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ORIGINAL ARTICLE

A simplified numerical and analytical study for assessing the seismic response of a gravity concrete lock

Um estudo numérico e analítico simplificado para a avaliação da resposta sísmica de uma eclusa em concreto gravidade

Neander Berto Mendes[a]
Lineu José Pedroso[a]
Paulo Marcelo Vieira Ribeiro[b]

[a]Universidade de Brasília – UnB, Departamento de Egenharia Civil e Ambiental, Programa de Pós-graduação em Estruturas e Construção Civil – PECC, Grupo de Dinâmica e Fluido-Estrutura– GDFE, Brasília, DF, Brasil
[b]Universidade Federal de Pernambuco – UFPE, Departamento de Egenharia Civil, Recife, PE, Brasil

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Abstract: This work presents the dynamic response of a lock subjected to the horizontal S0E component of the El Centro earthquake for empty and completely filled water chamber cases, by coupled fluid-structure analysis. Initially, the lock was studied by approximation, considering it similar to the case of a double piston coupled to a two-dimensional acoustic cavity (tank), representing a simplified analytical model of the fluid-structure problem. This analytical formulation can be compared with numerical results, in order to qualify the responses of the ultimate problem to be investigated. In all the analyses performed, modeling and numerical simulations were done using the finite element method (FEM), supported by the commercial software ANSYS.

Keywords: lock, seismic analysis, fluid-structure, finite elements, analytical solutions.

Resumo: Este trabalho apresenta a resposta dinâmica de uma eclusa submetida à componente horizontal S0E do terremoto ocorrido em El Centro, para os casos de câmara de água vazia e completamente cheia, por meio de uma análise acoplada fluido-estrutura. Inicialmente, estudou-se a eclusa de forma aproximada, considerando-a semelhante ao caso de um pistão duplo acoplado a uma cavidade acústica bidimensional (tanque), que representa um modelo analítico simplificado do problema fluido-estrutura. Esta formulação analítica pode ser comparada com resultados numéricos, com vistas a qualificar as respostas do problema final a ser investigado. Em todas as análises realizadas, o modelamento e as simulações numéricas foram feitos pelo método dos elementos finitos (MEF), com apoio do software comercial ANSYS.

Palavras-chave: eclusa, análise sísmica, fluido-estrutura, elementos finitos, soluções analíticas.

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1 INTRODUCTION

Locks are hydraulic structures used to transpose vessels in channels where there is sharp natural unevenness, such as rapids, or artificial unevenness, such as dams. The locks consist basically of a chamber, upstream and downstream gates, filling and emptying system and upstream and downstream accesses. The locks can be submitted to various types of actions, such as, for example, the seismic stresses. There is an interesting case in the event of dynamic interaction between the lock walls and the chamber fluid, during an earthquake. It is a fluid-structure phenomenon, where the movement of the structure produces stresses on the fluid, which in turn interacts, producing hydrodynamic pressures on the structure. If the fluid is considered to be acoustic, the problem will be treated as an acoustic-structural interaction,

Corresponding author: Neander Berto Mendes. E-mail: neanderberto@hotmail.com
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where the lock walls will function as a flexible wall and the chamber fluid will be associated with an acoustic compartment.

On locks, Ables [1] investigated the lock filling and emptying system in Bay Springs, Tennessee-Tombigbee waterway, in Mississippi, USA, using a reduced model. Novak and Cábelka [2] presented models in hydraulic engineering, their physical principles and applications thereof in design. Studies on characteristics and seismic responses of a lock through a 3-D dynamic analysis, taking into account the soil-structure interaction by the finite element method, were conducted by Ming [3]. The influence of soil stiffness, variation, and non-linearity on the dynamic characteristics and seismic responses of the lock system is discussed. The following conclusions are drawn: (1) the influence of soil-structure interaction on the dynamic characteristics and seismic responses of the lock is quite significant and (2) the lock system can generally be analyzed as a linearly elastic system, unless the lock is on soft ground or subjected to a strong earthquake. According to Novak [4], the headwaters of channeled rivers and waterways are usually surpassed by navigation locks. The main components of the locks are the lock gates, the crosspiece, the lock valves, and the filling systems. Trentini et al. [5] presented the result of the finite element modeling of one of the most complex structures of the Tucuruí unevenness transposition system, which is the lock downstream head 1. The modeling was carried out with the help of the ANSYS program, which allowed high quality and fidelity in the structure discretization. With 3D structure modeling to the smallest detail, it was possible to make both global and local assessments, allowing the verification of global stability and stress concentrations in the most critical regions of the structure, resulting from the interaction across all the elements and the foundation. Based on the structure of a lock, Lin and Yong-he [6] adopted a porch structural model and used the elastic foundation model of the ground behavior proposed by Winkler, characterizing the ground as a series of springs. A reverse iteration method is used to calculate the vibration frequency of the lock structure. With finite element programming, the Xinming lock in Shanghai is analyzed. The results are close to the measurements. Novak et al. [7] presented general studies on locks – locks with direct filling and indirect filling and emptying, the hydraulics of the locks, etc. Modeling is a vital part of all engineering projects and Novak et al. [8] provide guidance on simulations of hydraulic models, such as locks, and how they should be related to the prototype. By investigating the damage caused by the Wenchuan earthquake on the GuansongPeng lock, Xiao [9] summarized the main anti-earthquake measures in force in China. For reinforced concrete locks, when earthquake intensity was less than 8 degrees, a minor seismic damage can occur. The work also analyzes the specifications for the seismic design of hydraulic structures of the lock and points out possible serious secondary disasters, indicating that the anti-earthquake criteria are insufficient and should be changed in the future. Maltidis [10] investigated the seismic loading of hydraulic structures, with an emphasis on the dynamic pressures of water and ground and provided information for the design and analysis of navigation locks, considering the ground-structure and fluid-structure interactions.

In the context of studies on gravity structures, Oliveira [11] analyzed the stresses and the global stability of concrete gravity dams. Ribeiro [12] developed an analytical methodology for assessing the stress field in concrete gravity dams during earthquakes. Silva [13] studied the dynamic dam-reservoir interaction using analytical and numerical models. Melo [14] studied reservoir-gate coupling under seismic actions.

In the scope of the simplified model used here, Pedroso [15], [16] presented the complete development of the formulation for the uncoupled and coupled free vibrations of the 2D double-piston-acoustic cavity case. Souza [17] showed an application of finite element and finite differences methods to the fluid-structure interaction. Souza [18] developed a fluid-structure coupled analysis methodology for acoustic cavities with flexible walls. Ribeiro [19] developed analytical solutions for two-dimensional acoustic cavities applicable to the study of dam-reservoir dynamic interaction problems. Intartaglia et al. [20] studied the flexural vibrations of two thin beams coupled through a quiescent viscous fluid. Rezaiee-Pajand et al. [21] provided a brief bibliographic review on the dynamic behavior of liquid storage containers. The free vibration of two-dimensional, deformable rectangular tanks completely filled with a compressible fluid was analytically investigated, and exact solutions were obtained by them. A closed solution was also developed by them to assess the fundamental frequency of flexible 2D rectangular tanks filled with compressible liquid and an approximate formula to determine the corresponding pressure distribution on the tank walls was suggested.

As previously mentioned, "hydraulic structures" refer to a particular set of structures, such as dams, dikes, locks, quay walls, port entrance towers, etc. However, when referring to the navigation locks themselves, there is very limited number of technical-scientific engineering publications, in comparison with other hydraulic structures.

But the number of scientific contributions on the analysis of locks can be significantly increased if they are treated as retention structures and/or as tanks with fluids or water reservoirs. On the other hand, despite the large number of publications related to the analysis and design of those structures, addressed as such, there are few of these contributions related to the analysis of seismic fluid-structure behavior, if they are not considered similar to retention structures but are effectively treated as navigation locks. This is an aspect that highlights a significant gap in scientific activities in this field of knowledge, to which our work intends to pay a modest contribution.

Thus, the purpose of this work is to investigate the uncoupled and coupled fluid-structure (walls-water chamber) dynamic behavior of a navigation lock, when subjected to seismic action, under the conditions of the empty and completely filled water chamber. Initially, the lock was studied in an approximate way, considering it similar to the case of a double piston coupled to a two-dimensional acoustic cavity (tank), which represents a simplified analytical model of the coupled fluid-structure problem, but which sheds light on important aspects of the phenomenology involved, providing preliminary relevant information for the design and analysis of those structures. This analytical formulation is compared with numerical results, to qualify the answers to the ultimate problem that shall be investigated. In all the analyses performed, modeling and numerical simulations were done using the finite element method (FEM), with the support of the commercial software ANSYS. The analytical solution, although limited to predict the upper coupled modes of additional mass, is capable of satisfactorily responding to the first coupled modes and vibration frequencies, which represent the most significant part of the responses sought in the dynamic analysis of those structures.

2 MATHEMATICAL FORMULATION

The fluid-structure interaction problem under study is subject to some considerations: the solid has a linear elastic behavior, consisting of isotropic, homogeneous material, with a constant elasticity module and subjected to small displacements, when compared with the dimensions of the structure. It is also assumed that the fluid is invisible, compressible, and that the process is adiabatic.

The numerical analysis of the fluid-structure coupling by the finite element method is based on the acoustic formulation (U-P), with pressure being the variable in the fluid domain and displacement being the unknown factor of the structure.

The equation of the structure motion for the coupled problem is given by the Formula 1, whose vector of forces $\{F\}$ can be broken down into two other vectors: a vector of generic forces $\{F^E\}$ and another vector of forces at the interface $\{F^I\}$, which corresponds to the pressures of the fluid on the region contacting the solid.

The equation of solid motion is given by:

$$[M_s]\{\ddot{U}\}+[C_s]\{\dot{U}\}+[K_s]\{U\}=\{F^E\}+\{F^I\} \tag{1}$$

where:

$$[K_S]=\int_{V_s}[Bu]^T[C_s]\cdot[Bu]\cdot dV_S \text{ Solid stiffness matrix} \tag{2}$$

$$[M_S]=\int_{V_s}\rho[Nu]^T\cdot[Nu]\cdot dV_S \text{ Solid mass matrix} \tag{3}$$

where: $[Bu]$ = element shape functions derivatives matrix; $[C_s]$ = = elastic stiffness matrix or stress-strain matrix; ρ = specific mass of the structure; $[Nu]$ = structural element shape functions matrix.

The vector of forces at the interface $\{F^I\}$ applied in the region of the coupling is obtained by integrating the pressure on the fluid-structure interface surface, as follows:

$$\{F^I\}=\int_S\{N_u\}P\{n\}dS \tag{4}$$

where: $\{N_u\}$ = vector of shape functions used to discretize the displacement components u, v, w; $\{n\}$ = vector in the normal direction.

The displacement field of the solid element is given by the Equation 5:

$$U = \{N_u\}^T \{U_e\} \tag{5}$$

For the fluid element, the pressure at a certain point in the element can be expressed by:

$$P = \{N_p\}^T \{P_e\} \tag{6}$$

where the vector $U_e = [u_1 \quad v_1 \quad w_1 \quad u_2 \quad v_2 \quad w_2 \quad \dots \quad u_N \quad v_N \quad w_N]$ represents the nodal displacements, where ui, vi and wi correspond to the displacements of node i, in the x, y and z directions, respectively. While $P_e = [p_1 \quad p_2 \quad p_3 \quad \dots \quad p_N]$ represents the pressures and is associated with the nodal variables of the fluid.

Substituting Equation 6 in the force equation in the interface region (4), we obtain:

$$\left\{F^I\right\} = \int_S \left\{N_p\right\}\left\{N_u^T\right\}\{n\}\, dS \{P_e\} \tag{7}$$

It is observed that $\int_S \left\{N_p\right\}\left\{N_u^T\right\}\{n\}\, dS$ represent the fluid-structure matrix $\{FS\}$ for the coupled system, as also shown in Sousa [17] and in the summary of the method in the ANSYS program manual, then:

$$\left\{F_e^I\right\} = [FS]\{P_e\} \tag{8}$$

Substituting Equation 8 in Equation 1 the outcome is the structure motion equation for the coupled problem:

$$[M_s]\left\{\ddot{U}\right\} + [C_s]\{\dot{U}\} + [K_s]\{U\} - [FS]\{P_e\} = \left\{F^s\right\} \tag{9}$$

Applying the Galerkin method to the wave equation and using finite element discretization, the outcome is the dynamic equation for the acoustic cavity of the coupled problem, according to Sousa [17]).

$$\left[M_f\right]\left\{\ddot{P}\right\} + \left[C_f\right]\{\dot{P}\} + \left[K_f\right]\{P\} + \rho[FS]\left\{\ddot{U}\right\} = \{0\} \tag{10}$$

where:

$$\left[K_f\right] = \int_{\Omega_f} [Bp]^T \cdot [Bp] \cdot d\Omega_f \text{ Fluid stiffness matrix} \tag{11}$$

$$\left[M_f\right] = \int_{\Omega_f} [Np]^T \cdot [Np] \cdot d\Omega_f \text{ Fluid mass matrix} \tag{12}$$

$$\left[C_f\right] = \frac{1}{c}\int_{\Gamma 4} [Np]^T \cdot [Np] \cdot d\Gamma_4 \text{ Fluid damping matrix} \tag{13}$$

$$[FS]^T = \oint_{\Gamma 1} [Nu]^T \vec{n}.[Np].d\Gamma_1 \text{ Fluid-structure coupling matrix} \tag{14}$$

being $[Bp]$ = matrix operator (gradient) applied to the fluid element shape functions; $[Np]$ = fluid element shape function for pressure.

The shape functions used are for 2-D solid quadrilateral 4-node and axisymmetric elements, both for the structure and the fluid.

Equations 9 and 10 describe the problem of fluid-structure interaction and when placing them in the matrix form, the formulation presented in the ANSYS program is reproduced in an analogous way, as follows:

$$\begin{bmatrix} [M_s] & [0] \\ \rho[FS] & [M_f] \end{bmatrix} \begin{Bmatrix} [\ddot{U}] \\ [\ddot{P}] \end{Bmatrix} + \begin{bmatrix} [C_s] & [0] \\ [0] & [C_f] \end{bmatrix} \begin{Bmatrix} [\dot{U}] \\ [\dot{P}] \end{Bmatrix} + \begin{bmatrix} [K_s] & -[FS] \\ [0] & [K_f] \end{bmatrix} \begin{Bmatrix} [U] \\ [P] \end{Bmatrix} = \begin{Bmatrix} [F^E] \\ [0] \end{Bmatrix} \tag{15}$$

Or yet:

$$\boldsymbol{M}\ddot{X} + \boldsymbol{C}\dot{X} + \boldsymbol{K}X = \boldsymbol{F} \tag{16}$$

Equation 16 corresponds to the complete system for the fluid-structure interaction problem and represents the classic form of the equation of motion in forced vibrations.

3 APPROXIMATE ANALYTICAL SOLUTION FOR THE SIMPLIFIED LOCK MODEL

This development is based on the formulation of Pedroso [16] apud Souza [18], who presents the complete theoretical foundation for the case of the 2D double-piston-acoustic cavity problem in free vibrations. This study presents a consistent and reference methodology for understanding and comprehension of the phenomena associated with the fluid-structure coupling problem, in addition to providing an analytical formulation for the uncoupled structure, uncoupled fluid and coupled fluid-structure cases. Figure 1 shows the 2D double-piston-acoustic cavity system superimposed on the real lock-tank system, assumed in a 1st approach as a rigid-mobile structure.

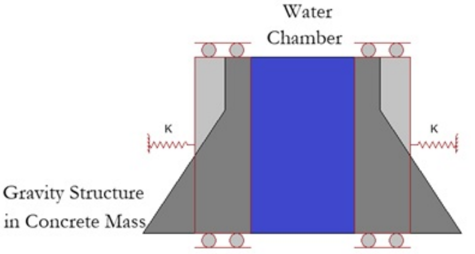

Figure 1. 2D double piston-acoustic cavity system superimposed on the supposed rigid-mobile lock-tank system

3.1 Uncoupled structure

In a first approach, the behavior of the rigid-mobile lock can be compared to a "piston" (straight beam) on an elastic base, represented in Figure 2a that works as if it was a beam on springs with elastic constant k_f separated by Δ.

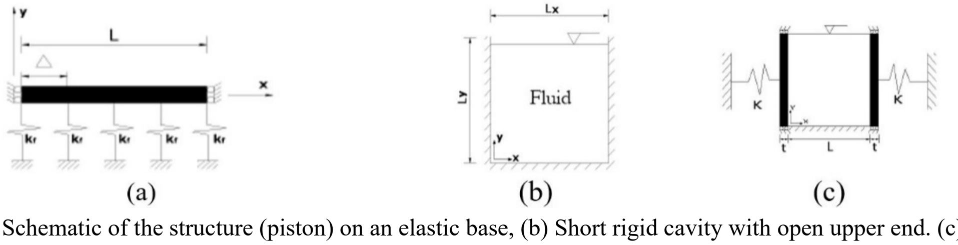

Figure 2. (a) Schematic of the structure (piston) on an elastic base, (b) Short rigid cavity with open upper end. (c) 2D double piston-cavity model with open boundary.

The analytical frequencies of the structure in Figure 2, in Hz, can be obtained by combining the dynamic behavior of a rectangular section beam on an elastic base - springs (m) - considering the bending (f) and shear (c) deformations. Equation 17 provides these frequencies for the uncoupled structure.

$$f_{m+f+c} = \sqrt{f_m^2 + f_{f+c}^2}; i = 1, 2, 3, ... \tag{17}$$

where,

$$f_m = f_{i=0} = (1/2\pi)\sqrt{\bar{k}/\bar{m}} \text{ = frequency of the springs;} \tag{18}$$

$$f_{f+c} = \frac{1}{(1/f_f)^2 + (1/f_c)^2} \text{, beams with comparable bending and shear deformations;} \tag{19}$$

$$f_f = (i^2\pi/2)\sqrt{EI/(\bar{m}L^4)} \text{ = beam bending frequencies;} \tag{20}$$

$$f_c = (i/2)\sqrt{KGA/(\bar{m}L^2)} \text{ = beam shear frequencies;} \tag{21}$$

$\bar{k} = K/L$ = spring stiffness per elastic base length unit; $\bar{m} = m_e/L$ = mass per beam length unit; $K = Nk_f$ = spring stiffness; N = number of springs; L = beam/elastic base length; me = mass of the structure; b = base of the beam cross-section (equal to 1m); E = Young's modulus of the material; ν = Poisson's ratio of the material; $I = bt^3/12$ = moment of inertia of the beam cross-section; EI = stiffness to the part bending; t = thickness/height of the beam cross-section; $K = 10(1+\nu)/(12+11\nu)$ = shear coefficient; $A = bt$ = cross-section area of the beam; $G = E/[2(1+\nu)]$ = transverse elasticity modulus of the material; KGA = transverse stiffness of the part.

The analytical frequencies of the structure, considering the normal deformations (n) of tension and compression are given by:

$$f_n = (i/2)\sqrt{EA/(\bar{m}L^2)}; i = 1, 2, 3, ... \tag{22}$$

where, EA = axial stiffness of the part.

3.2 Uncoupled cavity

To obtain the natural frequencies of a one-dimensional acoustic cavity, the transfer matrix method (MMT), Pedroso [15], is used, which is developed based on the wave equation, in terms of pressure:

$$\frac{\partial^2 p}{\partial x^2} + (\omega/c)^2 p = 0 \tag{23}$$

where, p = pressure; $\omega = 2\pi f$ = natural frequency, in rad/s; c = speed of sound.

The solution of Equation 23 is given by:

$$p(x) = A\cos(\omega x/c) + Bsen(\omega x/c) \tag{24}$$

With the acoustic flow being equal to

$$q = -\left(S/\left(i\omega\right)\right)\frac{\partial p}{\partial x} \tag{25}$$

where, S = cross-section area; i = imaginary unit.

Thus:

$$q\left(x\right) = \left(iS/c\right)\left[-Asen\left(\omega x/c\right) + B\cos\left(\omega x/c\right)\right] \tag{26}$$

For x = 0, at the input of the cavity, the result is:

$$A = p_e \tag{27}$$

$$B = \left(c/\left(iS\right)\right)q_e \tag{28}$$

And for x = L, at the output of the cavity, the result is:

$$p_s = p_e\cos\left(\omega L/c\right) + \left[c/\left(iS\right)\right]q_e sen\left(\omega L/c\right) \tag{29}$$

$$q_s = -\left(iS/c\right)p_e sen\left(\omega L/c\right) + q_e\cos\left(\omega L/c\right) \tag{30}$$

Where L = length of the cavity; pe = pressure at the input of the cavity; q_e = flow at the input of the cavity; p_s = pressure at the output of the cavity; q_s = flow at the output of the cavity.

Associating Equations 29 and 30 in a matrix form, we have:

$$\begin{bmatrix} p_s \\ q_s \end{bmatrix} = A\begin{bmatrix} p_e \\ q_e \end{bmatrix} \tag{31}$$

where,

$$A = \begin{bmatrix} \cos\left(\omega L/c\right) & \left(c/\left(iS\right)\right)sen\left(\omega L/c\right) \\ -\left(iS/c\right)sen\left(\omega L/c\right) & \cos\left(\omega L/c\right) \end{bmatrix} \tag{32}$$

this is the so-called transfer matrix.

In the case of open boundary, there is zero pressure and in the case of closed boundary, zero flow. Thus, the natural frequencies, in Hz, of closed-open and closed-closed 1D cavities filled with fluid are those given in Table 1.

Table 1. Natural Frequency of Acoustic Cavities

Cavity	Natural Frequency, f (Hz)	Cavity	Natural Frequency, f (Hz)
Closed-Open	$\frac{ci}{4L}, i = 1,3,5,...$	Closed-Closed	$\frac{ci}{2L}, i = 0,1,2,...$

c = speed of sound; L = width of the cavity.

For the rectangular geometry of the 2D cavity, the resulting frequency uncoupled from the cavity is obtained by combining the natural frequencies uncoupled in x and y (1D), which can be expressed as follows:

$$f = \sqrt{f_x^2 + f_y^2} \tag{33}$$

For the case closed-closed in x and closed-open in y, which are the boundary conditions of the model used in this article, shown in Figure 2b, we have:

$$f = c\sqrt{\left(\frac{i}{2L_x}\right)^2 + \left(\frac{j}{4L_y}\right)^2};\begin{cases} i = 0,1,2,\ldots \\ j = 1,3,5,\ldots \end{cases} \tag{34}$$

3.3 Coupled structure-cavity

This section presents the theoretical basis for the coupled free vibrations in the case of a "2D Double-Piston-Acoustic Cavity" (a simplified representation of the lock) for the first phase (F) mode of the pistons, which are characterized by the additional mass effect (fluid incompressibility) and for all the cavity mastery modes, which induce additional stiffness in the system.

The additional mass (MA) effect reduces the natural frequency of the coupled fluid-structure system in relation to the natural frequency of the corresponding uncoupled structural system, with the coupled mode being a structure dominant (DE) mode. The effect of additional stiffness (RA) increases the natural frequency of the coupled fluid-structure system in relation to the natural frequency of the corresponding uncoupled fluid system, with this coupled mode being a cavity dominant (DC) mode.

The referred case is characterized vertically because it has a boundary of the cavity opened to the atmosphere (p = 0) and the other rigid (bottom). The left and right ends have two rigid-mobile plates, supported by springs with elastic constant K. The structure has freedom of movement in the x-direction, as shown in Figure 2c.

Equation 35 allows, in a simplified way, the calculation of the first two fundamental frequencies of additional mass and incompressible fluid (i.e., with a compressibility parameter λ less than 1) for the two moving plates:

$$f = (1/2\pi)\sqrt{K/(m_e + m_{ad})} \tag{35}$$

on what: m_{ad} = additional mass of the chamber fluid; L = horizontal length of the cavity.

Equation 35 is for the first phase (F) mode, where each piston drags half the total mass of the fluid (m_f) in the chamber ($m_{ad} = m_f/2$). In this case, with the movement in phase opposition (OF) of the cavity walls, there is a confinement of the fluid without compressing it, which adds more mass to each piston than $m_f/2$. Equation 35 is still valid, but the additional mass needs to be evaluated by another process. As the fluid is incompressible, and there is the piston movement in OF, the movement of each piston in the opposite direction works as if more mass (virtual mass) was added to the opposite piston, an aspect not captured by the adopted double-piston model.

For the cavity domain modes, which generate additional stiffness in the coupled system, another formulation must be used, Pedroso [15]. Displacement of the xs piston imposes a mass flow source, given by:

$$q = i\omega\rho_f S x_S \tag{36}$$

Applying the coupled piston boundary condition in Equation 31 with the input being ($q_e = i\omega\rho_f S x_{Se}$) and the output being ($q_s = i\omega\rho_f S x_{Ss}$), we obtain:

$$p_e = \omega\rho_f c\left[x_{Se}\cot(\omega L/c) - x_{Ss} cossec(\omega L/c)\right] \tag{37}$$

$$p_s = \omega \rho_f c \left[x_{Se} cossec(\omega L / c) - x_{Ss} \cot(\omega L / c) \right] \tag{38}$$

The equations for the dynamic balance of the two pistons are given by:

$$\left(K_e - \omega^2 m_e\right) x_{Se} = - p_e S \tag{39}$$

$$\left(K_e - \omega^2 m_e\right) x_{Ss} = p_s S \tag{40}$$

where, K_e = stiffness of the structure; me = mass of the structure.

Observing that for pistons in phase, $x_{Se} = x_{Ss}$ and for pistons out of phase, $x_{Se} = -x_{Ss}$, Equation 39, results in:

$$\lambda^2 \left[\mu - (1/\lambda)(\cot\lambda \mp cossec\lambda) \right] = \alpha \tag{41}$$

where, $\mu = m_e / m_f$ = ratio between the mass of the structure and the mass of the fluid; $\alpha = K / K_f$ = ratio between the stiffness of the structure and the stiffness of the fluid; $\lambda = \omega L / c = 2\pi f L / c$ = compressibility parameter; $K_f = \rho_f c^2 S / L$ = cavity stiffness with fluid; L = width of the cavity; ρf = fluid density; S = cavity area.

Therefore, for the cavity domain modes, which generate additional stiffness, Equation 41 allows the calculation of the natural frequencies of the problem coupled in phase (F) (- signal), except the first root; and phase opposition (OF) (+ signal), Pedroso [16]. For the 1st mode in phase (F), $x_{Se} = x_{Ss}$, with λ <<< 1, Equation 41 results in Equation 35.

The resulting frequency coupled for the 2D double-piston-cavity case with the open upper end (Figure 2c) is the combination of the natural frequency coupled in the x-direction with the natural frequency uncoupled in the y-direction. It is given by Equation 42. Therefore, the analytical frequency of the coupled system for the case of Figure 2c can be expressed as follows, Pedroso [16]:

$$f = \sqrt{f_{acop,x}^2 + f_{des,y}^2} \tag{42}$$

with, for the closed-open case,

$$f_{des,y} = (j/4)(c/L);\ j = 1,3,5,\ldots \tag{43}$$

The larger frequencies and modes of additional mass (structure deformational modes) in this simplified analytical formulation cannot be calculated (the theory does not foresee this possibility, since the formalism used is for a 1D acoustic cavity problem, and the rigid-mobile structure); however, this case could be supported by a more comprehensive theory (additional mass dependent on the deformation mode structure, Silva [13], Ribeiro [19]), which will not be used in this work.

However, if the additional mass is calculated by the frequency reduction, obtained by the numerical analysis via FEM, and introduced in the frequency formulas of the structure, making it similar to a deformed one that produces a fluid displacement equivalent to the mass displaced by the coupled deformed one, we will have the analytical results marked by $\overline{A}$, for being analyzed by this process. It is a hybrid process in which the additional mass is calculated by reducing the frequencies of the structure by the presence of the fluid via FEM and then introduced in the analytical equation. However, it is important to clarify that the procedure indicated above was performed to prove the potential of the simplified model adopted, in capturing the coupled frequencies of the reservoir dominant, and with very good precision the 1st coupled frequency of the mode in phase, of additional mass of the structure, which is the one of interest for a previous evaluation of the designer. Therefore, it is not a matter of recommending a more complex and difficult calculation via FEM, to find the additional mass, and then inject it into the analytical model to obtain the modal frequencies and deformed of the relevant system, which would not be an adequate and rational procedure.

The additional mass is given by the following Equation:

$$\overline{m}_{ad} = \overline{m}\left[\left(f / f^{fe}\right)^2 - 1\right] \tag{44}$$

where f^{fe} is the numerical fluid- structure frequency.

The frequencies of additional mass (structure deformation modes) are calculated based on Equations 17, 18, 19, 20, 21 and 22, adding the additional mass portion due to the fluid per unit of beam length ($\overline{m}_{ad}$) to the mass portion of the structure per unit of beam length ($\overline{m}$). The fluid-structure (fe) spring (m), bending (f), shear (c) and normal (n) frequencies are, therefore, given by:

$$f_m^{fe} = \left(1/2\pi\right)\sqrt{k/\left(\overline{m}+\overline{m}_{ad}\right)} \tag{45}$$

$$f_f^{fe} = \left(i^2\pi/2\right)\sqrt{EI/\left[\left(\overline{m}+\overline{m}_{ad}\right)L^4\right]} \tag{46}$$

$$f_c^{fe} = \left(i/2\right)\sqrt{KGA/\left[\left(\overline{m}+\overline{m}_{ad}\right)L^2\right]} \tag{47}$$

$$f_n^{fe} = \left(i/2\right)\sqrt{EA/\left[\left(\overline{m}+\overline{m}_{ad}\right)L^2\right]} \tag{48}$$

4 RESULTS AND DISCUSSIONS

4.1 Description of the model and resulting actions

The results presented in this article are the outcome of a case study little addressed and explored in the technical literature: the problem of the dynamic behavior of a navigation lock in free and forced vibrations.

Due to the importance of the work and the rare studies found on this topic, this numerical simulation accompanied by a simplified solution is intended to provide designers with a simple way of approaching the issue.

The dimensions used in the concrete structure of the lock correspond to data from real life structures, namely: crest width, 8.00 m, base width, 34.00 m, total height, 55.00 m and height of the vertical wall in the dry part, 16.50 m. The width of the chamber is equal to 33.00 m. Figure 3a represents the left part of the lock-tank system, which is symmetrical in its dimensions.

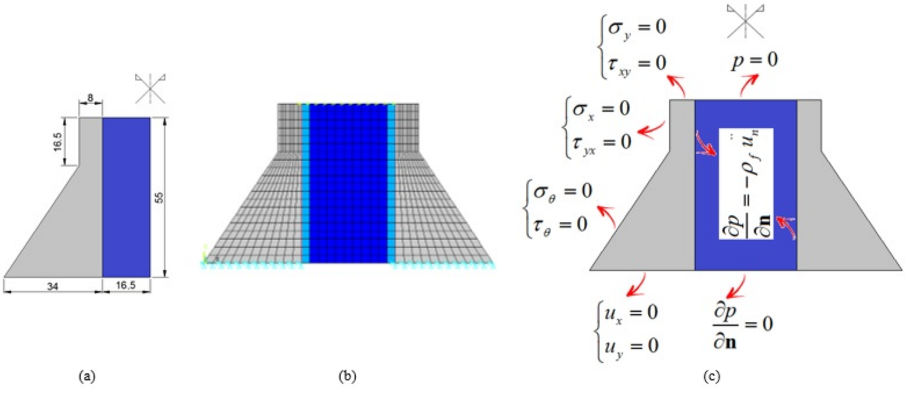

Figure 3. (a) Left part of the symmetrical lock-tank system. Measured in meters. (b) Model of the lock system. (c) Boundary conditions adopted in the lock model.

Numerical modeling using the finite element method (MEF) was performed using the ANSYS software. Figure 3b shows the discretized two-dimensional model of the complete system.

The physical properties adopted are: Young's modulus E = 25 GPa, Poisson's ratio $\nu = 0.25$ and specific mass $\rho = 2400$ kg/m^3 for gravity structures in concrete mass and speed of sound c = 1500 m/s and specific mass $\rho_f = 1000$ kg/m^3, for the fluid (water).

The boundary conditions adopted are: for the structures, clamped on the base and free on the top and dry side faces; for the cavity, closed at the base and open at the top, and the wall-chamber interface, fluid-structure condition. Figure 3c indicates the boundary conditions of the model.

The models were discretized so that the length of the elements at the edges of the structure and acoustic cavity in contacts, in general, would be 3.0 m (at the base of the gravity structures and water chamber), totaling 440 (220 + 220) PLANE182 elements for the structures and 220 FLUID29 elements for the cavity. The finite elements mentioned are from the library of the ANSYS program.

The ground movement selected for the dynamic analysis of the system was the S0E horizontal component of the ground movement recorded in El Centro, California, USA, during the earthquake verified in the Imperial Valley irrigation district, California, USA on May 18, 1940. The component of the ground movement and its peak acceleration are shown in Figure 4. The acceleration is given in g, where $g \approx 9.81$ m/s^2.

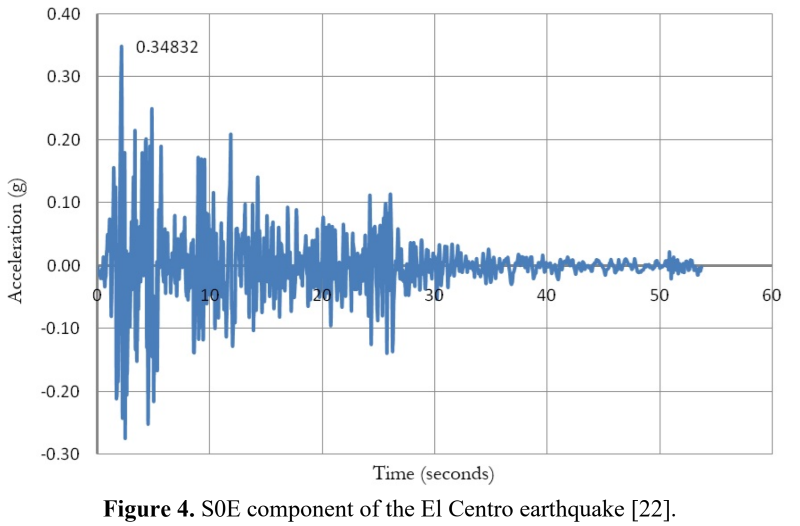

Figure 4. S0E component of the El Centro earthquake [22].

The natural frequencies, the free vibration modes and the time variations of the horizontal displacement at a point on the lock crest, where the maximum horizontal displacement and the envelopes of maximum normal vertical stress of tension and compression, due to the earthquake, are analyzed for the cases of empty and totally full water chamber.

In this work, for the numerical-computational structural analysis, transient dynamic analysis was used, which is a technique used to determine the dynamic response of a structure, under the action of any general time-dependent loads. Two methods of analyzing the transient response are available in ANSYS. We have applied the full method, which uses the complete matrices of the system to calculate the transient response (without matrix reduction).

4.2 Case studies

In this section, two cases that have similarities will be studied:

1st) Simplified lock and made comparable (modeled with the closest possible characteristics to the original lock) to a rigid-mobile structure (double-piston) of constant thickness (simplified method presented). 2nd) The Lock itself (actually modeled by the FEM).

Case 1 - Free Vibration of the Simplified Lock (Straight Thick Beam)

To qualify the results of the free vibrations of the lock system, a preliminary analytical-numerical study (2D double-piston-cavity) was carried out, considering the following dimensions: piston height, 55.00 m; piston thickness, 17.10 m (in order to maintain the same cross-section area as the gravity structure, 940.50 m^2) and cavity width, 33.00 m. The physical properties of the materials are the same as the lock.

The spring stiffness adopted is equal to 2.1 x 10^9 N/m. This value was reached by applying a horizontal unit load (P = 1N) at the end of a free-standing clamped vertical Timoshenko beam, with the physical and geometric properties of the piston (Figure 5a), obtained through analytical formula for beam deflection (Equation 49) the value of its horizontal displacement (w = 4.8 x 10^{-10} m) at that point (x = 0 m), resulting in a stiffness (K = P/w) of 2.1 x 10^9 N/m, so that the spring stiffness approximately represents the stiffness of the lock.

$$w(x) = P(L-x)/(KAG) - Px/(2EI)\left(L^2 - x^3/3\right) + PL^3/(3EI) \quad (49)$$

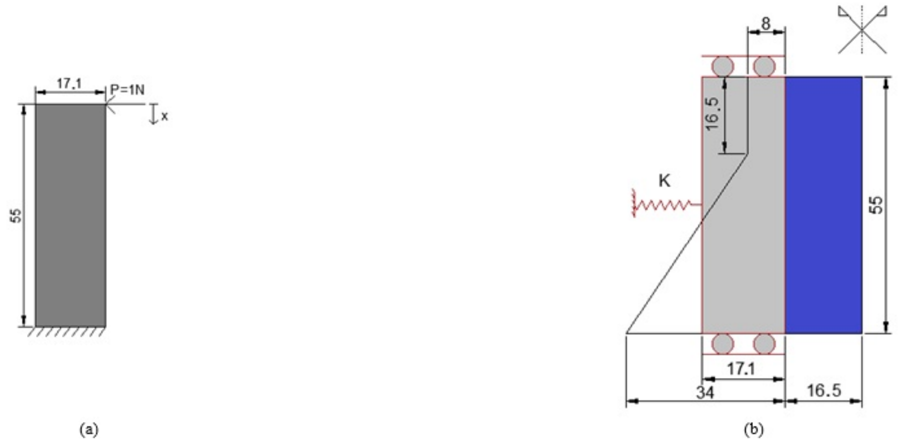

Figure 5. (a) Timoshenko beam clamped-free of unitary width, with its dimensions in meters. (b) Left part of the 2D double-piston-acoustic cavity system superimposed on the left part of the lock-tank system, with its dimensions. Both systems are symmetrical. Measured in meters.

Figure 5b shows the left part of the 2D double-piston-acoustic cavity system superimposing the left part of the lock-tank system, with its dimensions, both systems being symmetrical.

Figure 6 shows the first five modes of vibration of the uncoupled structure (a), of the uncoupled cavity (b) and the first ten modes of the 2D coupled double-piston-cavity system (c). The numerical and analytical results of the natural frequencies are presented, in Hz, the percentage difference between the analytical and numerical values in relation to the analytical values and the numerical modes of vibration obtained by the FEM via ANSYS, where i and j represent the modal indices, the sub-indices m, f, c and n mean spring, bending, shearing and normal, respectively, MA, additional mass, RA, additional stiffness, DE, dominant structure, DC, dominant cavity, F, phase and OF, opposed phase. The analysis by the MEF of the simplified system is carried out in order to compare these results with the analytical results obtained by the presented formulas.

Analyzing these results, there is good match between the analytical and numerical results with maximum differences of 2.31% for the third uncoupled structural mode, 0.65% for the fifth uncoupled fluid mode and 8.54% for the fifth coupled fluid-structure mode.

In the coupled case, among the presented modes that have analytical solution, mode 2 is the first mode, in phase (F), of the pistons (additional mass effect with fluid incompressibility ($\lambda_{numeric}$ = 0.58)), and the mode 5 is the first cavity domain and structure in opposed phase (OP) mode. Once the simplified analytical formulation has been validated, with the numerical model for these modes, it is possible to show the other modes (1, 3, 4, 6, 7 and 10) obtained by ANSYS to complement the sequence.

From the numerical values, it is observed that the coupled modes 1 and 2 are fundamental modes of additional mass with the incompressible fluid (λ = 0.39 and 0.58), at opposed phase (OF) and phase (F), reducing the first uncoupled frequency of the structure by 42.2% and 13.9%, respectively. Coupled modes 3 and 4 are modes of additional mass, dominant of the structure and compressible fluid (λ = 1.01 and 1.09), at opposed phase (OF) and phase (F) reducing the second uncoupled frequency of the structure by 16.6% and 9.84%, respectively. Mode 5 is an additional stiffness mode, caused by a certain compressibility of the fluid. It is a dominant mode of the cavity and the structure is opposed phase (OF), raising the first uncoupled frequency of the cavity by 72.1%. Coupled modes 6 and 7 are modes of additional mass, dominant of the structure and compressible fluid (λ = 2.73 and 2.97), at opposed phase (OF) and phase (F), reducing the third uncoupled frequency by 16.7% and 9.32%, respectively. Mode 8 is an additional stiffness mode, caused by the compressibility of the fluid. It is a dominant mode of the cavity and the structure is opposed phase (OF), raising the second uncoupled frequency of the cavity by 17.2%. Mode 9 is an additional stiffness mode, dominant cavity, with the structure in phase (F) following the deformed modes of the fluid, raising the third uncoupled frequency of the cavity by 10.6%. Coupled mode 10 is an additional mass mode, dominant of the structure and compressible fluid (λ = 4.04), normal, reducing the fourth uncoupled frequency of the structure by 0.29%.

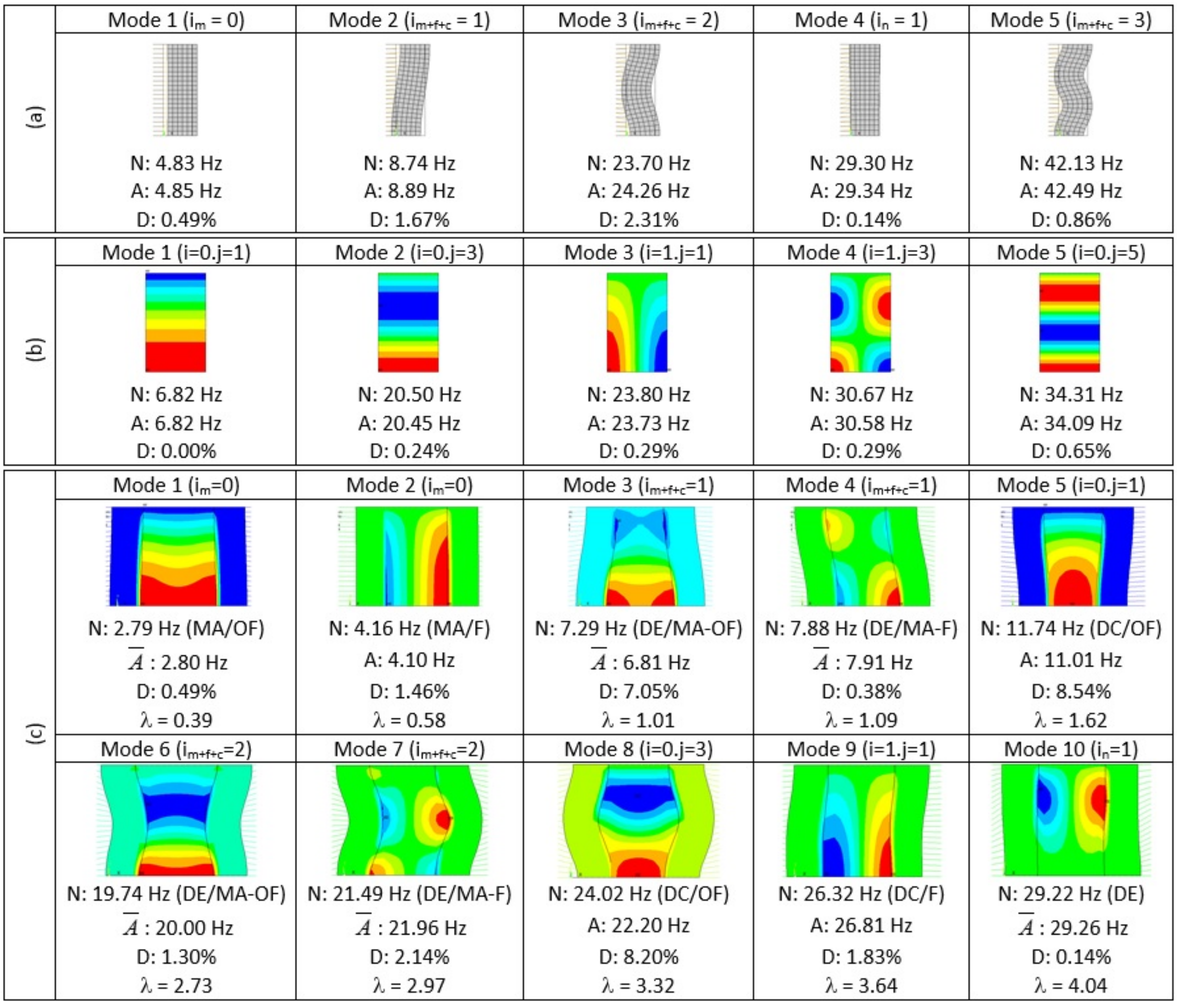

Figure 6. Natural numerical (N) and analytical (A) frequencies, in Hz, the percentage difference between the values (D) and the corresponding vibration modes of the 2D acoustic cavity piston case: (a) uncoupled structural system, (b) uncoupled fluid system and (c) coupled fluid-structure system. λ is the compressibility parameter. $\overline{A}$ is the coupled frequency obtained by the hybrid process.

As it can be seen in these results, the simplified analytical model can predict all the dominant cavity coupled modes (with additional stiffness and fluid compressibility - modes 5, 8 and 9) regardless of the deformed mode of the structure, which in these cases accompanies the deformed mode of the fluid. To make the explanations clearer, dominant modes (structure or cavity), are the modes of the media (solid and/or fluid) that control (dominate) the response, and the other medium adapts, fits, follows the deformed mode that predominates in the process. The coupled deformed modes corresponding to the simplified analytical formulation are not presented in this article but can be found in Pedroso [16], apud Souza [18].

Figure 6c depicts a greater number of coupled modes than in uncoupled cases (a) and (b), to show the effectiveness of the simplified formulation in predicting the coupled modes of the cavity (5, 8, 9) for additional stiffness. These dominant cavity modes reproduce the uncoupled cavity modes at a larger frequency ($f_1^{des} \rightarrow f_5^{acop}$, $f_2^{des} \rightarrow f_8^{acop}$, $f_3^{des} \rightarrow f_9^{acop}$). In this figure, the match between the uncoupled and coupled modal form can be seen, a fact that makes it possible to highlight the modal form mobilized by the structure in the coupled problem.

Case 2 - Free Vibrations of the Lock

Figure 7 shows the first five modes of vibration in this case. The numerical results of natural frequencies, in Hz, and the numerical modes of vibration obtained by the MEF via ANSYS are presented.

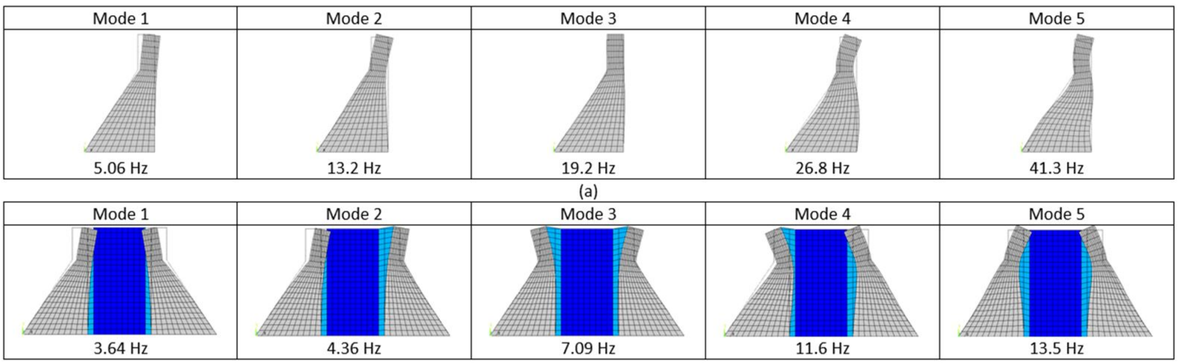

Figure 7. Vibration modes of the lock: (a) uncoupled structural system (left structure) and (b) coupled fluid-structure system.

Figure 8 shows a view with the first two modes of vibration of the double-piston system compared to those of the real life lock for the uncoupled structural case and the first four modes of vibration of the double-piston-acoustic cavity system compared with those of the lock-tank for the coupled fluid-structure case. The natural frequencies, in Hz, and the numerical modes of vibration obtained by the FEM via ANSYS are presented.

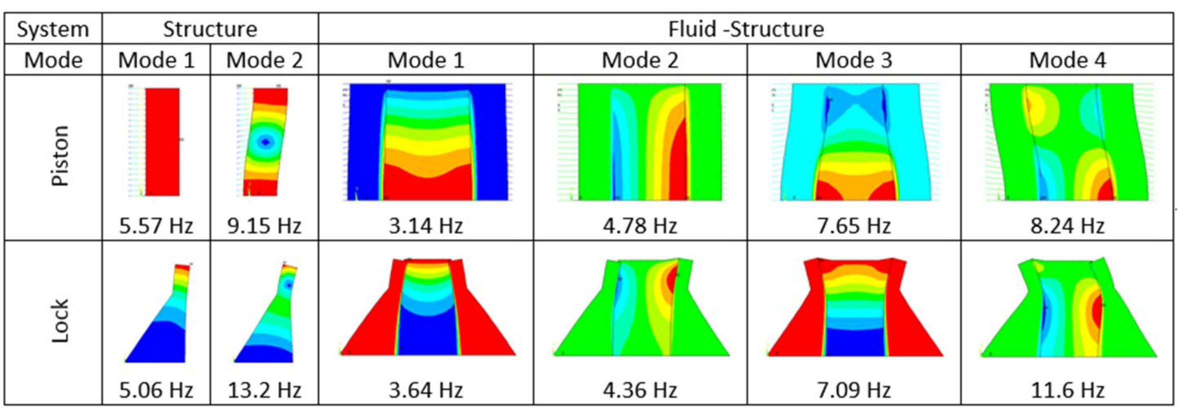

Figure 8. Natural frequencies, in Hz, and numerical vibration modes of the Piston compared to the lock.

The analytical study of the 2D double-piston-acoustic cavity system is done to show the relevance of the simplified model. Although a closed analytical formulation for the frequencies and modal deformations of the lock-tank system has not been presented, it is possible "as an approximation" (preliminary assessment), to use the 2D double-piston-acoustic cavity model. As it is a simplified and easy to apply model, it cannot be expected that it reproduces with great precision the numerical results obtained with a sophisticated program based on FEM. However, for the first frequencies (in particular the 1st one, the most important for this problem category), the simplified model presents results within the magnitude of 10%, which is one of the characteristics expected for a quick assessment on practical design problems in engineering, especially when a preliminary estimate of the case is sought. Thus, the methodology developed here becomes applicable to a preliminary study of the tank-lock problem.

Further analyzing the presented results, it is observed that the natural frequencies and the modes of vibration of the simplified model present good correlation with those obtained for the real life lock, concluding, therefore, that the two studied models do match.

Case 3 - Forced Excitation (Seismic) of the Lock

In this section, the dynamic response of the lock to a seismic request will be studied. In this case, the El Centro earthquake (component S0E) was adopted.

For the calculation of the parameters that relate frequencies to the damping rates, in the analysis in forced vibration with seismic load, $\zeta = 5\%$ was considered for $\omega_1 = 31.80$ rad/s and $\omega_2 = 82.94$ rad/s (1st and 2nd modes), resulting in Rayleigh's damping constants, $\alpha = 2.30$ and $\beta = 8.72 \times 10^{-4}$, for the uncoupled structural case and also $\zeta = 5\%$ for $\omega_1 = 22.87$ rad/s and $\omega_2 = 27.39$ rad/s (1st and 2nd modes), resulting in Rayleigh's damping constants, $\alpha = 1.25$ and $\beta = 1.99 \times 10^{-3}$, for the coupled fluid-structure case.

Figure 9a shows the points chosen for observing responses in the seismic analysis of the lock. Figure 9b shows the relative horizontal displacement in time of the upper extreme point of the left side of the inner (wet) wall (Point A of Figure 9a) of the lock, for the uncoupled (a) and coupled (b) problem.

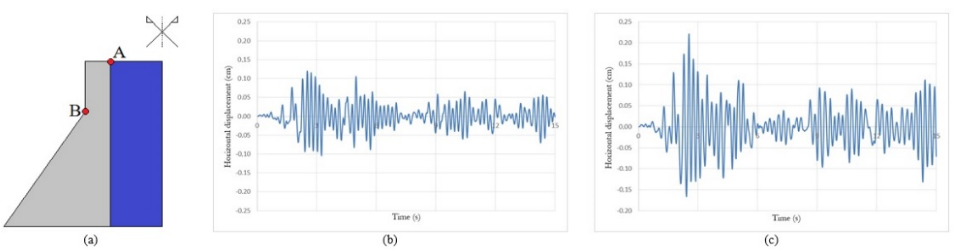

Figure 9. (a) Chosen points for the responses in the seismic analysis of the lock. Horizontal displacement in time (in cm) of the upper extreme point of the wall on the inner side of the left gravity structure of the lock (Point A) for the (b) uncoupled structural and (c) coupled fluid-structure cases.

Figure 10 show the envelopes of the maximum vertical tension and compression normal stresses in the lock structures, respectively, for the uncoupled (a) and coupled (b) case. The diameters of the circles represent the magnitude of the stresses, the values of which can be taken from the color palettes (vertical scale) also shown in the figures.

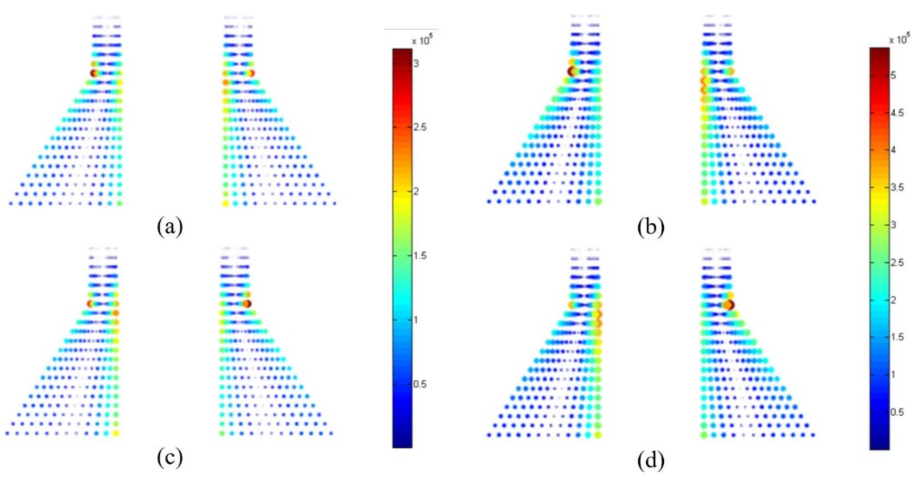

Figure 10. The envelope of the maximum normal vertical stresses of tension (a, b) and compression (c, d) in the gravity structures of the lock for the (a, c) uncoupled structural and (b, d) coupled fluid-structure cases, in Pa.

The maximum horizontal displacement at the upper point of the wet side of the lock gravity structure (Point A) in Figure 9 was 0.12 cm for the uncoupled structural case and 0.22 cm for the fluid-structure coupled case, as shown Figure 9b. For the design of gravity structures, the magnitude of normal stresses in the vertical direction will be critical. The maximum value of the normal tension and compression stresses was 311.65 kPa for the uncoupled structural case and 537.13 kPa for the coupled fluid-structure case, both in the transition between the vertical and inclined outer (dry) sides of the concrete structures of the lock (Point B) of Figure 9a, as shown in Figure 10. The fluid-structure coupled case presents a displacement, 84.8%, and stresses, 72.4% greater than the uncoupled structural case. Thus, a significant increase in responses to the earthquake is observed when the lock water chamber is completely full; this aspect emphasizes the importance of taking into account the coupled calculation when assessing the dynamic response in this type of problems.

The apparently significant tension stress values for a concrete mass structure need to be relativized due to some aspects, such as load combination and the weighting coefficients to consider, the probabilistic nature of earthquakes, among others. In any case, the values obtained in this preliminary analysis would be lower than the design values used in such a structure.

5 CONCLUSIONS AND PERSPECTIVES

This work aimed to perform a preliminary dynamic analysis (uncoupled and coupled) of a lock subjected to a reference earthquake (El Centro earthquake). The studies were done with numerical modeling by FEM from results obtained via ANSYS. In addition, the use of a simplified model (2D double-piston-cavity) was proposed, which presents an analytical solution to address the coupled lock-tank problem in its first natural frequencies.

In numerical analysis, the verification and validation of a problem could be described as processes that develop and present evidence of the accuracy of the results. To measure accuracy, accurate reference values are required. Within this perspective, it should be noted that a critical element in certain cases is the precision estimate when no reference value is available. Thus, simplified models, such as the one presented here, contribute significantly so that the designer has, in principle, the magnitude of the expected results. It has been shown that the 2D double-piston-cavity model consistently targets the results of the lock-tank system. To do this, the height of the pistons must be set equal to the height of the lock; the thickness of the pistons has to be determined in order to maintain the same cross-section area of the gravity structures and the width of the cavity equal to the width of the chamber. Of course, the physical properties of the materials must be the same as those of the lock. The determination of the spring stiffness was obtained by applying a horizontal unit load to the free end of a free-clamped beam, obtaining the value of its horizontal displacement at that point by using the analytical Timoshenko beam displacement formula, thereby calculating the stiffness of the relevant structure.

As for the forced excitation, the response results presented were: the variations in the time of the horizontal displacement at the point of the crest of the structure where the maximum horizontal displacement occurred and the maximum stress envelopes, due to the earthquake, for the cases of the empty completely and completely full of water. It was concluded that the maximum horizontal displacement occurs at the upper point of the wet side of the lock structure, and for the coupled fluid-structure case the displacement was 83.3% greater than for the uncoupled structural case. For the design of gravity structures, the magnitude of normal stresses in the vertical direction will be critical. The maximum value of normal tension and compression stresses occurs at the transition between the vertical and inclined walls of the outer (dry) side of the concrete structures of the lock (known as the bottleneck), and for the coupled fluid-structure case, the stress was 72.4% higher than for the uncoupled structural case. The results highlight the importance of considering fluid-structure coupling when analyzing this type of problems.

The relevance of this study lies in the fact that it highlights the influence of aspects of fluid-structure interactions on the dynamic behavior of a gravity structure system - water chamber subjected to an earthquake, thus contributing to an expansion of the limited bibliography available on this theme, and for the development of an appropriate methodology that provides technical support for the practice of safe and reliable projects.

Continuing this work, it is still possible to consider the interaction across the 3 media: structures (lock)-tank-foundation, considering in the modeling of the latter its inertia, damping and flexibility, an aspect that leads to a complex problem of wave propagation in unlimited domains, with reflection in the boundary.

As for the analytical solutions for the relevant case and which translate the additional masses associated with the modes of deformation of the dominant modes of the structure, these can be obtained through solving the wave equation with flexible boundaries that represent the appropriate modes of the vibrant structure in a fluid medium.

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