 1988; 24(6):741-754. [http://dx.doi.org/10.1016/0005-1098\(88\)90050-7](http://dx.doi.org/10.1016/0005-1098(88)90050-7)
4. Asano K and Morari M. Inteaction measure of tension-thickness control in tandem cold rolling. *Control Engineering Practice*. 1998; 6:1021-1027. [http://dx.doi.org/10.1016/S0967-0661\(98\)00101-4](http://dx.doi.org/10.1016/S0967-0661(98)00101-4)
5. Lee WH and Lee SR. Computer Simulation of Dynamic Characteristics of Tandem Cold Rolling Process. *KSME International Journal*. 1999; 13(8):616-624.
6. Pittner J, Samaras NS and Simaan MA. *A Simple Rolling Mill Model with Linear Quadratic Optimal Controller*. IEEE, IAS; 2002. p. 142-149.
7. Kim SS, Kim J-S, Yang S-Y, Lee B-R, Ahn K-K. H^∞ Control System for Tandem Cold Mills with Roll Eccentricity. *KSME International Journal*. 2004; 18(1):45-54.
8. Hemerly EM, Breckenfeld, R. P. Camolesi, V. J. Subspace Identification for Open and Closed Loop Petrochemical Plants. In: *Proceedings of 21st Brazilian Congress of Mechanical Engineering – ABCM*; 2011; Natal. Natal; 2011.
9. Qin SJ. An overview of subspace identification. *Computers & Chemical Engineering*. 2006; 30:1502-1513. <http://dx.doi.org/10.1016/j.compchemeng.2006.05.045>
10. Favoreel W, De Moor B and Van Overschee P. Subspace state space system identification for industrial processes. *Journal of Process Control*. 2000; 10:149-155. [http://dx.doi.org/10.1016/S0959-1524\(99\)00030-X](http://dx.doi.org/10.1016/S0959-1524(99)00030-X)
11. Pittner J and Simaan MA. *Tandem Cold Metal Rolling Mill Control - Using Practical Advanced Methods*. London: Springer-Verlag; 2011. <http://dx.doi.org/10.1007/978-0-85729-067-0>
12. Roberts WL. Mathematical models related to rolling force. In: Roberts WL. *Cold rolling of steel*. New York: Marcel Dekker; 1978.
13. Alves PG, Moreira LP and Castro JA. Dynamic Simulator for Control of Tandem Cold Metal Rolling. In: *Proceedings of 21st Brazilian Congress of Mechanical Engineering – ABCM*; 2011; Natal. Natal; 2011.
14. Katayama T. *Subspace Methods for System Identification*. London: Springer-Verlag; 2005.
15. Larimore WE. Canonical variate analysis in identification, filtering and adaptive control. In: *Proceedings of the 29-th Conference on Decision and Control*; 1990; Hawai. Hawai; 1990. p. 596-604.
16. Van Overschee P and De Moor B. N4SID: Subspace algorithms for identification of combined deterministic-stochastic systems. *Automatica*. 1994; 30:75-93. [http://dx.doi.org/10.1016/0005-1098\(94\)90230-5](http://dx.doi.org/10.1016/0005-1098(94)90230-5)
17. Verhaegen M and Dewilde P. Subspace model identification, part i: The output-error state-space model identification class of algorithms. *International Journal of Control*. 1992; 56:1187-1210. <http://dx.doi.org/10.1080/00207179208934363>
18. Alves PG and Hemerly EM. Modeling and numerical simulation of the deformation for calculations of set-up parameters in processes of cold rolling. In: *Proceedings of the 44th Rolling Seminar – Processes, Rolled and Coated products – ABM*; 2007; São Paulo. São Paulo; 2007.

Appendix 1. Theil coefficients for tol=1e-9 (a) and 1e-6 (b).

	U	UB	UV	UC
Output 1:	0.567580	0.007105	0.317475	0.675421
Output 2:	0.565649	0.000128	0.187938	0.811933
Output 3:	0.627186	0.023985	0.201655	0.774360
Output 4:	0.574484	0.006695	0.187044	0.806260
Output 5:	0.798843	0.019087	0.208902	0.772011
Output 6:	0.556228	0.002733	0.261293	0.735975
Output 7:	0.327011	0.001680	0.097975	0.900345
Output 8:	0.472826	0.010553	0.134884	0.854563
Output 9:	0.807412	0.001816	0.636091	0.362093
Output 10:	0.536586	0.005577	0.263294	0.725129
Output 11:	0.571612	0.001000	0.305553	0.693448
Output 12:	0.348290	0.005680	0.090454	0.903866
Output 13:	0.459018	0.001470	0.091679	0.906850
Output 14:	0.751597	0.001903	0.669019	0.329079

(a)

	U	UB	UV	UC
Output 1:	0.430993	0.000572	0.088007	0.911421
Output 2:	0.413674	0.012932	0.035204	0.951863
Output 3:	0.535990	0.015806	0.062827	0.921367
Output 4:	0.561794	0.012110	0.099453	0.888438
Output 5:	0.551276	0.019112	0.100683	0.880205
Output 6:	0.276944	0.003669	0.004303	0.992028
Output 7:	0.273381	0.006997	0.002109	0.990894
Output 8:	0.274753	0.001181	0.000073	0.998746
Output 9:	0.248868	0.000618	0.002862	0.996520
Output 10:	0.406679	0.000207	0.065911	0.933881
Output 11:	0.329182	0.004496	0.008018	0.987487
Output 12:	0.273092	0.002611	0.000640	0.996749
Output 13:	0.317150	0.001510	0.000633	0.997857
Output 14:	0.307267	0.000292	0.001970	0.997738

(b)

Appendix 2. Identified system matrices.

A=

0.7730	0.2783	0.0282	-0.1456	-0.0053	-0.1389	-0.1414	-0.0279	0.1422	-0.0735
0.2563	0.2699	-0.2303	0.0168	-0.1958	0.1923	-0.0407	-0.1531	-0.3017	0.1775
0.0019	-0.0360	0.7033	-0.1208	0.1569	-0.0102	0.1096	-0.0125	-0.0203	0.1281
-0.0066	-0.0561	-0.0319	0.7530	-0.0318	0.1492	-0.1881	-0.1354	0.0565	-0.0187
-0.0480	-0.1180	-0.0213	-0.0637	0.7215	0.0135	-0.1846	-0.0768	-0.0481	-0.0105
-0.0212	0.0998	0.1033	0.1081	-0.0123	0.6523	0.1224	0.1418	-0.0078	0.0034
0.0036	0.0103	0.0083	-0.0584	0.0020	0.1164	0.7455	-0.0674	0.0223	-0.0763
-0.0783	0.0955	0.0032	-0.2326	0.0831	0.3975	-0.3499	0.5402	0.1911	-0.1892
0.0130	-0.0016	0.0487	0.1821	-0.0900	-0.2038	0.1073	0.1413	0.8050	0.0726
-0.0035	0.0313	0.0251	-0.0522	0.0452	0.1593	-0.1323	-0.1153	0.1018	0.8515

B=

-3.8383	2.3731	-1.1001	7.2072	-3.7129	0.5050	2.2059	0.4906	0.5930	-1.2175
4.4073	-5.8420	4.1902	-7.0818	5.1284	-0.6186	-4.3770	3.1302	-3.6631	-0.0702
-5.4548	11.4234	-9.3562	2.7594	1.1733	0.8048	0.9276	2.5921	-3.5118	1.5182
-3.5584	-0.9888	7.5311	-3.4156	0.0809	2.7017	2.3230	3.2061	0.2691	1.0547
0.6295	-1.7874	3.0458	3.5189	-4.7136	3.3900	-2.7897	0.4308	0.1996	-2.2752
-1.2628	3.4944	-2.0955	5.1996	-5.3869	-0.0071	1.7499	-2.8746	-2.4579	-4.4674
2.3655	-4.6963	-0.0126	5.1324	-3.2961	-0.0172	-0.6176	3.5581	3.2820	-1.7671
1.4910	-4.5668	1.8657	-0.8076	0.4768	-1.3281	-0.1135	3.6240	1.0167	2.0741
-4.5495	1.5083	9.0428	-5.6531	-0.0415	-0.0645	-2.9071	-0.3951	-2.5686	-1.8867
-10.3212	10.2513	-0.1450	-4.2444	3.6312	-1.2112	0.0547	0.2708	3.0145	-0.5151

C=

-0.0008	-0.0024	0.0020	0.0050	0.0092	-0.0010	0.0009	-0.0036	-0.0002	-0.0032
-0.0037	-0.0017	-0.0014	0.0035	-0.0020	0.0036	-0.0027	-0.0005	-0.0050	-0.0002
-0.0034	-0.0003	0.0039	0.0017	-0.0002	-0.0007	0.0011	0.0023	0.0018	-0.0006
-0.0018	-0.0015	-0.0016	-0.0001	0.0016	-0.0001	0.0016	-0.0003	-0.0007	0.0031
-0.0008	-0.0013	0.0008	0.0006	-0.0004	-0.0019	-0.0021	0.0013	-0.0001	-0.0007
0.0143	0.0231	-0.0167	0.0960	-0.0758	0.0516	-0.0236	-0.0349	-0.0572	0.0235
0.0111	0.0013	0.0393	0.0906	-0.1098	-0.0136	0.1359	-0.0130	0.0966	-0.0595
0.0008	-0.0198	-0.0125	-0.0140	-0.0264	-0.0914	0.1537	-0.1337	-0.0597	0.0429
0.0136	-0.0308	-0.0113	0.0335	-0.0662	-0.2301	-0.0914	-0.0968	0.0098	-0.0248
0.0010	0.0038	-0.0032	-0.0130	-0.0146	-0.0000	-0.0009	0.0081	0.0025	0.0051
-0.0024	-0.0031	0.0039	-0.0199	0.0165	-0.0106	0.0013	0.0065	0.0119	0.0003
-0.0015	-0.0002	-0.0105	-0.0105	0.0139	0.0060	-0.0237	0.0019	-0.0137	0.0072
-0.0009	0.0038	0.0079	0.0024	-0.0001	0.0159	-0.0141	0.0169	0.0085	-0.0095
-0.0029	0.0054	-0.0028	-0.0037	0.0079	0.0247	0.0159	0.0012	0.0006	0.0095